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REGULAR PAPER

# Estimation of longitudinal aerodynamic parameters using recurrent neural network

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## Abstract
The aerodynamic modelling is one of the challenging tasks that is generally established using the results of the computational fluid dynamic software and wind tunnel analysis performed either on the scaled model or the prototype. In order to improve the confidence of the estimates, the conventional parameter estimation methods such as equation error method (EEM) and output error method (OEM) are more often applied to extract the aircraft's stability and control derivatives from its respective flight test data. The quality of the estimates gets influenced due to the presence of the measurement and process noises in the flight test data. With the advancement in the machine learning algorithms, the data driven methods have got more attention in the modelling of a system based on the input-output measurements and also, in the identification of the system/model parameters. The research article investigates the longitudinal stability and control derivatives of the aerodynamic models by using an integrated optimisation algorithm based on a recurrent neural network. The flight test data of Hansa-3 and HFB 320 aircraft were used as case studies to see the efficacy of the parameter estimation algorithm and further, the confidence of the estimates were demonstrated in terms of the standard deviations. Finally, the simulated variables obtained using the estimates demonstrate a qualitative estimation in the presence of the noise.

### Nomenclature

| | |
|---|---|
| $a_x,\ a_z$ | linear accelerations along x and z axis |
| $C_D,\ C_L,\ C_m$ | coefficients of drag, lift and pitching moment |
| $C_{D_0},\ C_{L_0}, C_{m_0}$ | coefficients of drag, lift and pitching moment at zero angle-of-attack |
| $C_{D_X},\ C_{L_X}, C_{m_X}$ | coefficients of drag, lift, and pitching moment w.r.t. X motion or control variable |
| $C_X,\ C_z$ | body force coefficients along x and z axis |
| $n_x,\ n_y$ | number of input, and output variables |
| $n_h, n_\Theta$ | number of hidden layer neurons; number of unknown parameters |
| $N$ | number of the data samples |
| $R$ | measurement noise covariance matrix |
| $T$ | engine thrust |
| $V,\ V_0$ | true velocity of aircraft, nominal velocity of aircraft |
| $\bar{V},\ \bar{W},\ \bar{H}$ | input weight matrix, output weight matrix, hidden layer output vector |
| $\bar{x}_{max},\ \bar{x}_{\min}$ | max. and min. value of specified range for normalisation of data samples |
| $Y, Z$ | predicted output vector, measured output vector |
| $\alpha,\ \theta,\ q$ | angle-of-attack, pitch angle, pitch rate |
| $\Theta$ | unknown parameter vector |
| $\hat{\Theta}$ | estimated parameter vector |
| $z^{-1}$ | a time delay in the data sample |
| $\bar{v}_{ij}$ | the connecting weight element between $i^{th}$ input variable and $j^{th}$ hidden layer neuron |

$\bar{w}_{jk}$ the connecting weight element between $j^{th}$ hidden layer neuron and $k^{th}$ output variable
$Z_{avg.}$ the mean of the measured output data samples, $Z$

## 1.0 Introduction

The performance evaluation of an aircraft is an essential step for its airworthiness and certification. This evaluation is usually performed from the flight-testing of the aircraft in order to acquire the behavioural knowledge based on the external aerodynamic forces and moments generated on the aircraft with respect to the deflections in the control surfaces as well as the change in the throttle settings [1–3]. Therefore, the determination of the aerodynamic model becomes a very much essential step for the operation of aircraft in the autopilot mode or providing trainings to the pilots using the flight simulator or designing of a flight controller [4], etc. Generally, the flight data collected from the predetermined flight manoeuvers under investigation are analysed to obtain higher confidence level in the estimates of the stability and control derivatives using the least-square and maximum likelihood estimators [5, 6]. The state-space formulations of the linear and non-linear aerodynamic models are more often employed for low and high angle-of-attack regimes [7, 8] and the variants of these estimators have been applied in order to obtain the better estimates in the presence of the process and measurement noises [9]. Due to the online requirement of the estimation in the reconfigurable flight control law design, the recursive parameter estimation methods were also applied in the estimation of the derivatives [10–12]. The accuracy of these estimators is limited either by a-prior information of these derivatives or the noise content in the flight test data.

Neural-networks have proven as a useful modelling tool in the applications of flight control law design, flight simulators for training of the pilots, etc. The experimental data of the aerodynamic forces and moments obtained from wind tunnel testing of a scaled model were modeled with the corresponding motion variables using neural network and support vector machines in order to represent a nonlinear aerodynamic model [13–15]. Estimation techniques based on the trained neural models of the aerodynamic coefficients of forces and moments were investigated in the extraction of the stability and control derivatives [16–18]. Raol et al. [19] represented the recurrent neural network (RNN) as a state space model in order to retain the physical insight for investigation of the aerodynamic derivatives. Numerical methods-based approaches were also investigated in the estimation of the aerodynamic derivatives from the trained feed forward neural network (FFNN) [20, 21]. An integrated optimisation method with FFNN and its variants were also applied to extract the derivatives from the real flight data in the sequential steps of modelling and estimation [22, 23]. The accuracy of the estimates obtained using the conventional FFNN depends primarily on the generalisation capability of the network which may be affected due to improper selection of the network parameters, the training procedures, convergence criteria, etc. This may further lead to a large number of iterations and thereby, more computational efforts are required. In a few cases the training may become more tedious and cumbersome task due to the inappropriate numbers of the layers and their nodes. Huang et al. [24, 25] introduced a training procedure for a single hidden layer neural network, which has ability to generate a generalised FFNN at a lower computational cost. Such neural networks have also been applied in the investigation of the aerodynamic parameters from the real flight data [26–28]. The variant of FFNN, namely recurrent neural network, has also been used to solve the real-world problems of forecasting in order to achieve better neural models at a lower computational effort [29, 30].

