ON A RIEMANNIAN MANIFOLD M_{2n} WITH AN ALMOST TANGENT STRUCTURE

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Professor Eliopoulous studied almost tangent structures on manifolds M_{2n} in [1; 2]. An almost tangent structure F is a field of class C^{∞} of linear operations on M_{2n} such that at each point x in M_{2n}, F_x maps the complexified tangent space T_x^c into itself and that F_x is of rank n everywhere and satisfies that $F^2 = 0$. In this note, we consider a $(1, 1)$ tensor field $F_{i}^{\ j}$ on a Riemannian M_{2n} which satisfies $F_{i}^{\ j} F_{j}^{\ k} = 0$ everywhere and is such that the rank of F is n everywhere. Such $F_{i}^{\ j}$ gives an almost tangent structure F on M_{2n}. We are able to connect such a structure F to an almost product and almost complex structure on M_{2n} (Theorem 1). Furthermore, we study the integrability of F and obtain two results (Theorems 2 and 3). The basic materials used to prove the theorems are stated in §1 and §2, which were developed in Yano [5].

1. Let M_{2n} be a Riemannian manifold of class C^{∞}. We assume that a field of class C^{∞} of linear operations F_x is defined on M_{2n} such that at each point $x \in M_{2n}$, F_x maps the tangent space T_x of M_{2n} at x into itself; moreover, F_x is of rank n everywhere in M_{2n}, and it satisfies the relation

$$F_x^{\ 2} = 0$$

for every $x \in M_{2n}$. Let the image $F_x(T_x)$ be B_x; then B_x is a

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distribution of dimension n on M_{2n}. Let C_x be the orthogonal distribution to B_x with respect to the Riemann metric. Thus

$T_x = B_x \oplus C_x$. Since $F^2 = 0$, we have $F_x(B_x) = 0$ and $B_x = F_x(T_x) = F_x(B_x \oplus C_x) = F_x(C_x)$. In any local coordinate neighborhood at the point x, one can write the operator F_x and distributions B_x, C_x in the tensorial notations:

$$F_i^{\ j} \qquad (i, j = 1, 2, \ldots, 2n);$$

$$B_\alpha^{\ i} \qquad (i = 1, \ldots, 2n; \ \alpha = 1, 2, \ldots, n);$$

$$C_{\overline{\alpha}}^{\ i} \qquad (i = 1, \ldots, 2n; \ \overline{\alpha} = \alpha + n).$$

One can always arrange that

$$(0) \qquad F(C_{\overline{\alpha}}) = B_\alpha \quad \text{or} \quad F_j^{\ i} C_{\overline{\alpha}}^{\ j} = B_\alpha^{\ i},$$

where here and in the following Greek indices α, β, γ, ... run over the range 1, 2, ..., n; $\overline{\alpha} = \alpha + n$, and Latin indices i, j, k, ... run over the range 1, 2, ..., 2n.

The matrix $(B_\alpha^{\ i}, C_{\overline{\alpha}}^{\ i})$ has an inverse, say $\begin{pmatrix} B_i^{\ \alpha} \\ C_i^{\ \overline{\alpha}} \end{pmatrix}$ then

$$(1) \qquad B_\alpha^{\ j} B_j^{\ \beta} = \delta_\alpha^{\ \beta}, \quad C_{\overline{\alpha}}^{\ i} B_i^{\ \beta} = 0, \quad B_\beta^{\ i} C_i^{\ \overline{\alpha}} = 0, \quad C_{\overline{\alpha}}^{\ i} C_i^{\ \overline{\beta}} = \delta_\alpha^{\ \beta},$$

and

$$(2) \qquad B_\alpha^{\ j} B_i^{\ \alpha} + C_{\overline{\alpha}}^{\ j} C_i^{\ \overline{\alpha}} = \delta_i^{\ j}.$$

Here and in the following $B_\alpha^{\ j} B_i^{\ \alpha} = \sum_{\alpha=1}^{n} B_\alpha^{\ j} B_i^{\ \alpha}$, $C_{\overline{\alpha}}^{\ j} C_i^{\ \overline{\alpha}} = \sum_{\overline{\alpha}=n+1}^{2n} C_{\overline{\alpha}}^{\ j} C_i^{\ \overline{\alpha}}$.

