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Does the Options Market Underreact to Firms' Left-Tail Risk?

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Abstract

We show that firms' left-tail risk positively predicts future returns of crash insurance. We proxy crash insurance with bear spreads, an option trading strategy that profits when extreme negative returns occur. Crash insurance for high (low) left-tail risk firms earns positive (negative) returns, suggesting that the downside protection it provides is not adequately priced. Our results are mainly explained by two types of underreaction: volatility underreaction in high left-tail risk portfolios and underreaction to the persistence of left-tail risk. Disagreement partially explains our results, but a risk-based approach does not.

I. Introduction

Loss aversion plays an important role in economic decisions. The utility of a loss-averse investor is steeper for losses than for gains (Kahneman and Tversky (1979), Tversky and Kahneman (1991)). Berkelaar, Kouwenberg, and Post (2004) and Jarrow and Zhao (2006) show that optimal portfolios for loss-averse investors include hedging and insurance against left-tail risk. Hedging demand for assets providing insurance against left-tail risk potentially impacts their prices. A growing body of literature investigates the impact of left-tail risk on the cross section of stock returns. Lu and Murray (2019) use bear spread returns constructed from Standard & Poor's (S&P) 500 index options to capture bear market risk and find that it is priced in the cross section of stock returns. Kelly and Jiang (2014) construct a tail-risk factor by identifying the common fluctuation of crash events for individual firms

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and find that stocks with higher loadings on the tail-risk factor earn higher future returns. By contrast, Atilgan, Bali, Demirtas, and Gunaydin (2020) show that the risk–return trade-off between firms' left-tail risk and stock returns breaks down because firms' left-tail risk and future stock returns have a negative relation. These findings are explained by the persistence of left-tail risk or left-tail momentum that indicates that equity investors underreact to firms' left-tail risk.

In this article, we study the performance of cross-sectional crash insurance, a financial instrument that provides coverage when extreme negative returns occur. In a world with two identical firms where one has higher left-tail risk than the other, a risk-averse investor buying crash insurance would expect returns on crash insurance for the high left-tail risk firm to be lower than those for the low tail-risk firm. Our empirical findings provide evidence against this claim.

We sort firms in deciles each month based on their left-tail risk and study the subsequent 1-month crash insurance return. We proxy crash insurance with a tradable option strategy (a bear put spread) that buys an out-of-the-money (OTM) put option and sells a deeper OTM (DOTM) put option. We delta-hedge our option strategy so that our results are not driven by movements in the underlying asset. Two counterintuitive findings emerge. First, we document a positive relation between firms' left-tail risk and future bear spread returns. Crash insurance for low tail-risk firms earns negative returns, as one would expect. However, the portfolio of firms in the lowest left-tail risk, decile 1, reports the most negative bear spread returns across all portfolios. The "10–1" long-short bear spread portfolio earns positive and significant returns. Second, bear spread returns for firms with the highest left-tail risk, deciles 9 and 10, as well as those for deciles 5–7 report, positive returns. Positive average returns mean that investors profit when buying insurance for left-tail risk, while insurance sellers incur losses on these assets.

To understand our results, we first explore potential volatility underreaction when pricing bear put spreads. Volatility underreaction is documented by Poteshman (2001), Cheng (2020), Barrero (2022), and Lochstoer and Muir (2022). Since bear spread prices depend on two implied volatilities (IVs), OTM and DOTM put options' IVs, we decompose these volatilities into the level of the implied volatility smile (the at-the-money (ATM) implied volatility) and the slope of the smile (option skew). Using ex ante measures of the ATM IV and the option skews from statistical models, we find that bear spread prices for firms with high left-tail risk are undervalued. This effect is generated by the underestimation of the ATM IV. Since our forecasts do not include variance risk premia, these findings suggest the existence of volatility underreaction for firms with high left-tail risk. For firms with low left-tail risk, crash insurance appears to be fairly priced, with premiums exceeding their ex ante estimates. Using naïve ex ante estimators based on the past 3-, 6-, or 12-month averages of the OTM and DOTM, implied volatilities confirm the volatility underreaction effect.

An alternative explanation of our results is the potential underestimation of the persistence of left-tail risk, our sorting variable. Atilgan et al. (2020) document that left-tail risk is highly persistent in the cross section of stock returns. We find that optionable stocks in our sample exhibit even stronger left-tail persistence: 77% of the firms in the highest left-tail risk portfolio (decile 10) remain in deciles 9 and 10 a year later. Empirically, we show that the positive relation between left-tail risk and crash insurance returns is more pronounced when left-tail risk increases or when the stock price is near its 52-week low. We argue that investors underestimate left-tail risk momentum, leading to underpricing of crash insurance on high left-tail risk stocks, thus making the positive relation stronger. These results are consistent with Chan (2003), Easterwood and Nutt (1999), and Hong, Lim, and Stein (2000), who document that underreaction is more pronounced for left-tail events. Furthermore, our anomaly becomes stronger in periods of high information uncertainty or high investor sentiment. These findings align with an underreaction explanation since information uncertainty amplifies investors' underreaction (Zhang (2006a), (2006b)) and investor sentiment leads to the mispricing of risky assets (Baker and Wurgler (2006), Stambaugh, Yu, and Yuan (2012), and Byun and Kim (2016)).

Our results are robust to different methodologies and control variables such as size, book-to-market, firm leverage, momentum, reversal, Amihud (2002) illiquidity, and idiosyncratic volatility. The positive relation between crash insurance returns and left-tail risk is not a compensation for variance risk premium, jump risk, or uncertainty of stock volatility risk. Our findings cannot be explained by information asymmetry, option demand pressure, option return predictors, or ex ante exposures of individual bear spread returns to systematic factors and left-tail factors. Using double sorting and Fama–MacBeth (1973) regressions, we show that the positive relation between crash insurance returns and left-tail risk is robust to all control variables. Only disagreement provides a partial explanation of our results.

We rule out a risk-based explanation by computing risk-adjusted long-short returns with a comprehensive list of risk factors. The alphas of long-short bear spread returns remain positive and significant after adjusting for these risk factors, suggesting that a risk-based explanation does not explain our findings. Our results also hold across different time periods, portfolio weighting schemes, daily delta-hedging, alternative definitions of the bear spread, modified Fama–MacBeth (1973) regressions proposed by Brennan, Chordia, and Subrahmanyam (1998), and the weighted least squares (WLS) parameter estimation proposed by Asparouhova, Bessembinder, and Kalcheva (2013). Transaction costs reduce the performance of our bear spread strategy, but the strategy remains profitable when disregarding options with large bid–ask spreads as in Heston, Jones, Khorram, Li, and Mo (2023) and trading at effective spreads of the magnitude paid by algorithmic traders.

Our study contributes to the growing literature on tail risk and asset prices (Campbell, Hilscher, and Szilagyi (2008), Kelly and Jiang (2014), Van Oordt and Zhou (2016), Chabi-Yo, Ruenzi, and Weigert (2018), Lu and Murray (2019), Atilgan et al. (2020), and Chen, Gan, and Vasquez (2023)). We document a positive relation between firms' left-tail risk and future crash insurance returns. Contingent claims traded in the options market offer unique opportunities to isolate and insure crash risk. Using the bear spread option strategy as a proxy for crash risk insurance, we show that the risk–return trade-off breaks down as the options market underreacts to volatility and the persistence of firms' left-tail risk. Our study highlights that although investors frequently emphasize the importance of tail risk management, crash risk insurance is likely underpriced in the options market. The underpricing of bear spreads when downside risk is high shows that adequately pricing tail risk in options markets can be challenging.

The remainder of the article proceeds as follows: Section II describes the data and the construction of bear spreads. Section III presents the main empirical results. In Section IV, we analyze potential explanations for the main findings. Section V presents robustness tests, and Section VI concludes.