The current research paper investigates the longitudinal stability and control derivatives using a hybrid optimisation method incorporating a dynamic neural model and non-linear least-square technique. The dynamic neural model is developed using RNN for longitudinal aerodynamic parameter estimation only whose performance has been evaluated using the mean-square-error (MSE), and the determination coefficient ($R^2$). The aerodynamic parameters are further, optimised using a nonlinear least-square method namely Gauss-Newton from the real flight data of Hansa-3, and HFB 320 aircraft consisting of the moderate amount of noise. To validate the estimates, the conventional methods

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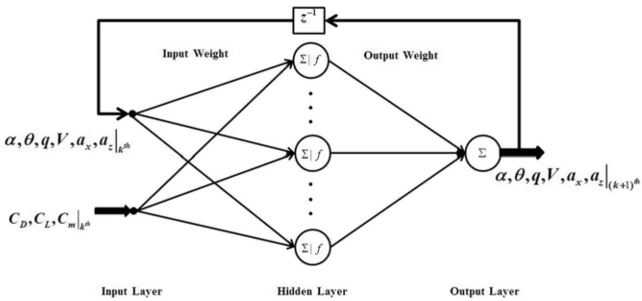


***Figure 1.*** *A recurrent neural model for longitudinal motion.*

have been applied to estimate the parameters from the same flight data. The research article has been organised as follows: the RNN-based optimisation method is demonstrated in the section 2.0 and the results obtained from the analysis of the case studies have been discussed in the section 3.0. Remarkable observations of the neural modelling and parameter estimation are highlighted in the conclusion section.

## 2.0 Recurrent neural network based parameter optimization

The longitudinal motion of aircraft is primarily governed by lift and drag forces, pitching moment about the centre of gravity, engine thrust, and weight, which can also be represented by the motion variables such as angle-of-attack, pitch angle, pitch rate, true velocity, linear accelerations, etc., the control surface deflection of elevator, and the engine throttle settings [1–3]. Such nonlinear functional relationship of the external forces and moment can be established with respect to the corresponding motion and control variables using a time series dynamic neural model [26–28]. In the present section, the parameter estimation procedure is described mathematically through two steps. The first step deals with the development of a dynamic neural model, and the second step deals with the optimisation procedure incorporating RNN.

### *2.1 System dynamic modelling using recurrent neural network*

The present subsection demonstrates about the recurrent neural network for estimation of the longitudinal parameters only. A time series dynamic neural model with one interval of time delay in the respective input and output variables can be represented by the following expression:

$$Y(k+1) = f(X(k), Y(k)) \tag{1}$$

Where X, and Y are the input and output variables of dimensions $X \in R^{n_x}$, and $Y \in R^{n_y}$, respectively. The recurrent neural network consists of a structure of a conventional Jordan network as shown in Fig. 1.

The coefficients of drag, lift and pitching moment of $k^{th}$ instant are considered as causing variables which can create an effect on the $(k+1)^{th}$ instant aircraft's motion variables such as angle-of-attack ($\alpha$), pitch angle ($\theta$), pitch rate ($q$), true velocity ($V$), linear accelerations ($a_x$, $a_z$). The input variables for the training of the network can be derived from the following relations:

$$\begin{aligned} C_D &= -C_X \cos(\alpha) - C_Z \sin(\alpha) \\ C_L &= \quad C_X \sin(\alpha) - C_Z \cos(\alpha) \\ C_m &= \quad [I_y\dot{q} - Z_{ENCG}T]/(\bar{q}S\bar{c}) \end{aligned} \tag{2}$$

Where, the body force coefficients ($C_X$, $C_z$) are computed from their respective linear accelerations ($a_x$, $a_z$).

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The normalisation of the input and output variables is essential for better nonlinear mapping of the network which is generally performed within a predetermined range ($\bar{x}_{max}$, $\bar{x}_{\min}$) [31]. Here, the number of hidden layer neurons is selected as of dimension $H \in R^{n_h}$. The input weight and biases of the hidden layer neurons as a matrix $\bar{V}$ of dimension $((n_x + n_y + 1) \times n_h)$ can be selected randomly for the given input and output variables of dimensions $X \in R^{n_x}$, and $Y \in R^{n_y}$, respectively. Therefore, the output of the hidden layer neuron can be expressed as follows:

$$\bar{h}_j = g\left(\sum_{i=1}^{n_x} \bar{x}_i \bar{v}_{ij} + \sum_{i=n_x+1}^{n_y} \bar{y}_i \bar{v}_{ij} + \bar{v}_{(n_x+n_y+1)j}\right) \tag{3}$$

Where $j = 1, 2, 3, \ldots, n_h$, g() represents the log-sigmoid activation function of the hidden layer neuron. The first two terms of the function depict the processing of the data through the respective connecting weight element ($\bar{v}_{ij}$) from the input and feedback variables. And the last term $\bar{v}_{(n_x+n_y+1)j}$ depicts the bias of the j$^{th}$ hidden layer neuron.