Put

$$B_i^{\ j} = B_i^{\ \alpha} B_\alpha^{\ j}, \quad C_i^{\ j} = C_i^{\ \bar\alpha} C_{\bar\alpha}^{\ j},$$

then

$$(3) \qquad B_i^{\ j} + C_i^{\ j} = \delta_i^{\ j}, \quad B_i^{\ j} B_j^{\ h} = B_i^{\ h}, \quad C_i^{\ j} C_j^{\ h} = C_i^{\ h}.$$

The following equalities are easy to verify:

$$(4) \quad \begin{cases} B_i^{\ h} B_\alpha^{\ i} = B_\alpha^{\ h}, & B_i^{\ h} C_{\bar\alpha}^{\ i} = 0, \\ C_i^{\ h} B_\alpha^{\ i} = 0, & C_i^{\ h} C_{\bar\alpha}^{\ i} = C_{\bar\alpha}^{\ h}; \end{cases}$$

$$(5) \quad \begin{cases} B_i^{\ h} B_h^{\ \alpha} = B_i^{\ \alpha}, & B_i^{\ h} C_h^{\ \bar\alpha} = 0, \\ C_i^{\ h} B_h^{\ \alpha} = 0, & C_i^{\ h} C_h^{\ \bar\alpha} = C_i^{\ \bar\alpha}. \end{cases}$$

Let F, B, C be the operators on T_x by the tensors $F_i^{\ j}$, $B_i^{\ j}$, $C_i^{\ j}$; then it is easy to see that $FB = 0$, $BF = F$, $CF = 0$ and $FC = F$. That is:

$$(6) \quad \begin{cases} B_j^{\ i} F_h^{\ j} = F_h^{\ i}, & F_j^{\ i} B_h^{\ j} = 0, \\ C_j^{\ i} F_h^{\ j} = 0, & F_j^{\ i} C_h^{\ j} = F_h^{\ i}. \end{cases}$$

2. Let

$$(7) \qquad A_{ji} = B_j^{\ \alpha} B_i^{\ \alpha} + C_j^{\ \bar\alpha} C_i^{\ \bar\alpha} = \sum_{\alpha=1}^{n} B_j^{\ \alpha} B_i^{\ \alpha} + \sum_{\bar\alpha=n+1}^{2n} C_j^{\ \bar\alpha} C_i^{\ \bar\alpha}.$$

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then A_{ji} is symmetric and

$$(8)\quad \begin{cases} A_{ji} B_\alpha^{\ j} B_\beta^{\ i} = \delta_{\alpha\beta}, & A_{ji} C_{\overline{\alpha}}^{\ j} C_{\overline{\beta}}^{\ i} = \delta_{\overline{\alpha}\overline{\beta}}, \quad A_{ji} B_\alpha^{\ j} C_{\overline{\beta}}^{\ i} = 0; \\ A_{ji} B_\alpha^{\ i} = B_j^{\ \alpha}, & A_{ji} C_{\overline{\alpha}}^{\ i} = C_j^{\ \overline{\alpha}}. \end{cases}$$

These identities show that (A_{ji}) is non-singular and that $(B_\alpha^{\ i}, C_{\overline{\alpha}}^{\ i})$ form an orthonormal frame with respect to A_{ij}.

Let

$$(9)\qquad B_{ji} = B_j^{\ h} A_{hi}, \quad C_{ji} = C_j^{\ h} A_{hi}.$$

Then

$$(10)\qquad B_{ji} = B_j^{\ \alpha} B_i^{\ \alpha}, \quad C_{ji} = C_j^{\ \overline{\alpha}} C_i^{\ \overline{\alpha}}, \quad B_{ji} + C_{ji} = A_{ji},$$

B_{ji}, C_{ji} are both symmetric and

$$(11)\qquad B_j^{\ t} B_i^{\ s} A_{ts} = B_{ji}, \quad B_j^{\ t} C_i^{\ s} A_{ts} = 0, \quad C_j^{\ t} C_i^{\ s} A_{ts} = C_{ji}.$$