II. Data

A. Sample Construction

Our sample period is from Jan. 1996 to Dec. 2017. We obtain stock price and accounting data from CRSP and Compustat. Option data from OptionMetrics include daily closing bid and ask prices, open interest, volume, implied volatility, and option Greeks. To avoid the bid–ask bounce, the mid points of bid and ask prices are used to compute option returns. Five filters are applied to process the option data and are described in Section A of the Supplementary Material.

B. Bear Spread Construction

A popular form of crash insurance available in the options market is a bear spread. A bear spread is constructed by taking a long position in one OTM put option, denoted PUT_1, with price P_1, strike price K_1, and delta Δ_1, and a short position in a DOTM put option, denoted PUT_2, with price P_2, strike price K_2, and delta Δ_2 ($K_1 > K_2$ and $\Delta_1 < \Delta_2$). The bear spread generates a payoff of $K_1 - K_2$ when the stock price at expiration is below K_2 and 0 when the stock price at expiration is above K_1. The bear spread payoff linearly decreases from $K_1 - K_2$ to 0 when the stock price is between K_2 and K_1.

Choosing K_1 and K_2 in empirical studies deserves careful consideration. As discussed in Lu and Murray (2019), if the bear region boundary K_2 is set to be at a constant percentage below the forward price, the bear region would correspond to left-tail events with different probabilities when the underlying assets possess different volatility levels. To address this issue for index option bear spreads, Lu and Murray (2019) set K_2 and K_1 to 1.5 and 1.0 standard deviations below the index forward price.

Since equity options have more sparse strike prices compared to index options, we use option deltas instead of strike prices to select put options. Extensive literature uses Black–Scholes deltas to identify options with the same moneyness across assets because the absolute delta approximates the probability that an option will be in the money at expiration (Bollen and Whaley (2004), Driessen, Maenhout, and Vilkov (2009), Jin, Livnat, and Zhang (2012), Bali and Murray (2013), Kelly, Lustig, and Van Nieuwerburgh (2016a), and, Kelly, Pástor, and Veronesi (2016b)).

The typical ranges of OTM and DOTM put option deltas are $[-0.40, -0.20)$ and $[-0.20, 0]$ (e.g., Kelly and Jiang (2014), Muravyev (2016)). We construct bear spreads with a long (short) position in PUT_1 (PUT_2) as the OTM (DOTM) put option with Δ_1 (Δ_2) closest to -0.30 (-0.10), the midpoint of the OTM (DOTM) delta range. Our results hold when we consider using a simple average or a kernel-weighted average of put options with deltas between $[-0.40, -0.20)$ and $[-0.20, 0]$ as reported in Table A9 in the Supplementary Material.

A bear spread has a negative delta $(\Delta_1 - \Delta_2)$, embedding an equivalent short position in the underlying stock. Therefore, unhedged bear spread returns also capture movements in the underlying stock. Given that Atilgan et al. (2020) document that stocks' left-tail risk predicts stock returns, we delta-hedge the bear spread position so that our results are not driven by stock price movements.[1] We use static delta-hedging as in previous equity option studies (Goyal and Saretto (2009), Bali and Murray (2013), and Byun and Kim (2016)).[2] Following Goyal and Saretto (2009) and Kelly and Jiang (2014), we use 1-month options to construct bear spreads, and form delta-hedged bear spreads on the first trading day immediately following the third Saturday in month t and close all positions at the option maturity on the third Friday in month $t+1$. Our methodology avoids the forward-looking bias when selecting our options.

The delta-hedged return of the bear spread over $[t, t+1]$ is

$$\text{RETURN} = \frac{(\Delta_{2,t} - \Delta_{1,t})S_{t+1} + \max(K_1 - S_{t+1}, 0) - \max(K_2 - S_{t+1}, 0)}{(\Delta_{2,t} - \Delta_{1,t})S_t + P_1 - P_2} - 1,$$

where P_1 (P_2), $\Delta_{1,t}$ $(\Delta_{2,t})$, and K_1 (K_2) are the price, delta, and strike price of PUT1 (PUT2), the OTM (DOTM) put at time t, and S_t (S_{t+1}) is the price of the underlying stock at time t $(t+1, \text{maturity})$.

Our sample consists of 155,003 cross-sectional monthly returns of delta-hedged bear spreads.

C. Left-Tail Risk Measures

We estimate left-tail risk using two standard measures following Atilgan et al. (2020): value-at-risk (VAR) and expected shortfall (ES). VARx (ESx) is calculated as (the average of the observations that are less than or equal to) the x percentile of daily returns over the past 250 trading days. As the left-tail loss measures are typically negative, we multiply these measures by -1 so that a higher value of VAR or ES corresponds to higher left-tail risk.

D. Other Variables

We construct four groups of control variables. Some of these variables are commonly used in studies of the cross section of equity option returns (Goyal and Saretto (2009), Bali and Murray (2013), Cao and Han (2013), Byun and Kim (2016), and Vasquez (2017)).

First, we construct three variables related to firm characteristics. Firm size (SIZE) is the natural logarithm of the firm market capitalization observed at the end of month $t-1$. Book-to-market ratio (BTM) is the ratio of a firm's net assets' book value at the previous fiscal year-end to the market capitalization of the stock at the end of month $t-1$. Firm leverage (DTA) is the ratio of a firm's total liability to the

[1] Lu and Murray (2019) show in a theoretical model that only delta-hedged bear spread returns are exposed to the left-tail risk. We empirically confirm that the predictability of the right-tail measure is subsumed by the left-tail measure as reported in Table A1 in the Supplementary Material.

[2] Our results are robust to daily delta-hedging as reported in Table A9 in the Supplementary Material.

book value of total assets at the previous fiscal year-end. Vasquez and Xiao (2024) find a negative relation between leverage and future option returns.

Second, we construct six variables related to stock returns and stock trading activities. Momentum (MOM) is the cumulative stock return from month $t-6$ to month $t-2$. Short-term reversal (REV) is the stock return in month $t-1$. Stock return skewness (SKEW) and kurtosis (KURT) are calculated using last year's daily stock return data. Two additional variables (illiquidity and idiosyncratic volatility) predict the cross section of delta-hedged option returns according to Zhan, Han, Cao, and Tong (2022) and Cao and Han (2013). Illiquidity ratio (ILLIQ) is defined as the natural logarithm of the average ratio of the absolute daily stock return to its daily dollar trading volume multiplied by 10^8 in month $t-1$. Idiosyncratic volatility of stock returns (IVOL) is the standard deviation of the residuals of the daily stock excess returns regressed on daily market excess returns in month $t-1$.

Third, we construct variables related to options. Variance risk premium (VRP) is the difference between the average implied volatility of ATM short-term options (with moneyness between 0.95–1.05 and 10–60 day-to-maturity) and the annualized last-quarter's daily stock return standard deviation observed at the end of month $t-1$. Goyal and Saretto (2009) show that VRP predicts future option returns. Volatility of volatility (VOV), which predicts option returns (Cao, Vasquez, Xiao, and Zhan (2023)), is calculated following Baltussen, Van Bekkum, and Van Der Grient (2018) by scaling the standard deviation of ATM short-term option implied volatility by the average ATM short-term option implied volatility over month $t-1$. Risk-neutral skewness (RNS) at the end of month $t-1$ is calculated using OTM call and put option prices following Bakshi, Kapadia, and Madan (2003). Option demand is computed as the log difference between the total market value of all options and the market value of underlying stocks.

Fourth, we construct six systematic risk exposure measures, three of which are exposed to systematic left-tail risk. These beta exposures are computed using 60-month rolling windows up to month $t-1$ where we regress the bear spread return of each firm on each of the systematic risk measures. The beta exposures to the systematic risk measures are β_{BEAR}, the beta exposure to the bear market risk calculated following Lu and Murray (2019); β_{STRAD}, the beta exposure to the zero-beta straddle return of the S&P 500 computed as in Coval and Shumway (2001); β_{JUMP} and β_{VOL}, the beta exposures to the market jump and market diffusive volatility calculated following Cremers, Halling, and Weinbaum (2015); β_{TAIL}, the beta exposure to the tail-risk factor calculated following Kelly and Jiang (2014); and β_{DOWNSIDE}, the beta exposure to the downside risk factor calculated following Ang, Chen, and Xing (2006). In Section C of the Supplementary Material, we describe the construction of each of these beta exposures in detail.