The weight matrix $\bar{W}$ consisting of the weight elements between the hidden layer neurons and the output nodes can be assumed to obtain the output as follows:

$$\bar{y}_k = \sum_{j=1}^{n_h} \bar{h}_j \bar{w}_{jk} \tag{4}$$

Where $k = 1, 2, 3, \ldots, n_y$

The above Equations (3) and (4) depict a forward processing of the data through the recurrent neural network with a single hidden layer. For a given size of the neural network, the computational cost is completely dependent on the method of the training algorithm. Back-propagation and global optimisation-based methods have been employed in the training of the neural networks which may have led to the over-fitting as well as consumed more time. Huang et al. [24, 25] introduced a training procedure for a single hidden layer in order to overcome such issues of the conventional neural network. This leads to the objective of minimising the residual error as follows:

$$\min_{\bar{W}} || \bar{H}\bar{W} - \bar{Z} || \tag{5}$$

The output weight can be estimated with an assumption of using the minimum norm of the output weight of all possible linear solutions as follows:

$$\hat{\bar{W}} = \bar{H}^{\dagger} \bar{Z} \tag{6}$$

Where $\bar{H}^{\dagger}$, Moore-Penrose generalised inverse matrix is given by $\bar{H}^{\dagger} = \left(\bar{H}^T \bar{H}\right)^{-1} \bar{H}^T$.

For appropriate ranges of the input weights and biases, the norm of the output weight is found to be low, which is also pointed out by Bartlett [32] that the norm of the weights must be low for better generalisation of the network rather than having lower value of its residual error. Further, two statistical parameters namely mean-square-error (MSE), and determination coefficient ($R^2$) have been applied to show the efficacy of the network's capability in the prediction of the input samples as follows [33]:

$$MSE = \frac{1}{n_y N} \sum_{i=1}^{N} (Y_i - Z_i)^2 \tag{7}$$

$$R^2 = \frac{1}{n_y} \left[ 1 - \frac{\sum_{i=1}^{N} (Z_i - Y_i)^2}{\sum_{i=1}^{N} (Z_i - Z_{avg.})^2} \right] \tag{8}$$

### *2.2 Aerodynamic parameter estimation*

In order to optimise the longitudinal aerodynamic parameters ($\Theta$), the first step is to predict the motion variables of (k+1)$^{th}$ instant by using the trained recurrent neural network function expressed

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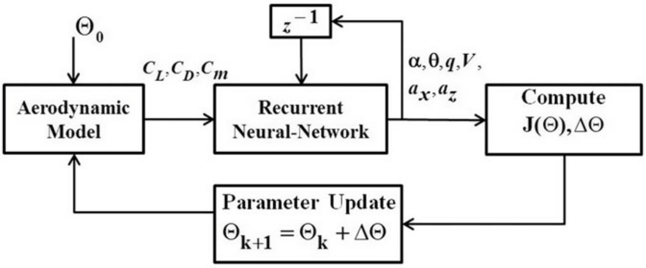


***Figure 2.*** *Block diagram of RNN-based optimisation method.*

as follows:

$$Y(k+1) = f(X(k), Y(k), \bar{V}, \bar{W}, \Theta) \tag{9}$$

The second step employs a cost function, $J$ to be defined based on maximum likelihood function with an assumption that the measurement error covariance matrix ($R$) is known a prior of the optimisation. Hence, the cost function $J(\Theta)$ can be expressed as follows [1, 27, 28]:

$$J(\Theta) = \frac{1}{2} n_y N + \frac{N}{2} \ln \{\det (R)\} + \frac{n_y N}{2} \ln (2\pi) \tag{10}$$

Where, the measurement error covariance matrix is given by $R = \frac{1}{N} \sum_{i=1}^{N} E_i E_i^T$ and the residual error is given by

$$E_i = [Z(i) - Y(i)].$$

The constant terms of the cost function can be ignored without affecting its minimum value and further, it can be expressed as follows:

$$J(\Theta) = \det(R) \tag{11}$$

The cost function has to be minimised by using a nonlinear least-square method namely Gauss-Newton for the unknown parameters ($\Theta$). The change in the parameter $\Delta\Theta$ can be expressed as follows:

$$\Delta\Theta = -\left[\left(\frac{\partial^2 J}{\partial \Theta^2}\right)_k\right]^{-1} \left(\frac{\partial J}{\partial \Theta}\right)_k \tag{12}$$

Where the first derivative of the cost function is called as the gradient vector and is expressed as follows:

$$\left(\frac{\partial J}{\partial \Theta}\right)_k = -\sum_{i=1}^{N} \left[\frac{\partial Y(i)}{\partial \Theta}\right]^T R^{-1} E_i \tag{13}$$

Where as the second derivative of the cost function is called as information or Hessian matrix and is expressed as follows:

$$\left(\frac{\partial^2 J}{\partial \Theta^2}\right)_k = \sum_{i=1}^{N} \left[\frac{\partial Y(i)}{\partial \Theta}\right]^T R^{-1} \left[\frac{\partial Y(i)}{\partial \Theta}\right] \tag{14}$$

The response gradient matrix $(\partial Y(i)/\partial \Theta)_{ij}$ is computed using the forward difference approximation as follows:

$$\left(\frac{\partial Y\,(i)}{\partial \Theta}\right)_{ij} \approx \frac{f\left(X\,(k)\,, Y\,(k)\,, \bar{V}, \bar{W}, \left(\Theta + \delta\Theta_j\right)\right) - f\left(X\,(k)\,, Y\,(k)\,, \bar{V}, \bar{W}, \Theta\right)}{\delta\Theta_j} \tag{15}$$

