

3. The integral  $\int e^{\lambda x} \sin(\alpha + \beta x) dx$  is deducible in the form  $e^{\lambda x} \sin(\alpha + \beta x - \theta) / \sqrt{\lambda^2 + \beta^2}$  by treating it as a limiting case of the summation cited in §2.  $\Delta$  is replaced by  $\delta$ ,  $t$  by  $x$ ,  $\gamma$  by  $e^\lambda$ , and the limits of  $\sqrt{R}/\delta x$  and  $\tan \theta$  are easily shown to be  $\sqrt{\lambda^2 + \beta^2}$  and  $\beta/\lambda$ .

The method of obtaining this integral in the above form by reversing the derivative process is given by Edwards (*Int. Calc.* p. 46).

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**To construct an ellipse whose focus F is given, which shall pass through three given points A, A', A'' (Halley's Problem).**

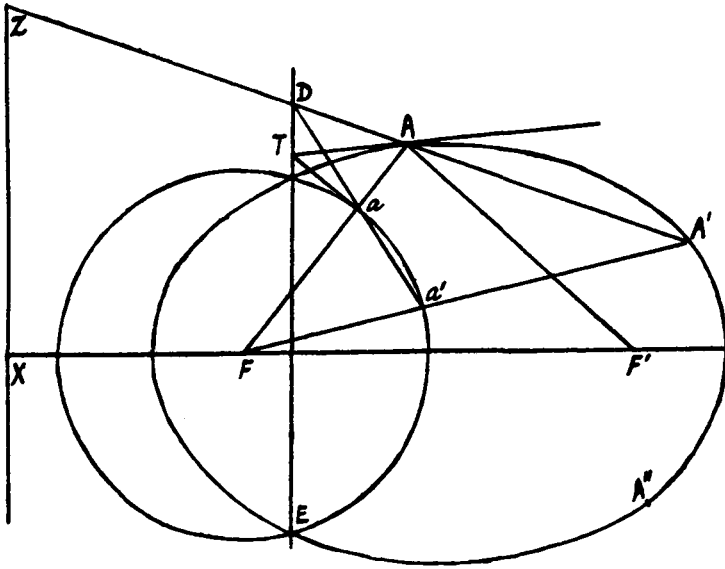
The following method is applicable to ellipses of small excentricity (such as the orbits of planets) and may be used as an alternative method to finding the directrix which in such cases is situated at a considerable distance.

With centre  $F$  describe a circle of any arbitrary and convenient radius  $r$ . If  $FA, FA', FA''$  intersect this circle in  $a, a', a''$ , then corresponding chords such as  $AA', aa'$  intersect on a fixed straight line  $DE$  perpendicular to the focal axis of the ellipse, which is in fact the common chord of the two curves and distant  $r/e$  from the  $S$ -directrix. For if  $AA'$  intersect the directrix in  $Z, SZ$  is the external bisector of the angle  $AF A'$  and is therefore parallel to  $aa'$ . Also  $Fa : FA = ZD : ZA =$  ratio of perpendiculars from  $D$  and  $A$  on directrix  $XZ$ . Hence the perpendicular from  $D$  on  $XZ = r/e$ .

Similarly by taking  $AA'', aa''$  together we get another point  $E$  on the common chord which is now completely determined. The line through  $F$  perpendicular to  $DE$  is the focal axis.

Now let  $x$  be any other point on the circle and let  $xa'$  meet  $DE$  in  $G$ ; then the intersection  $X$  of  $GA'$  and  $Fx$  is a point on the ellipse. It follows then that if the tangent at  $a'$  meets  $DE$  in  $T, TA'$  will be the tangent to the ellipse at  $A'$ . Hence the second

focus  $F'$  is got very easily since  $A'F$  and  $A'F'$  are equally inclined to  $A'T$ .



[See Housel's *Introduction à la Géométrie Supérieure*; Paris, Ganthiers-Villars, 1865, where a somewhat difficult proof depending on homology by de Jonquières is given.]

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### Common Logarithms calculated by simple multiplication.

The 4-place tables inform us that  $\log 3 = \cdot 4771$ . This means that  $3 = 10^{\cdot 4771}$  or  $3^{10000} = 10^{4771}$ ; in other words, that there are  $4771 + 1$  digits in  $3^{10000}$ . Hence to find  $\log 3$  we have merely to raise 3 to the 10000th power and count the digits in the result. This need not be so long a process as one might anticipate, if we use contracted multiplication, and arrange the work suitably. The figures given below form the actual calculation, which took