From option prices in the bear spread strategy, we can compute the Arrow–Debreu state price (AD_PRICE) of left-tail events (Lu and Murray (2019)). If we scale the option positions in the bear spread by K_1-K_2, we get a price of $(P_1-P_2)/(K_1-K_2)$ and a payoff of \$1 when $S_T<K_2$. Therefore, the price of the scaled bear spread should be equal to $e^{-rT}\widehat{\mathbb{E}}\left[\mathbf{1}_{\{S_T<K_2\}}\right]$, where $\mathbf{1}$ is the indicator function and $\widehat{\mathbb{E}}$ represents the expected value under the risk-neutral probability.

The scaled bear spread price can be interpreted as the discounted risk-neutral state probability of left-tail events.

We also report the quoted half bid–ask spread (BA_SPREAD) of the bear spread strategy. Since the bear spread involves buying an OTM put option and selling a DOTM put option, we compute its half bid–ask spread as

$$[(\mathrm{PUT}_{1,\mathrm{ASK}} - \mathrm{PUT}_{1,\mathrm{MID}}) + (\mathrm{PUT}_{2,\mathrm{MID}} - \mathrm{PUT}_{2,\mathrm{BID}})]/(\mathrm{PUT}_{1,\mathrm{MID}} - \mathrm{PUT}_{2,\mathrm{MID}}).$$

E. Summary Statistics

Table 1 reports summary statistics of variables in Panel A and characteristics of decile portfolios in Panel B. The cross-sectional correlation matrix is reported in Table A2 in the Supplementary Material.

In Panel A of Table 1, both the mean and median of delta-hedged bear spread returns are negative, consistent with the negative risk premium carried by the bear spreads as they provide insurance against left-tail risk. The 75th percentile is 6.77%, indicating that at least 25% of return observations are positive. PUT_1 and PUT_2 have median deltas of −0.304 and − 0.116 with moderate standard deviations, suggesting a satisfactory selection of option pairs in our bear spread sample. PUT_1's mean implied volatility is 48%, which is smaller than PUT_2's mean implied volatility of 53%. This is consistent with the typical shape of the option skew for put options (Xing, Zhang, and Zhao (2010)). VAR5 (VAR1) has a mean of 4.2% (7.0%), implying that on average there is a 5% (1%) probability that the daily loss that a firm experiences over the following day is 4.2% (7.0%) or higher. ES5 (ES1) has a mean of 6.2% (9.3%), which is larger than the mean of VAR5 (VAR1), as expected.

As for the summary statistics of the control variables, the median momentum and reversal are of similar magnitude to that reported by Byun and Kim (2016). The medians of illiquidity and idiosyncratic volatility are also similar to those reported by Cao and Han (2013) and Atilgan et al. (2020). The average AD_PRICE (the Arrow–Debreu security price implied by the bear spread strategy) is 21 cents, which is the crash insurance premium paid by investors to receive one dollar when extreme negative returns occur. The beta exposures to systematic tail-risk factors are, on average, positive for bear market risk, zero-beta market straddles, jump risk, and volatility market risk, and are negative for the market tail-risk factor and the downside risk factor.

In Panel B of Table 1, we present the mean values of characteristics across decile portfolios sorted by VAR5. VAR5 increases from 2.1% for decile 1 to 7.5% for decile 10. Firm size decreases from decile 1 to 10, while book-to-market, momentum, reversal, idiosyncratic volatility, and kurtosis increase from decile 1 to 10. The quoted half bid–ask spread is similar across decile portfolios and to its mean at around 22%. As expected, AD_PRICE increases from 17.6 cents for decile 1 to 26.1 cents for decile 10. As left-tail risk increases, the price of crash insurance against bear states becomes more expensive. Note that our article focuses on crash insurance returns, not prices.

Finally, beta exposures to the market bear spread, the jump factor, the volatility factor, and the tail factor report a monotonic pattern across decile portfolios.

TABLE 1

Summary Statistics

Panel A of Table 1 presents descriptive statistics for delta-hedged bear spread returns (in %), characteristics of put options in the bear spreads, the left-tail risk measures, and other variables that include control variables. Panel B reports the characteristics of 10 portfolios sorted by the 5% value-at-risk, VAR5, that corresponds to −1 times the 5th percentile of daily returns in the past year. The left-tail risk measures are the 5% (1%) value-at-risk, VAR5 (VAR1), that corresponds to 5th (1st) percentile of daily returns in the past year, and the expected shortfall, ES5 (ES1), is calculated as −1 times the average of the returns below the 5th (1st) percentile of daily returns in the past year. The characteristics are SIZE (market capitalization), BTM (book-to-market ratio), DTA (firm leverage), MOM (momentum computed as the return over the previous 6 months), REV (reversal which is the return over the previous month), ILLIQ (logarithm of Amihud illiquidity), IVOL (idiosyncratic volatility), and SKEW and KURT (skewness and kurtosis from 1 year of daily returns). We use 60-month rolling windows to compute the exposures of individual bear spread returns to the following systematic risk factors: the bear market factor (β_{BEAR}) following Lu and Murray (2019), zero-beta straddle (β_{STRAD}) from Coval and Shumway (2001), jump and volatility factors (β_{JUMP} and β_{VOL}) as in Cremers et al. (2015), the tail factor (β_{TAIL}) as in Kelly and Jiang (2014), and the downside factor (β_{DOWN}) following Ang et al. (2006). We also report the AD_PRICE (the Arrow–Debreu security price implied by bear spread strategy) and HALF_BA_SPREAD (the quoted half bid–ask spread of bear spread computed as $[(PUT_{1,ASK} - PUT_{1,MID}) + (PUT_{2,MID} - PUT_{2,BID})]/(PUT_{1,MID} - PUT_{2,MID})$). Statistics are computed as the time-series averages of the monthly cross-sectional means, standard deviations, and percentiles. The sample period is from Jan. 1996 to Dec. 2017 for stocks in the OptionMetrics database.

Panel A. Summary Statistics

Variables	Mean	Std. Dev.	25th	Median	75th
Delta–hedged bear spread returns					
	−0.166	13.224	−8.556	−1.740	6.775
Option characteristics					
PUT_1					
Delta	−0.304	0.047	−0.339	−0.304	−0.268
Implied volatility	0.476	0.204	0.333	0.439	0.574
PUT_2					
Delta	−0.116	0.032	−0.136	−0.111	−0.092
Implied volatility	0.527	0.221	0.374	0.487	0.629
Left–tail risk variables					
VAR5	0.042	0.016	0.030	0.040	0.052
VAR1	0.070	0.028	0.049	0.064	0.085
ES5	0.062	0.024	0.043	0.057	0.075
ES1	0.093	0.044	0.061	0.083	0.113
Other variables					
SIZE	22.230	1.576	21.094	22.179	23.303
BTM	4.507	49.701	0.349	0.689	1.427
DTA	0.185	0.194	0.023	0.143	0.281
MOM	0.151	0.338	−0.041	0.126	0.306
REV	0.028	0.133	−0.045	0.020	0.090
IILLQ	−7.960	1.563	−9.033	−8.017	−6.886
IVOL	0.022	0.013	0.013	0.019	0.026
SKEW	0.235	1.217	−0.176	0.181	0.590
KURT	8.605	11.117	4.156	5.464	8.635
β_{BEAR}	0.155	1.043	−0.356	0.097	0.624
β_{STRAD}	0.042	0.099	−0.002	0.041	0.085
β_{JUMP}	0.052	0.464	−0.134	0.048	0.232
β_{VOL}	0.027	0.825	−0.277	0.021	0.337
β_{TAIL}	−0.045	0.949	−0.462	−0.023	0.400
β_{DOWN}	−0.604	2.981	−1.203	−0.621	−0.018
AD_PRICE	0.210	0.054	0.174	0.206	0.241
HALF_BA_SPREAD	0.225	0.267	0.104	0.173	0.277