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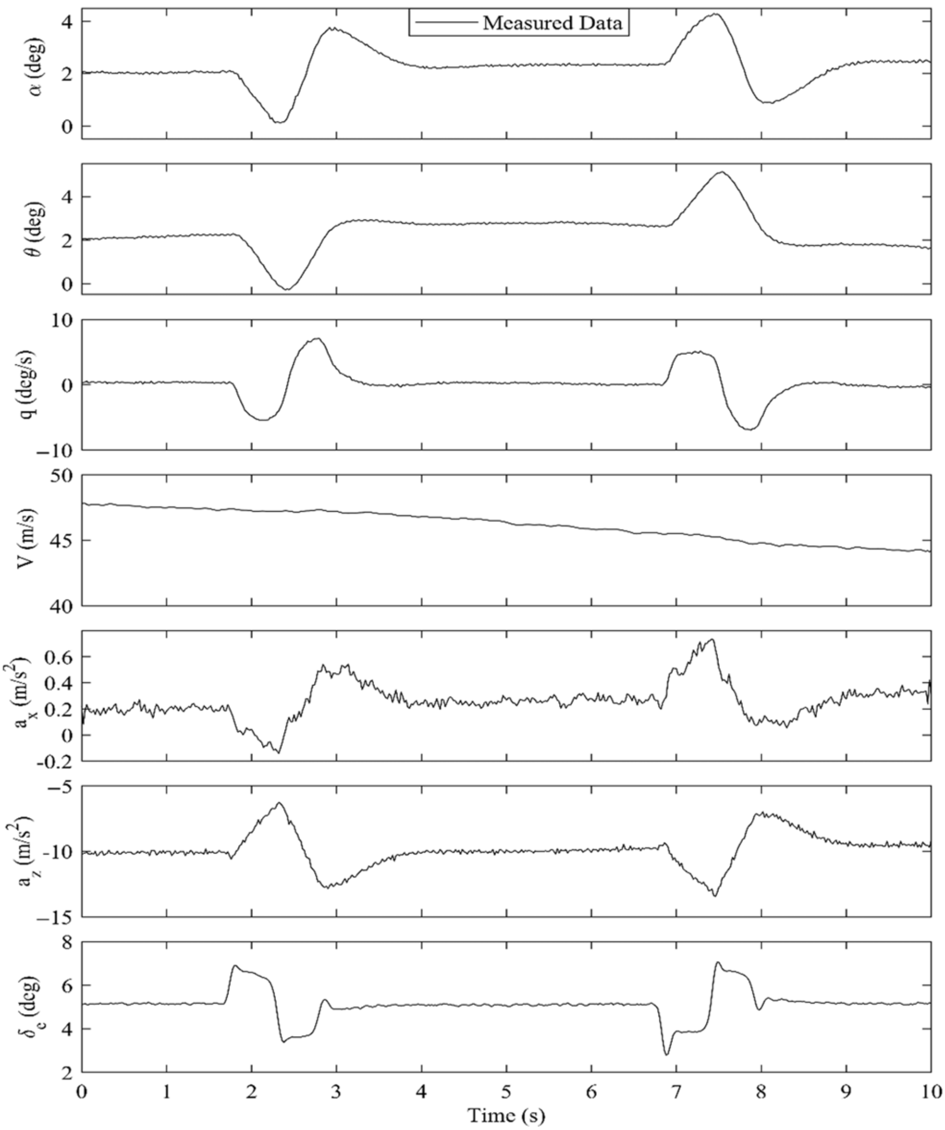


***Figure 3.*** *Longitudinal flight data of Hansa-3 aircraft: RFD01.*

Where, $i = 1, 2, 3, \ldots, N; j = 1, 2, 3, \ldots, n_\Theta$; and $\delta\Theta_j$ depict a small perturbation in the j[th] parameter of $\Theta$.

The next iteration parameter $\Theta_{k+1}$ is updated by the expression as follows:

$$\Theta_{k+1} = \Theta_k + \Delta\Theta \tag{16}$$

A detailed RNN-based optimisation method is represented in the block diagram as shown in Fig. 2. The optimisation method is initiated by analytical computation of the aerodynamic coefficients using the initial aerodynamic parameters ($\Theta_0$) and the longitudinal motion and control variables ($\alpha, q, \delta_e$). The new generated input samples are propagated through the trained RNN to obtain the corresponding observations. And further, the cost function and the change in the parameter vector are computed using Equations (11), and (12), respectively. The convergence of the optimisation method, which is set based

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*Table 1. Longitudinal derivatives of Hansa-3 aircraft*

| $\hat{\Theta}$ | Ref. Value [35] | EEM | MLE | RNN |
|---|---|---|---|---|
| $C_{D_0}$ | 0.035 | 0.0351 (0.0009) | 0.0356 (0.0009) | 0.0371 (0.0007) |
| $C_{D_\alpha}$ | 0.086 | 0.2259 (0.0077) | 0.2218 (0.0079) | 0.2076 (0.0072) |
| $C_{D_{\delta e}}$ | 0.026 | 0.1589 (0.0088) | 0.1554 (0.0090) | 0.1446 (0.0080) |
| $C_{L_0}$ | 0.354 | 0.3336 (0.0045) | 0.3292 (0.0046) | 0.3363 (0.0045) |
| $C_{L_\alpha}$ | 4.978 | 4.7764 (0.0447) | 4.7784 (0.0458) | 4.7300 (0.0452) |
| $C_{L_q}$ | – | 19.6299 (1.3529) | 19.6338 (1.3803) | 19.4638 (1.3116) |
| $C_{L_{\delta e}}$ | 0.265 | 0.4260 (0.0545) | 0.4837 (0.0561) | 0.4165 (0.0545) |
| $C_{m_0}$ | 0.052 | 0.1014 (0.0028) | 0.1034 (0.0032) | 0.1082 (0.0035) |
| $C_{m_\alpha}$ | −0.496 | −0.4285 (0.0279) | −0.3867 (0.0321) | −0.3509 (0.0391) |
| $C_{m_q}$ | – | −5.6274 (0.8435) | −7.1363 (0.9847) | −9.0479 (1.0510) |
| $C_{m_{\delta e}}$ | −1.008 | −0.8705 (0.0340) | −0.9093 (0.0396) | −0.9797 (0.0435) |

