# DYNAMICAL MODELS OF CIRCUMSTELLAR DUST SHELLS AROUND LONG–PERIOD VARIABLES

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**Abstract.** We present dynamical models of circumstellar dust shells around long-period variables which include time-dependent hydrodynamics and a detailed treatment of dust formation, growth and evaporation. Important effects due to the complex interaction between the dynamics of the pulsating atmosphere and the dust complex are demonstrated.

## 1. Introduction

We present results of our model calculations for circumstellar dust shells of long-period variable stars (LPVs). In particular, we will summarize the method and some results of the hydrodynamical calculations. What is our main aim and what can we learn from these models?

First, as we treat in detail the dynamics and physical processes of the dust shell, we can study its intrinsic structure, the distribution of the dust, the velocity field, the density structure, etc. which all are a result of the calculation.

Second, it turns out that it is absolutely necessary to take into account the relevant interactions among the physical processes. By doing so, we are able to study which processes contribute to the resulting structure and the dynamics of the model.

Third, as a result of the model we get the final outflow velocity, the mass loss rate, the dust–to–gas ratio, and the grain size distribution function. These values are a result of the calculation itself — they are *not* prescribed parameters — and therefore they can be directly compared with respective observations.

Finally, and perhaps most important, we can calculate the optical appearance of the model, the spectral energy distribution, light curves, bright-

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ness profiles, etc. and can compare these quantities with observations as well. If this approach is applied to a particular source and the theoretical spectral energy distribution, synthetic light curves, etc. match with the respective observations, we are able to determine the fundamental stellar parameters: stellar luminosity, stellar mass and temperature, and the elemental abundances. In this contribution we concentrate on the first three points and show typical results of the dynamics and intrinsic structure of the circumstellar dust shell.

The second part, the optical appearance, can be found elsewhere (Winters et al. 1994, 1995, 2000).

## 2. The Non–Linear Problem

In order to reach the objective described above we have to solve the following coupled problem:

**Time–dependent hydrodynamics.** The hydrodynamical description has to be time–dependent, to describe the variations of the velocity field and the occurrence of shock waves caused by the pulsation of the underlying star.

**Dust complex.** Since the formation of dust is a central phenomenon in these objects, it is necessary to use a theory which describes the formation of solid particles depending on the physical conditions, and which does not prescribe essential quantities such as the site where the dust forms.

**Chemistry.** Furthermore, we have to describe the chemistry of the gas phase in order to know at each instant of time and at each radial position how much condensable material is available.

**Radiative transfer.** Finally, we have to treat the radiative transfer problem in order to describe the influence of the radiation of the central object.

In contrast to the classical atmosphere problem, the dust complex now introduces a number of non-linear couplings and interactions which are of essential importance for the whole problem. A reliable theoretical modelling on the one hand requires a *physical* description of the various ingredients, and on the other hand, at least equally important, it requires taking into account all interactions among the different physical components.

It is evident from the list that the problem is non-linear and strongly coupled. Consider for instance the coupling between hydrodynamics and the dust complex: only if the density is high enough, is effective dust formation possible; but at the same time, since radiation pressure on dust enters into the equation of motion, dust formation immediately influences the velocity structure, which in turn alters the density stratification. The basic equations and the numerical method are described in detail in Fleischer et al. (1992) and Fleischer (1994).

## 3. A Typical Model Structure

The radial structure of a typical dynamical model is shown in Figure 1. It is evident that the dust is not distributed homogeneously across the shell but is concentrated in distinct layers such that the circumstellar dust shell exhibits an onion-like structure. Furthermore it can be seen that the dust quantities, e.g. the degree of condensation ($2^{nd}$ panel, dashed line), are intimately correlated with the hydrodynamical quantities, e.g. the velocity (upper panel, crosses). This suggests that there is a common mechanism which produces this structure. A more detailed analysis shows that the strong shocks propagating through the atmosphere are not produced by the interior pulsation alone but are a product of the re-amplification of the pulsational shocks caused by the radiative acceleration $\alpha$ on dust grains in the discrete dust layers ($1^{st}$ panel, dashed line). Since $\alpha$ exceeds unity close to the star, the material is accelerated to velocities above the escape velocity $v_{\rm esc}$ already at radii around $4\,R_*$. Due to its opacity, the dust also leads to a pronounced heating of the material inside the dust layers (backwarming), as is evident from the steps present in the temperature stratification around $3\,R_\star$ and $6\,R_\star$ (lower panel). In summary, we find that the dust completely determines the internal structure of the circumstellar shell, a result that is only revealed by the consistent treatment of the physical processes involved.