Panel B. Characteristics of Decile Portfolios

Variables	1	2	3	4	5	6	7	8	9	10
VAR5	0.021	0.026	0.029	0.033	0.037	0.041	0.046	0.051	0.059	0.075
SIZE	23.551	23.308	23.023	22.674	22.371	22.091	21.849	21.633	21.348	20.865
BTM	2.628	3.148	3.598	3.352	3.022	2.974	2.809	3.208	6.540	11.399
DTA	0.212	0.201	0.196	0.190	0.183	0.182	0.173	0.172	0.172	0.165
MOM	0.106	0.112	0.116	0.122	0.138	0.151	0.165	0.168	0.180	0.237
REV	0.021	0.022	0.025	0.026	0.032	0.034	0.034	0.040	0.038	0.050
ILLIQ	−9.072	−8.798	−8.557	−8.256	−8.018	−7.791	−7.563	−7.390	−7.150	−6.847
IVOL	0.011	0.013	0.015	0.017	0.019	0.021	0.023	0.026	0.030	0.037
SKEW	0.194	0.151	0.134	0.121	0.177	0.202	0.213	0.247	0.331	0.533
KURT	7.863	8.091	8.413	8.759	8.774	8.689	8.914	8.735	9.521	10.073
β_{BEAR}	0.090	0.083	0.119	0.110	0.133	0.144	0.172	0.198	0.205	0.301
β_{STRAD}	0.037	0.041	0.041	0.040	0.041	0.043	0.043	0.046	0.047	0.044
β_{JUMP}	0.032	0.043	0.046	0.029	0.033	0.049	0.062	0.061	0.063	0.104
β_{VOL}	0.026	0.021	0.031	0.011	0.013	0.002	−0.008	0.026	0.061	0.090
β_{TAIL}	−0.021	−0.002	−0.026	−0.009	−0.010	−0.033	−0.049	−0.105	−0.085	−0.108
β_{DOWN}	−0.486	−0.556	−0.567	−0.619	−0.732	−0.654	−0.670	−0.622	−0.681	−0.450
AD_PRICE	0.176	0.183	0.189	0.196	0.204	0.211	0.218	0.227	0.239	0.261
HALF_BA_SPREAD	0.242	0.218	0.222	0.217	0.217	0.223	0.225	0.223	0.225	0.233

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In Fama–MacBeth (1973) regressions and double sorts, we show that this monotonic relation does not subsume the predictability of VAR5 on bear spread returns.

Table A2 in the Supplementary Material reports the cross-sectional correlations of firm characteristics. The correlation among the four left-tail measures is high at above 75%. VAR5, the main left-tail measure used in our article, displays a high correlation with firm size (−51%), Amihud (2002) illiquidity (43%), idiosyncratic volatility (60.5%), and AD_PRICE (44%). The correlations (in absolute value) with the relative bid–ask spread, variance risk premium, and beta exposures to systematic left tail risk are below 10%.

In summary, we show that left-tail risk measures appear to be related to control variables including firm size, illiquidity, and idiosyncratic volatility. In Section III, we attempt to establish a cross-sectional relation between left-tail risk and bear spread returns.

III. Empirical Analysis

A. Univariate Portfolio Analysis

We conduct univariate portfolio analysis to examine the relation between left-tail risk and delta-hedged bear spread returns. In month *t*, we form decile portfolios of delta-hedged bear spreads by sorting firms based on one of the left-tail risk measures: VAR1, VAR5, ES1, and ES5. Decile 10 (decile 1) contains delta-hedged bear spreads on stocks with the highest (lowest) left-tail risk.

We report dollar-open-interest-weighted (DOI-weighted) returns. Our results are robust for equal-weighted returns, as reported in Table A3 in the Supplementary Material. The DOI-weighted returns put more weight on option strategies with higher dollar value and liquidity (open interest). Following Cao and Han (2013) and Gao, Xing, and Zhang (2018), we compute the DOI weight on the bear spread formation date by multiplying the bear spread cost with the minimum open interests of the two put options comprising that bear spread: $\mathrm{DOI} = (\mathrm{PUT}_1 - \mathrm{PUT}_2) \times \min(\mathrm{OI}_{\mathrm{PUT}_1}, \mathrm{OI}_{\mathrm{PUT}_2})$.

Table 2 reports a systematic pattern in which returns of DOI-weighted delta-hedged bear spreads generally increase across decile portfolios. Portfolios with low left-tail risk (deciles 1 and 2) report negative returns, while those with high left-tail risk (deciles 9 and 10) exhibit positive returns. In the first row, decile 1 portfolio (with the lowest VAR5) has an average monthly return of −0.59%, while decile 10 portfolio (with the highest VAR5) has an average monthly return of 0.45%. The "10–1" monthly return spread is 1.04% (*t*-stat = 2.44), and it is economically significant. The option dollar open interest percentage of portfolios 1 and 10 represents 23.1% of the total DOI, as reported in Panel A of Table A4 in the Supplementary Material. If all firms were identical, portfolios 1 and 10 should represent 20% of the total DOI. The returns and alphas of portfolios sorted on VAR1, ES5, and ES1 exhibit a similar pattern.

Crash insurance provided by bear spreads offers protection against stock price crashes. In theory, option traders should pay a fair price for such protection, accept a negative risk premium, and expect negative future returns. However, we find that high decile portfolios generate positive and higher returns than low decile portfolios

TABLE 2

Univariate Portfolio Analysis

Table 2 reports the time-series average monthly returns (in %) for the dollar-open-interest-weighted delta-hedged bear spread decile portfolios sorted on left-tail risk measures (VAR5, VAR1, ES5, and ES1), along with the return spreads ("10–1") between decile 10 and decile 1. Panel A reports the time-series average monthly returns for decile portfolios sorted by VAR5, along with their time-series standard deviation (in %), skewness, and kurtosis. Panel B reports the time-series average monthly returns for decile portfolios sorted by VAR1, ES5, and ES1. The left-tail risk measures are the 5% (1%) value-at-risk, VAR5 (VAR1), that corresponds to −1 times the 5th (1st) percentile of daily returns in the past year, and expected shortfall, ES5 (ES1), calculated as −1 times the average of the returns below the 5th (1st) percentile of daily returns in the past year. Each month t, decile portfolios of delta-hedged bear spreads are formed and held to maturity by sorting underlying stocks on one of the left-tail risk measures. The dollar open interest weight is calculated as the minimum of the open interests of the two puts in each bear spread, multiplied by the cost of the bear spread. Newey–West (1987) adjusted t-statistics are presented in parentheses. The sample period is from Jan. 1996 to Dec. 2017 for stocks in the OptionMetrics database.

Panel A. Sorted by VAR5

Statistics	1	2	3	4	5	6	7	8	9	10	10–1
Mean	−0.59	−0.38	−0.23	−0.18	0.16	0.06	0.12	−0.14	0.01	0.45	1.04
	(−3.99)	(−2.29)	(−1.24)	(−0.77)	(0.58)	(0.26)	(0.41)	(−0.40)	(0.04)	(1.02)	(2.44)
Std. Dev.	2.61	2.71	3.08	3.44	3.96	3.81	4.14	4.39	5.03	6.10	6.25
Skew	1.03	0.42	0.85	0.77	0.77	0.71	0.15	0.42	0.68	1.04	0.97
Kurt	5.74	4.49	4.13	4.86	4.60	3.95	5.03	3.85	3.61	5.72	5.72

Panel B. Sorted by VAR1, ES5, and ES1

	1	2	3	4	5	6	7	8	9	10	10–1
VAR1	−0.55	−0.53	−0.25	−0.05	0.33	0.13	−0.14	−0.15	0.52	0.27	0.83
	(−3.92)	(−3.23)	(−1.17)	(−0.23)	(1.20)	(0.44)	(−0.54)	(−0.52)	(1.43)	(0.71)	(2.26)
ES5	−0.53	−0.41	−0.40	0.10	−0.10	0.21	−0.10	0.09	0.12	0.46	0.99
	(−3.75)	(−2.67)	(−1.96)	(0.45)	(−0.42)	(0.67)	(−0.30)	(0.27)	(0.37)	(1.02)	(2.35)
ES1	−0.56	−0.38	−0.15	−0.04	0.07	−0.25	0.01	0.27	0.50	0.29	0.85
	(−3.96)	(−1.77)	(−0.86)	(−0.18)	(0.28)	(−0.93)	(0.02)	(0.88)	(1.36)	(0.80)	(2.61)

and the "10–1" returns are statistically and economically significant for all left-tail risk measures. These results indicate underpricing of bear spreads when left-tail risk is high.

