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LETTERS TO THE EDITOR

Dear Editor,

The queue length distribution for the $M/G/1$ queue under the D-policy

This is to notify you of a flaw in the result of Dshalalow (1998). Due to space limitation, we consider only a stationary $M/G/1$ (exhaustive FIFO) queue under the D-policy, which is the simplest queue among the class of queues considered by Dshalalow. We use Dshalalow's notation, omitting definitions.

Dshalalow's departure time queue length probability generating function can be expressed as

$$p(z) = p_{M/G/1}(z)\frac{1 - \mathrm{E}[z^{\nu_D}]}{(1 - z)\mathrm{E}[\nu_D]}, \tag{1}$$

where $p_{M/G/1}(z)$ is the departure time queue length probability generating function for a standard $M/G/1$ queue (where $D = 0$). The so-called decomposition structure of (1) would have been valid if service times of ν_D customers (who arrived during an idle period) were i.i.d. random variables.

Suppose the server has just finished serving all ν_D customers. The probability generating function for the number of customers in the system is then given by $B_{\nu_D}(\lambda - \lambda z)$. If this number is zero (with a probability $B_{\nu_D}(\lambda)$), the next idle period begins. Otherwise, the busy period continues. At this point in time, however, service times of those already in the system and of those yet to arrive until the end of the busy period are all i.i.d. random variables. Thus, from this point on, the departure process is identical to that of the $M/G/1$ queue with generalized vacations such that the probability generating function for the number of arrivals during an idle period is given by

$$X(z) = \frac{B_{\nu_D}(\lambda - \lambda z) - B_{\nu_D}(\lambda)}{1 - B_{\nu_D}(\lambda)}.$$

Then, based on the decomposition property, we claim that

$$p(z \mid B) = p_{M/G/1}(z)\frac{1 - X(z)}{(1 - z)\mathrm{E}[X]},$$

where $p(z \mid B)$ is the conditional probability generating function given that the departing customer is one of those who arrived during a busy period.

Due to the PASTA property (see Wolff (1982)), respective probabilities that an arriving customer finds the server busy and idle are $\rho\ (= \lambda b < 1)$ and $1 - \rho$. Thus, $p(z)$ can be expressed as

$$p(z) = \rho p(z \mid B) + (1 - \rho)\, p(z \mid I),$$

where $p(z \mid I)$ is the conditional probability generating function given that the departing customer is one of those who arrived during an idle period. We claim that (details can be found in Chae and Park (2000))

$$p(z \mid I) = \frac{1}{\mathrm{E}[\nu_D]}\left\{B_{\nu_D}(\lambda - \lambda z) + \sum_{n=1}^{\infty} z^n \int_0^D \mathrm{e}^{-(\lambda - \lambda z)t} \Pr(\nu_{D-t} = n)\, \mathrm{d}m(t)\right\}, \tag{2}$$

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where $m(\cdot)$ is the renewal function of the renewal process whose interrenewal times correspond to i.i.d. service times. Note that $B_{\nu_D}(\lambda - \lambda z)$ in (2) represents the number of customers in the system at the departing point of the 'last-comer' among ν_D customers. Note further that $\mathrm{d}m(t)$ in (2) is the probability that the accumulated work becomes $(t, t + \mathrm{d}t)$, $0 < t < D$, just after the arrival of any one among ν_D customers except the 'last-comer'. And, when this customer departs, she will leave behind ν_{D-t} customers (who arrive during the remaining idle period) plus those who arrive during the first t units of time of the busy period.

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