Consider now the tensor G_{ij} defined by:

$$G_{ji} = \tfrac{1}{2}(A_{ji} + F_j^{\ t} F_i^{\ s} A_{ts} + B_{ji}).$$

Noticing that $(F_j^{\ t} B_t^{\ \alpha} - C_j^{\ \overline{\alpha}}) B_\beta^{\ j} = 0$, $(F_j^{\ t} B_t^{\ \alpha} - C_j^{\ \overline{\alpha}}) C_{\overline{\beta}}^{\ j} = 0$ and $(B_\beta^{\ j}, C_{\overline{\beta}}^{\ j})$ is a basis for T_x, we have that $F_j^{\ t} B_t^{\ \alpha} = C_j^{\ \overline{\alpha}}$. Then by (0), (6) and (8) one has

$$(12)\qquad G_{ji} B_\alpha^{\ i} = B_j^{\ \alpha}, \quad G_{ji} C_{\overline{\alpha}}^{\ i} = C_j^{\ \overline{\alpha}}, \quad G_{ti} B_j^{\ t} = B_{ji}.$$

The first two relations of (12) show that (G_{ij}) is non-singular. Let (G^{ij}) be the inverse matrix of (G_{ji}), then

$$(12') \qquad G^{ij}B_j^{\alpha} = B_{\alpha}^{i}, \quad G^{ij}C_j^{\overline{\alpha}} = C_{\overline{\alpha}}^{i}.$$

Originally B_i^{α}, $C_i^{\overline{\alpha}}$ were defined in (1) from the matrix $(B_{\alpha}^{i}, C_{\overline{\alpha}}^{i})$, now we have found a Riemannian metric G_{ij} so that they are related by (12) and (12').

From (7), (10), (11) and (12) we have

$$(13) \qquad G_{st}B_i^{s}B_j^{t} = B_{ij}, \quad G_{st}B_i^{s}C_j^{t} = 0, \quad G_{st}C_i^{s}C_j^{t} = C_{ij}.$$

Finally, we consider the operations F_i^{j} and $C_i^{\overline{\gamma}}B_{\gamma}^{j} = \sum_{\gamma=1}^{n} C_i^{\gamma+n}B_{\gamma}^{j}$ on T_x, these two coincide because

$$F_i^{j}B_{\alpha}^{i} = 0, \quad F_i^{j}C_{\overline{\alpha}}^{i} = B_{\alpha}^{j}; \quad (C_i^{\overline{\gamma}}B_{\gamma}^{j})B_{\alpha}^{i} = 0, \quad (C_i^{\overline{\gamma}}B_{\gamma}^{j})C_{\overline{\alpha}}^{i} = B_{\alpha}^{j}.$$

Thus we have

$$(14) \qquad F_i^{k} = C_i^{\overline{\gamma}}B_{\gamma}^{k}.$$

If we defined $F_{ik} = G_{kj}F_i^{j}$ then by (12):

$$(15) \qquad F_{ik} = C_i^{\overline{\gamma}}B_k^{\gamma} = \sum_{\gamma=1}^{n} C_i^{\gamma+n}B_k^{\gamma}.$$

3. Let

$$\psi_{ik} = C_i^{\overline{\gamma}} B_k^{\gamma} + C_k^{\overline{\gamma}} B_i^{\gamma} = F_{ik} + F_{ki}$$

$$\varphi_{ik} = C_i^{\overline{\gamma}} B_k^{\gamma} - C_k^{\overline{\gamma}} B_i^{\gamma} = F_{ik} - F_{ki},$$

then

$$\psi_i^{\ k} \overset{\text{def.}}{=\!=} G^{kj}\psi_{ij} = C_i^{\ \overline{\gamma}} B_{\gamma}^{\ k} + B_i^{\ \gamma} C_{\overline{\gamma}}^{\ k},$$

$$\varphi_i^{\ k} \overset{\text{def.}}{=\!=} G^{kj}\varphi_{ij} = C_i^{\ \overline{\gamma}} B_{\gamma}^{\ k} - B_i^{\ \gamma} C_{\overline{\gamma}}^{\ k},$$