We conclude that DOI- and equal-weighted bear spread returns are positively related to left-tail risk. For the remainder of the article, we report results only for VAR5 given the high correlation among the four left-tail risk measures and that VAR5 is the measure with a distribution closest to normal compared to the other measures (Atilgan et al. (2020)). In Section III.B, we study risk-adjusted returns of bear spread portfolios when sorting by VAR5.

B. Risk-Adjusted Bear Spread Returns

We now compute the risk-adjusted returns and risk exposures of the decile portfolios and the "10–1" portfolio. The positive relation between VAR5 and bear spread returns potentially represents the presence of market-wide risk, systematic left-tail risk, or option-based systematic risk.

We regress the returns of decile portfolios and the "10–1" DOI-weighted bear spread returns on various linear pricing models. We include traditional equity pricing models consisting of the Fama and French (1993) three factors and the Carhart (1997) momentum factor. We also include four systematic option factors: i) the aggregate volatility factor measured by the zero-beta S&P 500 index ATM straddle return from Coval and Shumway (2001), ii) the jump and iii) volatility factors calculated as in Cremers et al. (2015), and iv) the VIX volatility index.

To control for systematic left-tail risk, we risk-adjust the returns with the bear market factor (AD_BEAR) computed as in Lu and Murray (2019), the tail factor by Kelly and Jiang (2014), the downside factor of Ang et al. (2006), and a factor constructed using coskewness following Harvey and Siddique (2000). Finally, we control for factors that explain the cross section of option returns. These include the illiquidity factor from options by Zhan et al. (2022), and three factors from options (size, idiosyncratic volatility, and variance risk premium) as suggested by Horenstein, Vasquez, and Xiao (2022). The last four factors, not to be confused with the stock factors, are computed as the long-short return of decile delta-hedged option returns sorted on each characteristic. We also risk-adjust using the coskewness model of Vanden (2006), which uses the market return, the square of the market return, as well as the bear spread return of the S&P 500 and its square, and the product of the market return and the market bear spread return. Section B of the Supplementary Material contains a detailed description of the construction of these factors.

Panel A of Table 3 reports that the alphas of the "10–1" returns are all larger in magnitude than the "10–1" DOI-weighted bear spread return reported in Table 2. Panel B reports that only the exposures to market risk are consistently significant for all decile portfolios including the long-short portfolio. These exposures are all negative and that of decile 10 is the most negative among all deciles. In market downturns, the portfolio with the riskiest stocks in terms of left-tail risk, decile 10, would deliver the highest return.

We conclude that stock and option systematic factors, left-tail systematic risk factors, and factors from the cross section of option returns do not explain the "10–1" DOI-weighted bear spread returns. The alphas for all models are larger than the raw returns. The exposures of decile portfolios to systematic factors show no consistent patterns. Only the exposures to the market factor show a monotonic relation with bear spread decile portfolio returns. However, the CAPM alpha of the "10–1" portfolio remains positive and significant. Similar results are obtained for equal-weighted returns, as reported in Table A5 in the Supplementary Material.

C. Bivariate Portfolio Analysis

We investigate whether the positive relation between the underlying stocks' left-tail risk measures and future bear spread returns can be explained by firm characteristics using the bivariate portfolio sorting method. In month t, we form decile portfolios by sorting based on one of the control variables. Then, within each decile, we further sort based on the left-tail risk measure VAR5. Each left-tail risk decile portfolio is then averaged across the control variable deciles. Control variables are SIZE (market capitalization), BTM (book-to-market ratio), DTA (firm leverage), MOM (momentum computed as the return over the previous 6 months), REV (reversal, which is the return over the previous month), ILLIQ (logarithm of Amihud illiquidity), IVOL (idiosyncratic volatility), and SKEW and KURT (skewness and kurtosis from 1 year of daily returns).

Table 4 reports that the "10–1" portfolio returns and their corresponding alphas are positive and statistically significant for all control variables. The "10–1" 5-factor alphas are statistically significant and range from 0.65% to 1.36% per month. The results for equal-weighted returns display a similar pattern to the DOI-weighted returns and are reported in Table A6 in the Supplementary Material.

TABLE 3

Risk-Adjusted Returns and Exposures to Systematic Factors

Table 3 reports the risk-adjusted returns and the exposures to systematic factors for the delta-hedged bear spread decile portfolios sorted on the left-tail risk measure VAR5. Panel A reports the time-series average dollar-open-interest-weighted monthly returns and alphas (in %) for the delta-hedged bear spread decile portfolios, along with the return spreads and the associated alpha spreads ("10–1") between decile 10 and decile 1. Panel B reports the post-formation exposures to the systematic left-tail risk measures of each delta-hedged bear spread decile portfolio sorted by VAR5 and the associated difference between the post-formation betas ("10–1") of decile 10 and decile 1 portfolios. The post-formation beta exposures are calculated from a regression of the delta-hedged bear spread portfolio returns on the contemporaneous market return and each factor return. VAR5 is the 5% value-at-risk that corresponds to −1 times the 5th percentile of daily returns in the past year. CAPM alphas are calculated after adjusting for CAPM market risk factor; 4-factor (4F) alphas are calculated after adjusting for Fama–French three factors and Carhart (1997) momentum factor; the ZB Straddle is the zero-beta straddle return from Coval and Shumway (2001); the jump and volatility factors (JUMP and VOL) are computed as in Cremers et al. (2015); the bear market factor (AD_BEAR) is calculated following Lu and Murray (2019); VIX is the monthly return of the VIX volatility index; the Tail factor is calculated following Kelly and Jiang (2014); Downside is calculated following Ang et al. (2006); Coskewness factor (COSKEW) is computed following Harvey and Siddique (2000); Illiquidity factor on options is calculated following Zhan et al. (2022); the three factors from options are the size, idiosyncratic volatility, and variance risk premium factors from delta-hedged option returns calculated following Horenstein et al. (2022); Vanden's alpha is computed following Vanden (2006). Newey–West (1987) adjusted *t*-statistics are presented in parentheses. The sample period is from Jan. 1996 to Dec. 2017 for stocks in the OptionMetrics database.