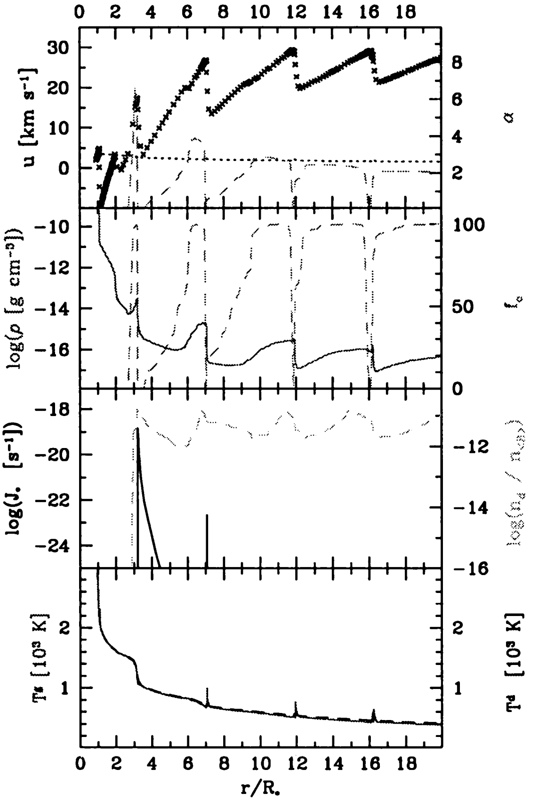

*Figure 1.* Typical radial structure of a dynamical model. The parameters of the model are $T_* = 2600\,\mathrm{K}$, $L_* = 10^4 L_\odot$, $M_* = 1.0 M_\odot$, $\epsilon_C/\epsilon_O = 1.80$, $P = 650\,\mathrm{d}$, $\Delta u = 2\,\mathrm{km\ s^{-1}}$.

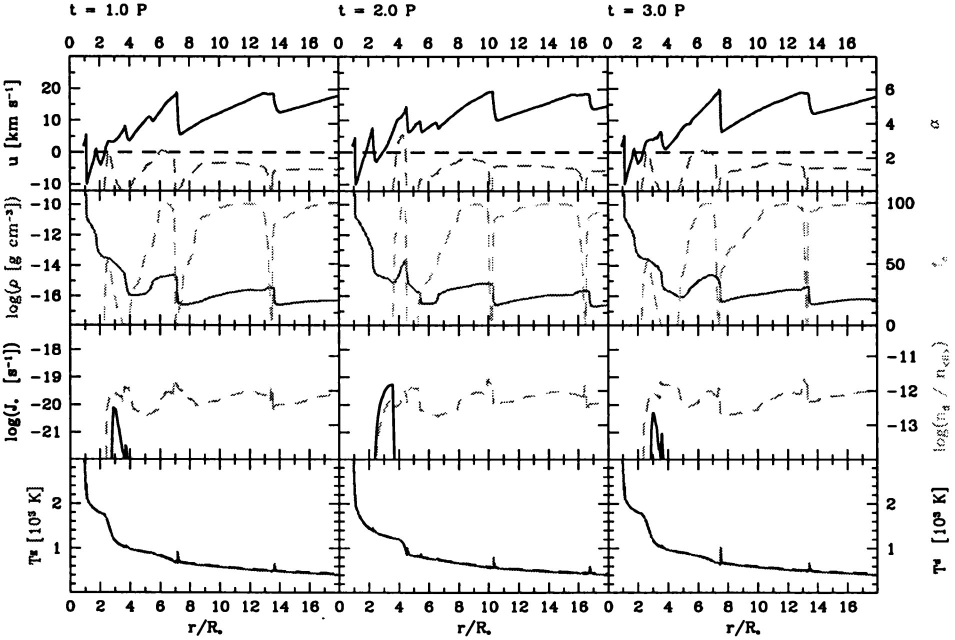

*Figure 2.* Same model as in Fig. 1 but with a reduced carbon-to-oxygen ratio of $\epsilon_C/\epsilon_O = 1.50$.