and

$$\psi_i^{\ k}\psi_k^{\ j} = (C_i^{\ \overline{\gamma}} B_{\gamma}^{\ k} + B_i^{\ \gamma} C_{\overline{\gamma}}^{\ k})\ (C_k^{\ \overline{\delta}} B_{\delta}^{\ j} + B_k^{\ \delta} C_{\overline{\delta}}^{\ j})$$

$$= C_i^{\ \overline{\gamma}} C_{\overline{\gamma}}^{\ j} + B_i^{\ \gamma} B_{\gamma}^{\ j} = C_i^{\ j} + B_i^{\ j} = \delta_i^{\ j},$$

$$\varphi_i^{\ k}\varphi_k^{\ j} = (C_i^{\ \overline{\gamma}} B_{\gamma}^{\ k} - B_i^{\ \gamma} C_{\overline{\gamma}}^{\ k})\ (C_k^{\ \overline{\delta}} B_{\delta}^{\ j} - B_k^{\ \delta} C_{\overline{\delta}}^{\ j})$$

$$= -C_i^{\ \overline{\gamma}} C_{\overline{\gamma}}^{\ j} - B_i^{\ \gamma} B_{\gamma}^{\ j} = -\delta_i^{\ j}.$$

Thus ψ is an almost product structure and φ is an almost complex structure on M_{2n}. Furthermore,

$$\varphi_i^{\ k}\psi_k^{\ j} = C_i^{\ j} - B_i^{\ j},\quad \psi_i^{\ k}\varphi_k^{\ j} = -C_i^{\ j} + B_i^{\ j},$$

that is,

(16) $$\varphi_i^{\ k}\psi_k^{\ j} = -\psi_i^{\ k}\varphi_k^{\ j}.$$

Conversely, if on M_{2n} there are almost product structure ψ and almost complex structure φ which satisfy (16), then if we define

$$F_i^{\ k} = \tfrac{1}{2}(\varphi_i^{\ k} + \psi_i^{\ k}),$$

$F_i^{\ k}$ is a tensor field on M_{2n} which satisfies $F^2 = 0$. Thus we have proved the following theorem.

THEOREM 1. <u>On</u> M_{2n} <u>there is a</u> (1.1) <u>tensor field</u> F <u>satisfying</u> $F^2 = 0$ <u>if and only if there are almost product structure</u> ψ <u>and almost complex structure</u> φ <u>which satisfy</u> $\varphi\psi = -\psi\varphi$.

Let

$$H_i^{\ k} = B_i^{\ \gamma} C_{\bar{\gamma}}^{\ k} \quad \text{or} \quad H_i^{\ k} = \tfrac{1}{2}(\psi_i^{\ k} - \varphi_i^{\ k});$$

then H satisfies the following :

$$(17) \quad \begin{cases} H_i^{\ k} H_k^{\ j} = 0 \\ H_i^{\ k} B_k^{\ \delta} = 0, \quad H_i^{\ k} C_k^{\ \bar{\alpha}} = B_i^{\ \alpha}, \end{cases}$$

and

$$(18) \quad \begin{cases} F + H = \psi, \quad F - H = \varphi, \\ H_i^{\ k} F_k^{\ j} + F_i^{\ k} H_k^{\ j} = \delta_i^{\ j} \end{cases}.$$

Thus H is again a tensor field on M_{2n} which satisfies $H^2 = 0$.

For the structure (ψ, φ) satisfying (16), Hsu [3, page 441, Corollary 2.4] showed that there is an affine connection Γ so that $\nabla_k \varphi_i^{\ j} = 0$, $\nabla_k \psi_i^{\ j} = 0$ where ∇ is the covariant differentiation with

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respect to Γ. In fact, let $\overset{\circ}{\nabla}$ be the covariant differentiation with respect to any given affine connection $\overset{\circ}{\Gamma}$, then one of such affine connection is the following:

$$\Gamma_{ji}^{\ \ h} = \overset{\circ}{\Gamma}_{ji}^{\ \ h} - \tfrac{1}{2}(\overset{\circ}{\nabla}_j \varphi_i^{\ a})\varphi_a^{\ h} + \tfrac{1}{4}(\overset{\circ}{\nabla}_j \psi_i^{\ a})\psi_a^{\ h} - \tfrac{1}{4}\varphi_i^{\ b}[(\overset{\circ}{\nabla}_j \psi_b^{\ d})\psi_d^{\ a}]\varphi_a^{\ h}.$$

Such a connection, satisfying $\nabla\varphi = 0$, $\nabla\psi = 0$, is called a (φ, ψ) connection.