Panel A. Risk-Adjusted Returns

Alphas	1	2	3	4	5	6	7	8	9	10	10–1	(*t*-Stat)
Raw return	−0.59	−0.38	−0.23	−0.18	0.16	0.06	0.12	−0.14	0.01	0.45	1.04	(2.44)
CAPM alpha	−0.47	−0.28	−0.10	−0.03	0.38	0.21	0.30	0.01	0.22	0.80	1.28	(3.02)
4F alpha	−0.50	−0.44	−0.16	−0.09	0.23	0.13	0.13	−0.20	0.03	0.86	1.35	(3.16)
4F + ZB Straddle alpha	−0.34	−0.28	0.00	0.08	0.39	0.27	0.23	−0.05	0.21	0.97	1.31	(3.15)
4F + Jump + Vol alpha	−0.41	−0.35	−0.10	0.06	0.41	0.17	0.26	−0.09	0.19	0.91	1.31	(3.13)
4F + AD_BEAR alpha	−0.45	−0.42	−0.06	−0.05	0.37	0.08	0.27	−0.16	0.23	0.94	1.39	(3.26)
4F + VIX alpha	−0.49	−0.42	−0.10	−0.04	0.31	0.20	0.15	−0.18	0.14	0.84	1.33	(3.15)
4F + Tail factor alpha	−0.49	−0.44	−0.14	−0.15	0.25	0.12	0.06	−0.23	0.03	0.83	1.32	(3.09)
4F + Downside alpha	−0.46	−0.40	−0.13	−0.07	0.29	0.13	0.16	−0.16	0.07	0.96	1.43	(3.28)
4F + Coskew alpha	−0.54	−0.43	−0.18	−0.07	0.25	0.12	0.11	−0.19	0.04	0.86	1.39	(3.27)
4F + Illiquidity alpha	−0.59	−0.76	−0.35	−0.45	0.04	0.11	−0.08	−0.07	0.07	1.24	1.83	(2.41)
4F + 3F Option's alpha	−0.18	−0.38	0.45	−0.45	0.61	0.45	0.01	0.23	0.33	2.10	2.28	(2.33)
All factors' alpha	0.21	−0.02	0.70	−0.02	1.12	0.46	0.58	0.88	0.71	3.12	2.90	(2.70)
Vanden's alpha	−1.09	−1.12	−0.54	−1.06	−0.39	−0.61	−0.07	−0.65	−0.29	0.30	1.39	(2.37)

Panel B. Exposures to Systematic Factors

Exposures	1	2	3	4	5	6	7	8	9	10	10–1
MKT	−0.177	−0.161	−0.195	−0.222	−0.320	−0.222	−0.272	−0.228	−0.308	−0.534	−0.357
	(−3.10)	(−2.43)	(−2.97)	(−2.28)	(−3.41)	(−3.37)	(−2.93)	(−2.63)	(−2.43)	(−4.47)	(−2.95)
ZB_ STRADDLE	0.021	0.020	0.016	0.019	0.022	0.011	0.019	0.026	0.017	0.023	0.002
	(3.69)	(3.80)	(2.45)	(3.23)	(3.01)	(1.57)	(2.50)	(2.69)	(2.02)	(2.31)	(0.19)
JUMP	0.020	0.023	0.008	0.038	0.048	0.006	0.041	0.033	0.045	0.007	−0.014
	(1.69)	(2.01)	(0.58)	(2.04)	(2.22)	(0.30)	(1.74)	(0.92)	(1.84)	(0.22)	(−0.42)
VOL	0.051	0.045	0.065	0.045	0.052	0.031	0.049	0.034	0.079	0.050	−0.001
	(2.18)	(1.94)	(2.32)	(1.34)	(1.58)	(0.94)	(2.00)	(0.74)	(1.94)	(0.88)	(−0.02)
AD_BEAR	0.052	0.006	0.115	0.046	0.151	−0.066	0.149	0.031	0.216	0.102	0.050
	(0.95)	(0.13)	(1.99)	(0.69)	(2.16)	(−0.98)	(1.94)	(0.41)	(2.13)	(1.12)	(0.61)
VIX	0.047	0.059	0.167	0.153	0.247	0.210	0.084	0.103	0.294	−0.033	−0.079
	(0.79)	(0.81)	(3.31)	(1.90)	(2.84)	(2.43)	(1.06)	(0.94)	(3.08)	(−0.27)	(−0.71)
TAIL	−0.037	−0.026	−0.057	0.068	−0.086	−0.001	0.093	−0.006	−0.043	0.065	0.102
	(−0.96)	(−0.62)	(−1.66)	(1.50)	(−2.05)	(−0.02)	(2.11)	(−0.09)	(−0.57)	(0.93)	(1.18)
DOWNSIDE	0.058	0.032	0.048	0.039	0.082	−0.004	0.010	0.025	0.024	0.164	0.106
	(2.47)	(1.14)	(1.83)	(1.11)	(2.60)	(−0.11)	(0.36)	(0.68)	(0.52)	(3.32)	(1.87)
COSKEW	−0.071	−0.014	−0.033	−0.013	−0.023	−0.044	−0.017	−0.012	−0.002	0.001	0.071
	(−2.02)	(−0.45)	(−0.93)	(−0.22)	(−0.32)	(−0.69)	(−0.39)	(−0.20)	(−0.04)	(0.01)	(0.74)

The results indicate that after controlling for firm characteristics, the strong positive relation between firms' left-tail risk measure VAR5 and future returns of delta-hedged bear spreads remains. The return predictability of VAR5 cannot be explained by control variables commonly used in the literature nor by bear spread exposures to systematic and left-tail risk factors.

TABLE 4

Bivariate Portfolio Analysis: Dollar-Open-Interest-Weighted

Table 4 presents results (in %) for dollar-open-interest- (DOI-) weighted delta-hedged bear spread portfolios based on bivariate dependent sorts of one characteristic variable and VAR5. In month t, decile portfolios of delta-hedged bear spreads are formed by sorting underlying stocks based on one of the characteristic variables. Then, within each decile, additional decile portfolios of delta-hedged bear spreads are formed by sorting underlying stocks based on VAR5. Each VAR5 decile portfolio is then averaged over the control characteristic deciles. This table reports the DOI-weighted returns for each decile portfolio, the "10–1" return spread, and the associated 5-factor alpha spreads (5F_ALPHA) of the "10–1" portfolio. 5F_ALPHA is calculated after adjusting for Fama–French three factors, Carhart (1997) momentum factor, and Coval and Shumway (2001) systematic volatility factor. VAR5 is the 5% value-at-risk that corresponds to -1 times the 5th percentile of daily returns in the past year. Firm characteristic and beta control variables include SIZE (market capitalization), BTM (book-to-market ratio), DTA (firm leverage), MOM (momentum computed as the return over the previous 6 months), REV (reversal is the return over the previous month), ILLIQ (logarithm of Amihud illiquidity), IVOL (idiosyncratic volatility), and SKEW and KURT (skewness and kurtosis from 1 year of daily returns). We use 60-month rolling windows to compute the exposures of individual bear spread returns to the following systematic risk factors: the bear market factor (β_{BEAR}) following Lu and Murray (2019), zero-beta straddle (β_{STRAD}) from Coval and Shumway (2001), jump and volatility factors (β_{JUMP} and β_{VOL}) as in Cremers et al. (2015), the tail factor (β_{TAIL}) as in Kelly and Jiang (2014), and the downside factor (β_{DOWN}) following Ang et al. (2006). Newey–West (1987) adjusted t-statistics are presented in parentheses. The sample period is from Jan. 1996 to Dec. 2017 for stocks in the OptionMetrics database.