## 4. Multiperiodicity

An important property of the model shown in the preceding section is that its radial structure repeats after *one* pulsational cycle. Figure 2 shows three successive starting points of the hydrodynamical cycle of a model with a reduced overabundance of carbon to oxygen. All other parameters are the same as in Fig. 1. From Fig. 2 one can see that the radial structure of the model with a reduced carbon overabundance repeats only after *two* pulsational cycles. Even tiny details of the radial structure, e.g. the multiple shocks present below $4\,R_*$, are reproduced after this period of time. Around $2.5\,R_*$ at $t = 1.0P$ and $t = 3.0P$, respectively, it can be seen that the radiative acceleration $\alpha$ causes a pertubation in the velocity structure which later on turns into a dominant shock wave that sweeps up the preceding pulsational shock: cf. $t = 2.0P$ between 4 and 5 $R_*$. Due to the reduced amount of condensable material it takes two pulsational cycles to form a new dust layer. The effect of *multiperiodicity* strongly depends on the $\epsilon_C/\epsilon_O$ ratio. Lowering this number causes longer intervals between the formation of two dust layers; an increase causes the opposite effect. The time interval between the formation of two dust layers is of course not necessarily an integral number, e.g. models with $\epsilon_C/\epsilon_O$ values between 1.8 and 1.5 form a

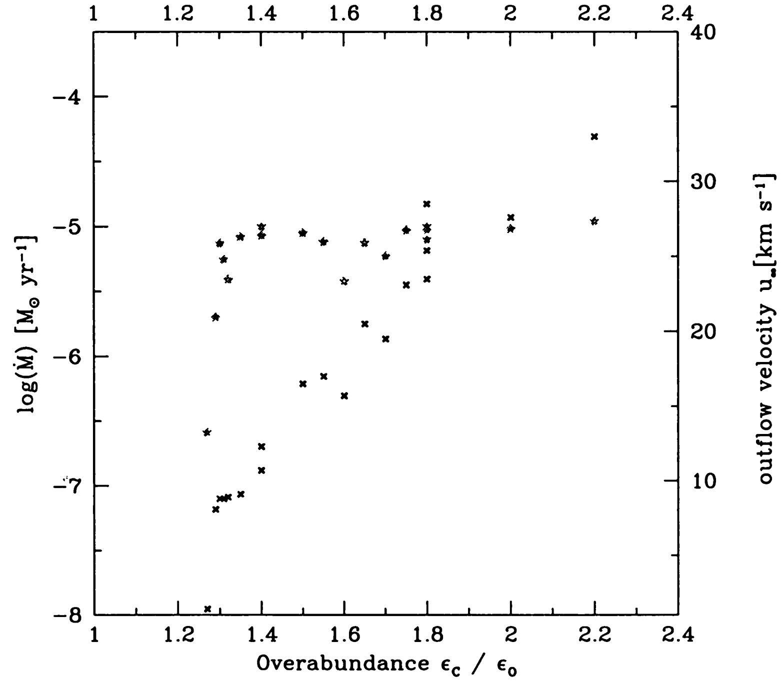

*Figure 3.* Dependence of the mass-loss rate $\dot{M}$ (asterisks, left axis) and the final outflow velocity $u_\infty$ (crosses, right axis) on the overabundance of carbon to oxygen $\epsilon_C/\epsilon_O$.

new dust layer on time scales larger than 1 and smaller than 2 pulsational periods. Depending on the remaining model parameters it is also possible that a dust layer forms on time scales of 3, 4 ... times the pulsational period, or even twice per pulsational cycle.

## 5. Dependence of $\dot{M}$ and $u_\infty$ on $\epsilon_C/\epsilon_O$

We have calculated a grid of models with the parameters given in the caption of Fig. 1 except for the overabundance of carbon to oxygen which is varied over a larger parameter range. The influence of this variation on the mass-loss rate and the final outflow velocity is shown in Fig. 3. Keeping all parameters except the $\epsilon_C/\epsilon_O$ ratio fixed results in a fairly constant mass-loss rate of $\sim 10^{-5}\mathrm{M}_\odot\mathrm{yr}^{-1}$ for all models with $\epsilon_C/\epsilon_O > 1.30$. In contrast, $u_\infty$ is proportional to $\epsilon_C/\epsilon_O$. Below the value of $\sim 1.30$ for $\epsilon_C/\epsilon_O$ there is just enough dust formed to drive a wind which is, however, very slow and less massive. Going to even lower values of $\epsilon_C/\epsilon_O$, pulsation and radiative acceleration on dust grains alone no longer can support an outflow. A fitting equation which relates a given set of the six model parameters to the resulting mass-loss rate is derived in Arndt et al. (1997).