Hence we have the following corollary:

COROLLARY 1. <u>There is an affine connection for which</u> $\nabla_k F_i^{\ j} = 0$, $\nabla_k H_i^{\ j} = 0$.

Furthermore, by Hsu [3, page 415, Theorem 3.1] with respect to the (φ, ψ) connection Γ, the holonomy group can be represented as the form $\begin{pmatrix} A_n & 0 \\ 0 & A_n \end{pmatrix}$ referred to a suitable basis where A_n is any $n \times n$ matrix. Thus we have the following corollary:

COROLLARY 2. <u>On</u> M_{2n} <u>with a</u> (1,1) <u>tensor field</u> F <u>such that</u> $F^2 = 0$, <u>with respect to the connection given in Corollary 1, the holonomy group can be represented as the form</u> $\begin{pmatrix} A_n & 0 \\ 0 & A_n \end{pmatrix}$ <u>referred to a suitable basis.</u>

4. The Nijenhuis tensor formed with $F_i^{\ k}$ is defined by:

$$t_{jk}^{\ \ i}(F) = F_j^{\ \ell}(\partial_k F_\ell^{\ i} - \partial_\ell F_k^{\ i}) - F_k^{\ \ell}(\partial_j F_\ell^{\ i} - \partial_\ell F_j^{\ i}).$$

<u>Definition.</u> The tensor field F is called integrable if $t_{jk}^{\ \ i}(F) = 0$.

Now we shall calculate $t_{jk}^{\ \ i}(F)$ by use of $F_i^{\ k} = C_i^{\ \gamma} B_\gamma^{\ k}$ and **(1)**.

$$F_j^{\ell}(\partial_k F_\ell^{\ i} - \partial_\ell F_k^{\ i}) = F_j^{\ell}(\partial_k C_\ell^{\ \bar\gamma} - \partial_\ell C_k^{\ \bar\gamma})B_\gamma^{\ i} - C_j^{\ \bar\delta} B_\delta^{\ \ell} C_k^{\ \bar\gamma} \partial_\ell B_\gamma^{\ i}.$$

$$(19)\quad \begin{cases} t_{jk}^{\ \ i}(F) = F_j^{\ell}(\partial_k F_\ell^{\ i} - \partial_\ell F_k^{\ i}) - F_k^{\ell}(\partial_j F_\ell^{\ i} - \partial_\ell F_j^{\ i}) \\ \qquad = C_j^{\ \bar\delta} B_\delta^{\ \ell}(\partial_k C_\ell^{\ \bar\gamma} - \partial_\ell C_k^{\ \bar\gamma})B_\gamma^{\ i} - C_k^{\ \bar\delta} B_\delta^{\ \ell}(\partial_j C_\ell^{\ \bar\gamma} - \partial_\ell C_j^{\ \bar\gamma})B_\gamma^{\ i} \\ \qquad\quad - C_j^{\ \bar\delta} B_\delta^{\ \ell} C_k^{\ \bar\gamma} \partial_\ell B_\gamma^{\ i} + C_k^{\ \bar\delta} B_\delta^{\ \ell} C_j^{\ \bar\gamma} \partial_\ell B_\gamma^{\ i} . \end{cases}$$

$$t_{jk}^{\ \ i} B_\alpha^{\ j} = -C_k^{\ \bar\delta} B_\alpha^{\ j} B_\delta^{\ \ell}(\partial_j C_\ell^{\ \bar\gamma} - \partial_\ell C_j^{\ \bar\gamma})B_\gamma^{\ i}$$