Variables	1	2	3	4	5	6	7	8	9	10	10–1	(t-Stat)	5F_ALPHA	(t-Stat)
SIZE	–0.76	–0.41	–0.28	–0.30	–0.39	–0.23	–0.11	0.08	0.17	0.21	0.97	(3.26)	1.07	(3.62)
BTM	–0.42	–0.38	0.01	0.04	–0.09	0.03	–0.12	0.12	0.08	0.28	0.50	(1.49)	0.65	(2.18)
DTA	–0.54	–0.20	0.06	–0.28	0.00	0.42	0.07	–0.01	0.08	0.18	0.62	(2.27)	0.68	(2.54)
MOM	–0.29	–0.32	–0.13	–0.14	–0.40	0.14	–0.08	–0.02	–0.26	0.56	0.85	(2.45)	1.04	(3.02)
REV	–0.45	–0.40	–0.17	–0.22	0.27	–0.21	0.12	0.01	0.07	0.60	1.06	(2.91)	1.17	(3.31)
ILLIQ	–0.90	–0.14	–0.42	–0.37	–0.28	0.06	–0.11	0.13	0.15	0.26	1.16	(3.44)	1.26	(3.92)
IVOL	–0.46	0.06	–0.32	–0.14	–0.07	–0.04	–0.27	–0.29	0.12	0.57	0.58	(2.42)	0.81	(3.11)
SKEW	–0.35	–0.27	–0.40	–0.16	0.05	0.04	0.14	0.03	0.01	0.47	0.82	(1.94)	1.09	(2.83)
KURT	–0.42	–0.29	–0.32	–0.25	0.01	0.35	0.14	0.13	0.06	0.21	0.63	(1.60)	0.96	(2.60)
β_{BEAR}	–0.57	–0.58	–0.42	–0.19	–0.26	–0.08	–0.17	–0.34	–0.39	0.26	0.82	(2.19)	1.20	(3.34)
β_{STRAD}	–0.59	–0.41	–0.26	–0.31	–0.30	–0.07	–0.37	–0.23	–0.26	0.14	0.73	(1.96)	1.04	(2.79)
β_{JUMP}	–0.49	–0.63	–0.33	–0.33	–0.01	–0.19	–0.20	–0.06	–0.44	0.05	0.55	(1.43)	0.93	(2.45)
β_{VOL}	–0.57	–0.31	–0.47	–0.47	0.18	–0.15	–0.32	–0.43	–0.44	0.28	0.86	(2.35)	1.36	(3.88)
β_{TAIL}	–0.51	–0.61	–0.21	–0.16	–0.39	–0.04	–0.24	–0.41	–0.10	0.12	0.63	(1.70)	0.78	(2.15)
β_{DOWN}	–0.60	–0.46	–0.52	–0.17	–0.42	–0.27	0.06	0.03	–0.56	0.11	0.71	(2.03)	1.12	(3.32)

D. MacBeth Regressions

We perform Fama–MacBeth (1973) regressions to formally test the positive cross-sectional relation between VAR5 and future bear spread returns. The dependent variable is the delta-hedged bear-spread monthly return formed in month t, and the variable of interest is VAR5. We use the same variables from the double sorting analysis.

Table 5 presents the regression results. In the first row, we perform a univariate regression on VAR5. The coefficient is 0.201 (t-stat = 4.58),

TABLE 5

Fama–MacBeth Regressions

Table 5 presents the results of Fama–MacBeth (1973) regressions of monthly delta-hedged bear spread returns on VAR5 and control variables. VAR5 is the 5% value-at-risk that corresponds to −1 times the 5th percentile of daily returns in the past year. Control variables are SIZE (market capitalization), BTM (book-to-market ratio), DTA (firm leverage), MOM (momentum computed as the return over the previous 6 months), REV (reversal is the return over the previous month), ILLIQ (logarithm of Amihud illiquidity), IVOL (idiosyncratic volatility), and SKEW and KURT (skewness and kurtosis from 1 year of daily returns). We use 60-month rolling windows to compute the exposures of individual bear spread returns to the following systematic risk factors: the bear market factor (β_{BEAR}) following Lu and Murray (2019), zero-beta straddle (β_{STRAD}) from Coval and Shumway (2001), jump and volatility factors (β_{JUMP} and β_{VOL}) as in Cremers et al. (2015), the tail factor (β_{TAIL}) as in Kelly and Jiang (2014), and the downside factor (β_{DOWN}) following Ang et al. (2006). The first 2 columns report univariate and bivariate regressions of delta-hedged bear spread returns on VAR5 and one control variable. The last 2 columns report the results of the multivariate regressions of delta-hedged bear spread returns on VAR5 and all the control variables using the full sample and the subsample with beta exposure measures. Coefficients are time-series averages, and the associated Newey–West (1987) t-statistics are reported in parentheses. The sample period is from Jan. 1996 to Dec. 2017 for stocks in the OptionMetrics database.

	Univariate/Bivariate Regressions			Multivariate Regressions	
Variables	Coefficient on VAR5	Coefficient on Control	Adj. R^2		
VAR5	0.201 (4.58)		1.32%	0.158 (3.38)	0.163 (3.05)
SIZE	0.210 (4.64)	0.0002 (0.82)	1.47%	−0.003 (−3.74)	−0.002 (−2.24)
BTM	0.118 (2.63)	0.0001 (1.04)	1.33%	0.0001 (0.58)	0.0001 (1.63)
DTA	0.128 (2.74)	−0.002 (−1.16)	1.41%	−0.002 (−1.13)	0.0001 (0.00)
MOM	0.172 (3.98)	−0.002 (−1.43)	1.90%	−0.001 (−0.85)	−0.004 (−1.76)
REV	0.186 (4.26)	−0.004 (−1.03)	1.83%	−0.001 (−0.15)	−0.005 (−1.51)
ILLIQ	0.239 (5.13)	−0.001 (−3.55)	1.54%	−0.003 (−4.44)	−0.002 (−2.79)
IVOL	0.235 (5.11)	−0.113 (−2.85)	1.81%	−0.135 (−3.25)	−0.205 (−4.58)
SKEW	0.182 (4.18)	−0.001 (−1.80)	1.50%	0.0002 (0.52)	0.0005 (0.97)
KURT	0.183 (4.15)	−0.0001 (−2.53)	1.53%	0.00001 (−0.58)	0.00001 (0.87)
β_{BEAR}	0.141 (2.78)	0.001 (2.12)	1.93%		0.001 (1.18)
β_{STRAD}	0.163 (3.00)	−0.002 (−0.43)	1.64%		0.0003 (0.06)
β_{JUMP}	0.159 (2.91)	0.0001 (0.07)	1.69%		−0.002 (−1.75)
β_{VOL}	0.159 (2.92)	0.0003 (0.60)	1.80%		0.001 (1.13)
β_{TAIL}	0.163 (2.98)	0.001 (1.89)	1.72%		0.001 (2.17)
β_{DOWN}	0.164 (2.99)	0.0004 (1.15)	1.70%		0.001 (1.94)
Adj. R^2				3.14%	4.32%

confirming the positive relation between VAR5 and future delta-hedged bear spread returns. In bivariate regressions with control variables, the coefficient of VAR5 decreases in most cases but remains positive and statistically significant. In the last 2 columns, we perform multivariate regressions on VAR5 and all control variables and risk exposures. The coefficient on VAR5 remains positive and statistically significant in all models.

The results in Table 5 indicate a strong and robust positive relation between firms' left-tail risk and future delta-hedged bear spread returns after controlling for various combinations of control variables. We also show that beta exposures of bear spread returns to systematic left-tail risk factors do not explain our results.

IV. Potential Explanations

Above, we document a positive relation between firms' left-tail risk and the returns of crash risk insurance measured with bear spreads. Our results cannot be explained by risk-based factor models, beta exposures to systematic left-tail risk or traditional control variables. In this section, we investigate potential explanations of our findings. The main potential explanations are reported below, and the remaining ones are reported in the Supplementary Material.

A. Volatility Underreaction

Underreaction to volatility is widely documented in the literature. Poteshman (2001) documents that investors underreact to individual daily changes in instantaneous variance and more so when there are daily changes with the opposite sign. Theoretical support is provided by Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (2001), and Hong and Stein (1999). Lochstoer and Muir (2022) find that expectations about volatility underreact to news about volatility. Cheng (2020) documents that volatility markets underreacted in the early stages of the COVID-19 pandemic. Trading strategies based on this underreaction profited from the subsequent increase in volatility.

In this analysis, we produce statistical estimates of DOTM and OTM implied volatilities to generate ex ante forecasts of bear spread prices that we compare with the actual observed prices in the market. Note that this statistical forecast can be generated by the investor before setting up the trade and contains no forward-looking bias. We expect our forecasted prices to be lower than those observed in the market since we do not include any risk premium in the forecasted price. By comparing our statistical forecasts with actual prices, we aim to determine whether there is potential volatility underreaction.

