$$\therefore \quad \lambda_1 \lambda_2 \lambda_3 = \begin{vmatrix} a, & h, & g \\ h, & b, & f \\ g, & f, & c \end{vmatrix}.$$

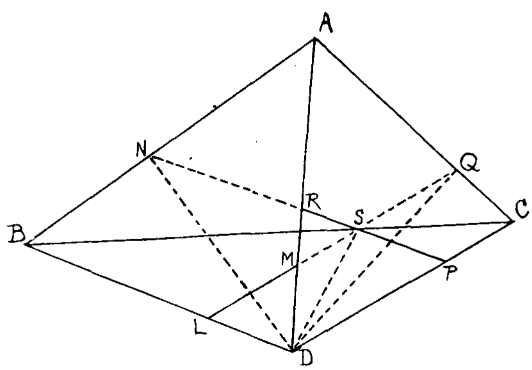
Using the values of a, b, ... above, it is easy to show that

$$\frac{al_1 + hm_1 + gn_1}{l_1} = \frac{hl_1 + bm_1 + fn_1}{m_1} = \frac{gl_1 + fm_1 + cn_1}{n_1} = \lambda_1$$

and so to obtain the result, but the direct method is of some interest.

R. J. T. Bell.

A Geometrical Proof for Hero's Formula.



The following proof is designed to link up Hero's formula geometrically with the formulae for the trigonometrical functions of $\frac{1}{2}A$ in a triangle.

From the bisector of angle A let AD be cut off equal to the mean proportional between AB and AC, and let N be the projection of D on AB. The formulae $AN = \sqrt{s(s-a)}$, $ND = \sqrt{(s-b)(s-c)}$ are easily established geometrically, and are assumed here. Thus triangle AND gives directly the formulae for $\sin \frac{1}{2}A$, $\cos \frac{1}{2}A$, $\tan \frac{1}{2}A$. Hero's formula is thus represented by $AN \,.\, ND$. It is required to prove therefore that twice the area of triangle AND is equal to the area of triangle ABC.

Join BD, DC and draw the pedal triangles LMN, PQR of the triangles ABD, ADC, which are similar since $AB : AD = AD : AC$. These triangles are then divided by their pedal triangles into similar component pairs, and the three triangles round a pedal triangle are all similar to the whole triangle. Now PR and QR are equally inclined to AD (a pedal property), and N is the image of Q in AD. $\therefore$ PR produced passes through N; similarly LM produced passes through Q. Let these lines cut BC in S and S'.

Again $PN \parallel DB$ since alternate angles BDR, DRP are corresponding angles of the similar triangles BDA, DRP; similarly $LQ \parallel DC$.

But $BS : SC = DP : PC$ (BD parallel to SP)

$= BL : LD$ (complete similarity of the figures)

$= BS' : S'C$ (LS' parallel to DC)

$\therefore$ S and S' coincide.

Now area of triangle NBS = area of triangle NDS (NS parallel to BD)

,, ,, QCS = ,, ,, QDS (QS ,, DC).

To the sum of these areas add area $ANSQ$.

$\therefore$ in area, triangle ABC = kite $ANDQ$ = twice triangle AND.

G. D. C. Stokes.

Editor's Note.—Mr John T. Brown suggests the following neat method of proving that the triangle ABC is twice the triangle AND:

Suppose DN produced its own length to E.

Then the angles EAD, BAC are equal,

and $AE . AD = AD^2 = AB . AC$.

Hence, by Euc. VI, 15, the triangles AED, ABC are equal; *i.e.* twice triangle AND = triangle ABC.

W. A.

A Proof of the Theorem of Pythagoras.

The triangle ABC has a right angle at B. On AC, on the same side as B, describe a square $ADEC$. Draw DF perpendicular to AB or AB produced.

The triangles ABC and DFA are congruent, having sides CA and AD equal, and the corresponding angles equal. Hence DF is equal to AB.