$$t_{jk}^{\ \ i} B_\alpha^{\ j} B_i^{\ \beta} C_{\bar\gamma}^{\ k} = -B_\alpha^{\ j} B_\gamma^{\ \ell}(\partial_j C_\ell^{\ \bar\beta} - \partial_\ell C_j^{\ \bar\beta})$$

$$\overset{\text{def.}}{=\!=} -\Omega_{\alpha\gamma}^{\ \ \bar\beta} .$$

For $\Omega_{\alpha\gamma}^{\ \ \bar\beta}$ there was the following consideration according to Yano [4]. An arbitrary contravariant vector dx^i in the tangent space $T_x(M_{2n})$ at x can be written in

$$(20)\qquad dx^i = B_\alpha^{\ i} dy^\alpha + C_{\bar\alpha}^{\ i} dy^{\bar\alpha}$$

where

$$(20')\qquad dy^\alpha \overset{\text{def.}}{=\!=} B_j^{\ \alpha} dx^j , \qquad dy^{\bar\alpha} \overset{\text{def.}}{=\!=} C_j^{\ \bar\alpha} dx^j .$$

Thus the distribution B is defined by

$$(21)\qquad dy^{\bar\alpha} = C_j^{\ \bar\alpha} dx^j = 0 .$$

The condition for the distribution B to be completely integrable is that

$$(\partial_k C_j^{\overline{\alpha}} - \partial_j C_k^{\overline{\alpha}}) dx^k \wedge dx^j = 0$$

be satisfied by any dx^i satisfying (21), that is

$$\Omega_{\alpha\gamma}^{\overline{\beta}} = B_\alpha^{\ j} B_\gamma^{\ \ell} (\partial_j C_\ell^{\overline{\beta}} - \partial_\ell C_j^{\overline{\beta}}) = 0 .$$

Using this fact and the relation between $t_{jk}^{\ \ i}$ and $\Omega_{\alpha\gamma}^{\overline{\beta}}$ given above, one has proved Theorem 2.

THEOREM 2. _If_ F _is integrable, then the distribution_ B _is completely integrable; if_ H _is integrable, then the distribution_ C _is completely integrable_.

The last part of this theorem is a parallel conclusion of the first part.

Conversely, if both distributions B and C are completely integrable, then there are functions $y^\alpha(x^i)$, $y^{\overline{\alpha}}(x^i)$ satisfying (20), (20') and hence

$$B_j^{\ \alpha} = \frac{\partial y^\alpha}{\partial x^j}, \quad C_j^{\overline{\alpha}} = \frac{\partial y^{\overline{\alpha}}}{\partial x^j}, \quad B_\alpha^{\ i} = \frac{\partial x^i}{\partial y^\alpha}, \quad C_{\overline{\alpha}}^{\ i} = \frac{\partial x^i}{\partial y^{\overline{\alpha}}} .$$

Noticing that $\partial_k C_\ell^{\overline{\gamma}} = \partial_k \partial_\ell y^{\overline{\alpha}}$ and $B_\delta^{\ \ell} \partial_\ell B_\gamma^{\ i} = \partial_\delta B_\gamma^{\ i} = \partial_\delta \partial_\gamma x^i$, by (19) we have

$$\begin{aligned} t_{jk}^{\ \ i}(F) &= -C_j^{\overline{\delta}} B_{\delta}^{\ \ell} C_k^{\overline{\gamma}} \partial_\ell B_\gamma^{\ i} + C_k^{\overline{\delta}} B_\delta^{\ \ell} C_j^{\overline{\gamma}} \partial_\ell B_\gamma^{\ i} \\ &= -C_j^{\overline{\delta}} C_k^{\overline{\gamma}} \partial_\delta B_\gamma^{\ i} + C_k^{\overline{\delta}} C_j^{\overline{\gamma}} \partial_\delta B_\gamma^{\ i} = 0 . \end{aligned}$$

Thus F is integrable. Similarly, it is easy to show that H is also integrable. Hence we have Theorem 3.

THEOREM 3. *If* B *and* C *are both completely integrable, then* F *and* H *are both integrable.*

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