*Note:* Values in parenthesis denote the standard deviations.

on the relative change in the consecutive values of the cost function, is checked in each of the iterations of the algorithm. The parameters and the corresponding aerodynamic coefficients are updated until the convergence of the algorithm is met. Finally, the confidence of the optimised parameters is computed in terms of the standard deviations as follows: $\sigma_{\Theta_j} = \sqrt{p_{jj}}$. Where $j = 1, 2, 3, \ldots, n_\Theta$;$p_{jj}$ depicts the diagonal elements of the estimation error covariance matrix, P, which is given approximately as the inverse of the Hessian matrix as follows:

$$P \approx \left\{ \sum_{i=1}^{N} \left[ \frac{\partial Y(i)}{\partial \Theta} \right]^T \left[ R^{-1} \right] \left[ \frac{\partial Y(i)}{\partial \Theta} \right] \right\}^{-1} \quad (17)$$

## 3.0 Results and discussion

The section demonstrates about the capability of the recurrent neural network in the estimation of the longitudinal aerodynamic parameters from the real flight data of Hansa-3 and HFB 320 aircraft.

### *3.1 Estimation of longitudinal derivatives of Hansa-3 aircraft*

The real flight data (RFD) of Hansa-3 aircraft was considered as the first case study whose short period dynamics mode was excited by two consecutive doublet commands with almost same magnitude but, with the opposite elevator deflections. The corresponding flight data was gathered at an altitude of 2000 m and at a trim speed of 46.8 m/s in an almost calm weather as shown in Fig. 3. The quality

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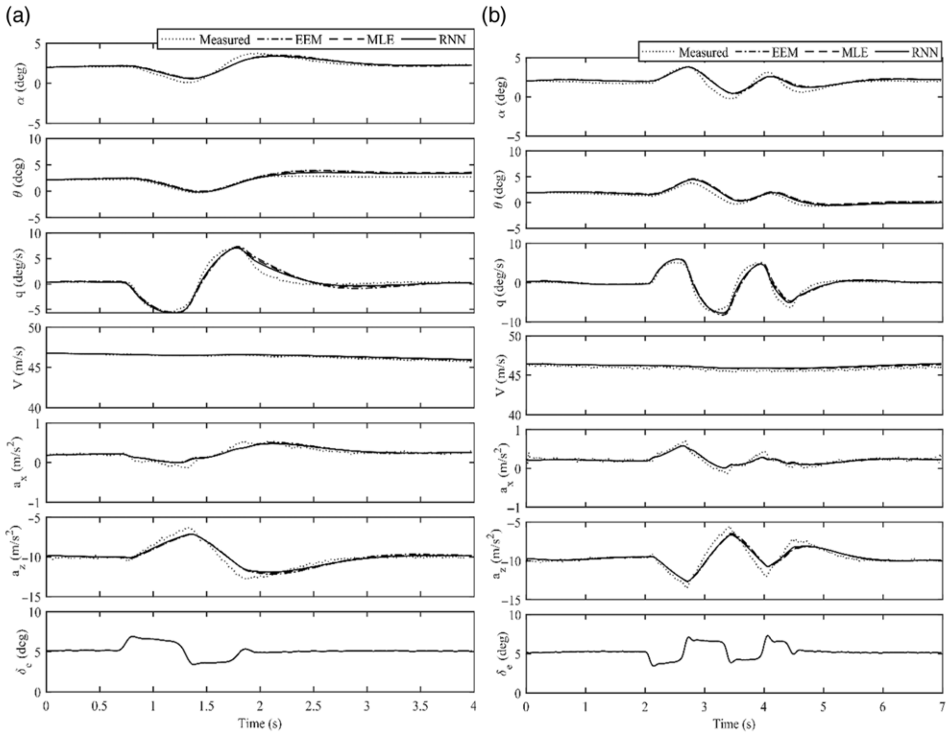


***Figure 4.*** *Validation of the estimated parameters using flight simulation.*

improved flight data was considered for investigation of the stability and control derivatives of a linear aerodynamic model with the consideration of the effective motion and control variables such as $\alpha,\ q\bar{c}/2V,\ \ \delta_e$ which is expressed as follows [34]:

$$\begin{aligned} C_D &= C_{D_0} + C_{D_\alpha}\alpha + C_{D_{\delta_e}}\delta_e \\ C_L &= C_{L_0} + C_{L_\alpha}\alpha + C_{L_q}(q\,\bar{c}/2V) + C_{L_{\delta_e}}\delta_e \\ C_m &= C_{m_0} + C_{m_\alpha}\alpha + C_{m_q}(q\,\bar{c}/2V) + C_{m_{\delta_e}}\delta_e \end{aligned} \tag{18}$$