## 6. Shortcomings of the Models

The models suffer from two major shortcomings: the use of an equilibrium chemistry and the treatment of the cooling of the gas behind the shock fronts. As strong velocity changes and short-term variations in the radiation field are usually encountered in LPV atmospheres, the standard situation of the gas phase is chemical non-equilibrium (cf. Sedlmayr & Winters 2000). To consider these effects in the models would require solving a time-dependent reaction network which is extremely CPU-time-consuming. However, calculations of rate networks show that in the inner shell region, where the dust forms, the situation can be approximated by the equilibrium case if one is only interested in the densities of the dust-forming species (cf. Beck et al. 1992). In future work, the treatment of the post-shock cooling processes in the models also has to be improved. Calculations in the isothermal as well as the adiabatic limit case show that the structure of the circumstellar dust shell appears in the way described above. Using LTE cooling laws, as done in the models described in this paper and in Höfner et al. (1996), yields models very close to the isothermal limit case. Incorporating the laws proposed by Bowen (1988) results in models with an unrealistic temperature structure as the re-heating behind the shocks proceeds much too slowly. A solution to the post–shock cooling problem could be the incorporation of the scheme developed in Woitke et al. (1996).

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**Discussion:**

**Luttermoser**: This work you have just presented is phenomenal! I have one problem with it, however. We see UV emission lines — and, in the case of Miras, strong hydrogen Balmer lines — in the spectra of these stars. Yet the shocks in your model are not hot enough to produce these lines. Can you comment on this?

**Fleischer**: The models I just presented include an LTE cooling law, which yields a rather efficient cooling of the gas behind the shocks. Currently we are working on the incorporation of a more realistic treatment of the heating/cooling processes by using the approach given in Woitke, Krüger & Sedlmayr (in press) [*A&A* 311, 927, 1996 – Ed.]. I do not expect major changes in the overall structure of the circumstellar dust shell, as I already checked the influence of the cooling law by varying the C–parameter of Bowen's (1988) parameterized law.

**Linsky**: I encourage you to include an important physical process that is probably not in your calculations. Manfred Cuntz has computed models with a stochastic distribution of piston amplitudes and periods. He finds that models including a stochastic distribution of wave properties rather than monochromatic waves have supershocks due to coalescence of individual shocks and greatly increased mass-loss rates.

**Fleischer**: As far as I know, the work of Cuntz deals with sound waves generated by the outer convective zone. Our models try to simulate the large-amplitude pulsation of a Mira star. Nevertheless, I agree that we have to get rid of this piston approximation in favor of a physical model of the interior pulsation which, of course, could include the contribution of sound waves.

**Mowlavi**: What is, from your model calculations, the dependence of the mass-loss rate on the C/O ratio?

**Fleischer**: The mass-loss rate is essentially <u>in</u>dependent of the C/O ratio (Fleischer 1994, Dissertation, TU–Berlin).

**Mowlavi**: What are then the other parameters that influence the mass-loss rate? The mass-loss rates you obtain reach values as high as $10^{-5}$ or more. Could you comment on the development of "superwinds" as are often supposed to occur at some stage of the ascent of the AGB phase?

**Fleischer**: The mass-loss rate mainly depends on stellar temperature, stellar luminosity, and mass. The mass-loss rate increases with increasing luminosity, decreasing temperature, and decreasing mass. Since during the AGB evolution the stellar luminosity increases, the temperature (slightly) decreases, and due to mass loss the stellar mass also decreases, the mass-loss rate produced by our models easily reaches values as high as $\sim 10^{-4}$ $M_{\odot}$/yr. Therefore, dust-driven mass loss as it results from our models shows the typical characteristics of the so-called "superwind."

**Gustafsson**: A good criterion of very good theoretical modelling is that all good models enthusiastically suggest further complications. Here is another one: Your strong and very impressive coupling between pulsations, dust formation and wind suggests that non-radial perturbations could have severe consequences. Do you dare to comment on that?

**Fleischer**: Most likely this would yield a cloudy or patchy structure of the dust distribution. However, a quantitative answer can only be given by a more-dimensional treatment which within our approach is beyond today's computing power.