In order to estimate the longitudinal parameters, a recurrent neural network is generated using the procedure followed in section 2.0. The values of MSE and $R^2$ are obtained as 2.02E-02, and 0.9896, respectively, which demonstrate a qualitative generalisation of the network. Further, the integrated optimisation method as discussed in section 2.0 is applied to compute the longitudinal parameters of the linear aerodynamic model from RFD01. The method is initiated using the guess values of the derivatives, which are chosen closer to the estimates of EEM. It is found that the optimisation algorithm converges in a few iterations and the optimal results are shown with their standard deviations in Table 1. To validate the estimates of RNN method, the conventional estimation methods based on least-square and maximum likelihood estimation have been applied to extract the parameters from the same flight data and also, the results are compared to the values of the parameters considered as given in the reference value [35]. EEM is the most commonly applied parameter estimation method in the investigation of the linear aerodynamic model parameters [6] and the estimates of EEM are presented with their corresponding standard deviations in Table 1. Further, MLE method has been applied in the optimisation of the parameters from their initial guess values and the optimal results are presented in Table 1. It is observed that the MLE method is found to be sensitive to the integration method, initial state conditions, initial parameter values, amount of noise percentage in the data, etc. In Table 1, it is observed that the estimates of

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***Table 2.*** *Longitudinal derivatives of HFB 320 aircraft*

| $\hat{\Theta}$ | Ref. Value [1] | EEM | MLE | RNN |
|---|---|---|---|---|
| $C_{D_o}$ | 0.1227 | 0.1225 | 0.1178 | 0.1200 |
| | | (0.0016) | (0.0032) | (0.0033) |
| $C_{D_\alpha}$ | 0.3200 | 0.3249 | 0.2348 | 0.3305 |
| | | (0.0038) | (0.0064) | (0.0080) |
| $C_{D_V}$ | −0.0645 | −0.0648 | −0.0517 | −0.0629 |
| | | (0.0014) | (0.0028) | (0.0028) |
| $C_{L_o}$ | −0.0929 | −0.0685 | −0.2047 | −0.0796 |
| | | (0.0203) | (0.0196) | (0.0162) |
| $C_{L_\alpha}$ | 4.3278 | 4.2573 | 3.2123 | 4.2875 |
| | | (0.0483) | (0.0394) | (0.0388) |
| $C_{L_V}$ | 0.1487 | 0.1324 | 0.3635 | 0.1397 |
| | | (0.0172) | (0.0173) | (0.0137) |
| $C_{m_o}$ | 0.1119 | 0.0919 | 0.1344 | 0.1073 |
| | | (0.0043) | (0.0051) | (0.0060) |
| $C_{m_\alpha}$ | −0.9678 | −0.9565 | −1.0024 | −0.9743 |
| | | (0.0129) | (0.0087) | (0.0179) |
| $C_{m_V}$ | 0.0039 | 0.0218 | −0.0139 | 0.0089 |
| | | (0.0039) | (0.0047) | (0.0054) |
| $C_{m_q}$ | −34.7098 | −42.0239 | −27.0570 | −37.3099 |
| | | (1.0164) | (0.6813) | (1.3358) |
| $C_{m_{\delta e}}$ | −1.5291 | −1.6032 | −1.4371 | −1.5749 |
| | | (0.0250) | (0.0174) | (0.0329) |
| L1(%) | – | 17.816 | 22.087 | 6.461 |
| L2(%) | – | 20.883 | 22.093 | 7.425 |

*Note:* Values in parenthesis denote the standard deviations.

RNN are in a close agreement to those of obtained from EEM, MLE, and reference value. It is seen that the derivatives $C_{D_0}$, $C_{L_0}$, $C_{m_0}$ are found to be approximately 0.03, 0.33, and 0.10, respectively, which demonstrate the consistency in the estimation as these are the base derivatives in the modelling of the drag, lift and pitching moment coefficients. The lift slope parameter $C_{L_\alpha}$ and the lift damping parameter $C_{L_q}$ have also shown a consistency in the estimation nearly to the values of 4.7 and 19, respectively, from EEM, MLE and RNN whereas the values of the other derivatives $C_{D_\alpha}$, $C_{D_{\delta e}}$, $C_{L_{\delta e}}$, $C_{m_\alpha}$, $C_{m_{\delta e}}$ are found within a closer bound. The value of the pitching moment damping derivative $C_{m_q}$ is found to be higher in contrast to the values of the other methods. Finally, the estimates using the aforementioned methods have been validated by a comparison of the measured variables with the simulated variables. These simulated variables are generated using the integration of the aircraft's dynamic equations with the same initial state conditions and the first doublet elevator control command as shown in Fig. 4 (a). A similar validation has been carried out by generating the simulated variables with the estimated parameters and a multistep elevator control command of another flight data [27]. It is observed that the simulated quantities are in a good agreement and follow the nature satisfactorily with their respective measured ones. However, small deviations can be observed at the intermediate instants of the manoeuver which might be due to the presence of noise in the measured variables.

### *3.2 Estimation of longitudinal derivatives of HFB-320 aircraft*

The current sub-section demonstrates the determination of the derivatives associated with the aerodynamic forces and moment coefficients ($C_L$, $C_D$, $C_m$) of the HFB-320 aircraft. The real flight data was gathered by sequential excitation of the short period and phugoid modes using a multi-step and pulse

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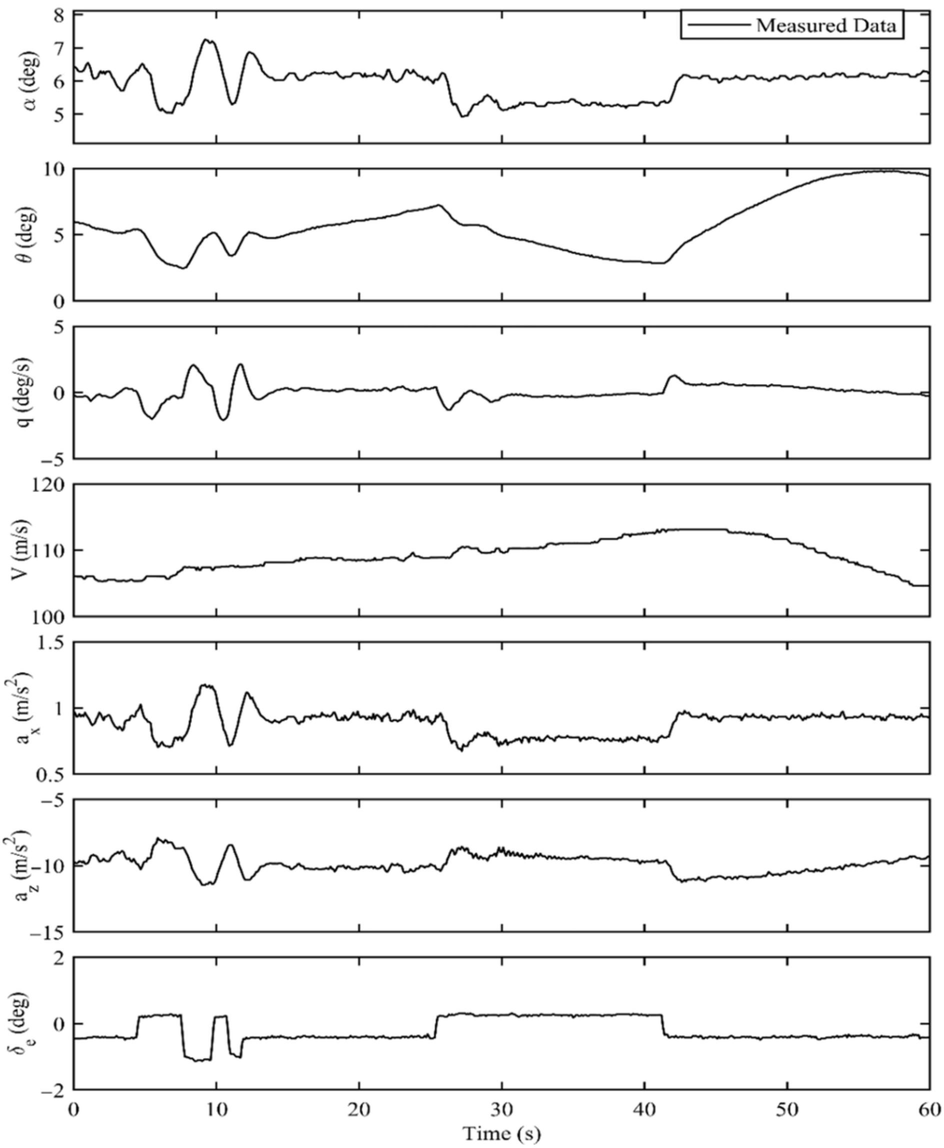


***Figure 5.*** *Longitudinal flight data of HFB-320 Aircraft: RFD02.*

input elevator command, respectively, and the quality improved flight data was considered in the estimation of the stability and control derivatives [1, 11] as shown in Fig. 5. In the modelling of the aerodynamic drag and lift forces, the parameters of zero angle-of-attack, angle-of-attack and normalised velocity have been used whereas the additional parameters of normalised pitch rate and elevator deflection have been used for modelling of the pitching moment coefficient.

The complete aerodynamic model, whose parameters are to be estimated, is considered as follows from Ref. [1, 11]

$$\begin{aligned} C_D &= C_{D_0} + C_{D_\alpha}\alpha + C_{D_V}(V/V_0) \\ C_L &= C_{L_0} + C_{L_\alpha}\alpha + C_{L_V}(V/V_0) \\ C_m &= C_{m_0} + C_{m_\alpha}\alpha + C_{m_V}(V/V_0) + C_{m_q}(q\bar{c}/2V) + C_{m_{\delta e}}\delta_e \end{aligned} \tag{19}$$

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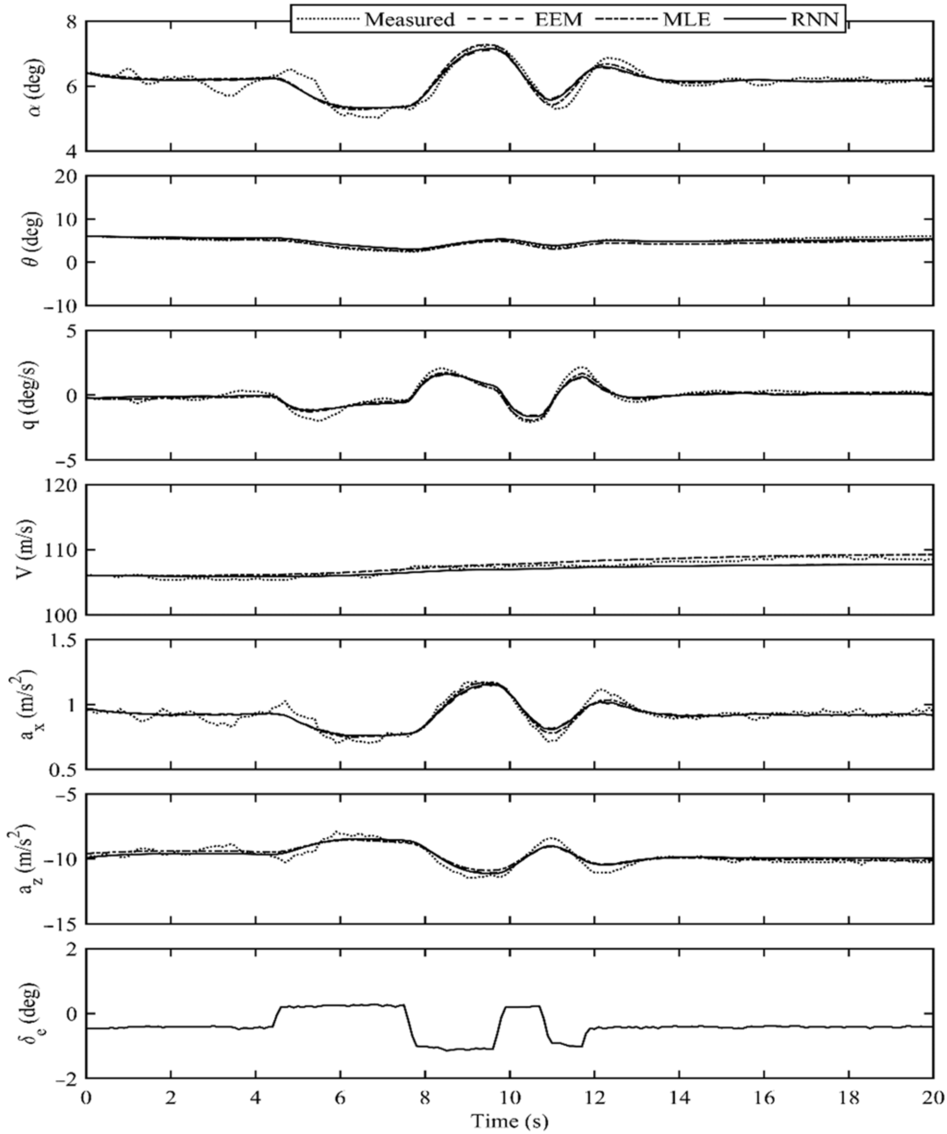


***Figure 6.*** *Validation of the estimated parameters using flight simulation.*

In order to estimate the aerodynamic parameters, a recurrent neural model is generated as described in section 2.0. The values of $R^2$ and MSE are obtained as 0.9908 and 2.65E-02, respectively from the dynamic neural models which demonstrate a qualitative generalisation of the network. Further, the optimisation method using the RNN model is applied to extract the aerodynamic parameters from the real flight data. It is found that the optimisation method gets converged in a low number of iterations and finally, the optimal parameters with their corresponding standard deviations are presented in Table 2. The validation of the obtained estimates has been carried out with a comparison of the reference value from Girish et al. [11] and the values of EEM and MLE. MLE-based estimates are obtained by applying a nonlinear least-square optimisation method based on the residual error minimisation principle with the help of integrating the dynamic equations of HFB 320 aircraft. As the real flight data carries moderate amount of noise, hence the reference values are considered as a result of filter error method applied to

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extract the same parameters from the real flight data. It is evident from the columns of the table that RNN method estimates the derivatives of lift, drag and pitching moment with lower standard deviations and the estimates are found to be in a good agreement with the estimates of the other methods. However, a small variation in the values of $C_{m_V}$ is observed due to being a weaker derivative. Further, the parameter estimation error norms (PEEN) in terms of L1 and L2 norms have been computed for the aerodynamic models consisting of the parameters obtained using EEM, MLE and RNN as per the formulations used in Ref. [36, 37]. From Table 2, it is seen that the values of L1 and L2 norms obtained for RNN are lower in contrast to the values of norms obtained from EEM and MLE.

To validate the estimates, the simulated variables are generated by integrating the aircraft's longitudinal equations of motion in wind axes form with the same initial conditions and are presented in Fig. 6. It is observed that the observation variables are in a good agreement with the measured motion variables of the flight data. However, small variations in true velocity of aircraft are observed due to the propagation of the residual error in the integration process.

## 4.0 Conclusion

In the present paper, a dynamic neural model based on RNN is studied in the estimation of the longitudinal parameters from the real flight dataset carrying short period and phugoid modes. The RNN-based neural model is well generated using appropriate selection of the network parameters such as initial weights, biases, number of the hidden layer neurons, activation function, time delay etc. that is well ensured in terms of MSE, and $R^2$ before to use in predictions of the input samples of the optimisation method. It is observed that a small and large number of hidden layer neurons may lead to either an under-fit or over-fit neural model, respectively. It is also observed that the time delay with more than one may also lead to the over-fitting of the recurrent neural model. The estimates are found to be within a good agreement in terms of the standard deviations with the estimates of the conventional methods. The final validation using the simulated variables also demonstrated the qualitative estimation for the present case studies. The advantage of such optimisation method is to estimate the derivatives at a lower computational effort due to the simple and faster training and also, the estimation does not need the dynamic equations of the aircraft, initial states, etc. a prior. Such dynamic neural model-based parameter estimation could be a better alternative where the physical phenomenon-based models are difficult to establish such as the model requirements at high angle-of-attack regime due to the unknown aerodynamic parameters.

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