We construct our statistical forecasts of bear spread prices in two steps. We first estimate the ATM implied volatility by forecasting the underlying stocks' realized volatility over the options' life. This implied volatility forecast is the lower bound of what this volatility should be since we do not include a volatility risk premium. Next, we estimate the option implied volatility skews of the OTM and DOTM puts relative to the ATM options. Adding the estimated ATM volatility to the estimated

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option skews, we obtain estimated OTM and DOTM implied volatilities to compute statistical forecasts of the bear spread price.[3]

First, statistical forecasts for the ATM implied volatility are obtained from forecasting physical volatility for each stock $E_t[\mathrm{RV}_{t+1}]$. We use the heterogeneous autoregressive model of Corsi (2009). We regress 1-month realized volatilities onto lagged realized volatilities computed over different frequencies:

$$\mathrm{RV}^M_{t,t+1} = \gamma_{0,i} + \gamma_{1,i}\mathrm{RV}^M_{t-1,t} + \gamma_{2,i}\mathrm{RV}^W_t + \gamma_{3,i}\mathrm{RV}^D_t + \epsilon_{t,t+1},$$

where $\mathrm{RV}^M_{t-1,t}$ is the realized volatility over the past month, and RV^W_t and RV^D_t denote realized volatilities over the past week and day, respectively. We estimate the coefficients using a 5-year rolling window. Each month, we estimate the model up to date t and estimate $E_t\left[\mathrm{RV}^M_{t+1}\right]$, the out-of-sample forecast of the 1-month-ahead realized volatility. We use $E_t\left[\mathrm{RV}^M_{t+1}\right]$ as the statistical forecast of the ATM implied volatility $E_t[\mathrm{IV}_{\mathrm{ATM},t+1}]$.

Second, we estimate the OTM and DOTM option skews with an ARMA(2,2) model using daily data over 5 years.[4] We first calculate the OTM (DOTM) option skews as the difference between OTM (DOTM) put implied volatility and the ATM option implied volatility, $\mathrm{IV}_{\mathrm{OTM}} - \mathrm{IV}_{\mathrm{ATM}}$ $(\mathrm{IV}_{\mathrm{DOTM}} - \mathrm{IV}_{\mathrm{ATM}})$. We separately model both option skews, OTM and DOTM, to capture the slope as well as the convexity of the implied volatility surface. We obtain the ARMA(2,2) model parameter estimates for each month. Using the estimated parameters, we compute the 1-month-ahead option skew forecasts on day t as $E_t[\mathrm{IV}_{\mathrm{OTM},t+1} - \mathrm{IV}_{\mathrm{ATM},t+1}]$ and $E_t[\mathrm{IV}_{\mathrm{DOTM},t+1} - \mathrm{IV}_{\mathrm{ATM},t+1}]$. Option skews are reestimated for each observation day t using a 5-year rolling window.[5]

We compute statistical forecasts of bear spread prices using the estimated OTM and DOTM put implied volatilities by adding ATM implied volatilities to the estimated OTM and DOTM put option skews for each option observation day t.

Table 6 reports actual and statistical forecasts of bear spread prices along with their corresponding implied volatilities. Panel A displays the actual and estimated OTM and DOTM implied volatilities; Panel B reports the actual and estimated IV spreads computed as the difference between the DOTM and OTM option skews; Panel C presents the actual and estimated ATM implied volatilities; and Panel D includes the actual and estimated scaled bear spread prices.

The main takeaway from Table 6 is that fair statistical forecasts of scaled bear spread prices are higher than actual prices for high left-tail risk portfolios as reported in Panel D. In particular, statistical forecasts of bear spread prices for deciles 7–10 are above actual prices by as much as 6.84%. For example, decile 10 bear spread forecasted prices are 2.24% higher than actual prices. Note that these statistical forecasts are conservative since we do not include any volatility risk premium in our

[3]Using a naïve estimator based on the average of the OTM and DOTM implied volatilities of the last 3, 6, or 12 observations observed in the previous 3, 6, or 12 months produces similar results. Table A7 in the Supplementary Material reports these results.

[4]We work with ARMA(2,2) instead of other specifications like ARMA(1,1) since it produces the lowest root-mean-squared error.

[5]Our results hold when using a 1-year rolling window as reported in Table A8 in the Supplementary Material.

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TABLE 6

Volatility Underreaction

Table 6 reports the comparison of the statistical forecasts of scaled bear spread prices, $E_t[\text{AD\_PRICE}_{t+1}]$, and the actual scaled bear spread prices, AD_PRICE_t. Panel A reports actual OTM and DOTM put implied volatilities ($\text{IV}_{\text{OTM},t}$ and $\text{IV}_{\text{DOTM},t}$) and the estimated OTM and DOTM put implied volatilities ($E_t[\text{IV}_{\text{OTM},t+1}]$ and $E_t[\text{IV}_{\text{DOTM},t+1}]$). The estimated volatilities are obtained by adding the estimated volatilities in Panel C with the estimated OTM and the DOTM option skews defined as $E_t[\text{IV}_{\text{OTM},t+1} - \text{IV}_{\text{ATM},t+1}]$ and $E_t[\text{IV}_{\text{DOTM},t+1} - \text{IV}_{\text{ATM},t+1}]$ (not reported). Panel B reports the volatility spread obtained as the difference between DOTM and OTM put implied volatilities ($\text{SPREAD}_t = \text{IV}_{\text{DOTM},t} - \text{IV}_{\text{OTM},t}$). The expected volatility spread, $E_t[\text{SPREAD}_{t+1}]$, is the difference between the estimated DOTM and OTM option skews obtained from an ARMA (2,2) model using a 5-year rolling window of daily volatilities. Panel C reports ATM implied volatility, $\text{IV}_{\text{ATM},t}$, and the estimated volatility, $E_t[\text{IV}_{\text{ATM},t+1}]$, that is obtained from a heterogeneous autoregressive model using a 5-year rolling window of daily volatilities. Panel D reports the actual scaled bear spread price and the forecasted scaled bear spread price computed with the expected OTM and DOTM volatilities from Panel A. The sample period is from Jan. 1996 to Dec. 2017 for stocks in the OptionMetrics database.

Statistics	1	2	3	4	5	6	7	8	9	10
Panel A. OTM and DOTM IV Estimation										
$\text{IV}_{\text{OTM},t}$	0.243	0.286	0.315	0.346	0.375	0.408	0.445	0.491	0.550	0.687
$E_t[\text{IV}_{\text{OTM},t+1}]$	0.242	0.285	0.314	0.345	0.376	0.408	0.445	0.491	0.550	0.685
$\text{IV}_{\text{DOTM},t}$	0.285	0.331	0.362	0.395	0.426	0.462	0.501	0.550	0.614	0.765
$E_t[\text{IV}_{\text{DOTM},t+1}]$	0.287	0.333	0.365	0.397	0.429	0.457	0.500	0.544	0.612	0.758
Panel B. IV Spread Estimation										
SPREAD_t	0.042	0.045	0.046	0.049	0.051	0.053	0.056	0.059	0.064	0.078
$E_t[\text{SPREAD}_{t+1}]$	0.045	0.048	0.050	0.053	0.053	0.050	0.056	0.054	0.062	0.073
$E_t[\text{SPREAD}_{t+1}] - \text{SPREAD}_t$	0.32%	0.35%	0.39%	0.36%	0.21%	−0.34%	−0.02%	−0.51%	−0.13%	−0.52%
Panel C. ATM Volatility Estimation										
$\text{IV}_{\text{ATM},t}$	0.221	0.263	0.292	0.322	0.352	0.384	0.420	0.465	0.520	0.649
$E_t[\text{IV}_{\text{ATM},t+1}]$	0.216	0.260	0.290	0.318	0.348	0.383	0.424	0.468	0.530	0.653
$E_t[\text{IV}_{\text{ATM},t+1}] - \text{IV}_{\text{ATM},t}$	−0.55%	−0.37%	−0.28%	−0.37%	−0.36%	−0.19%	0.47%	0.33%	0.99%	0.42%
Panel D. Scaled Bear Spread Prices										
AD_PRICE_t	0.159	0.168	0.175	0.181	0.187	0.193	0.201	0.207	0.217	0.238
$E_t[\text{AD\_PRICE}_{t+1}]$	0.147	0.159	0.167	0.174	0.185	0.209	0.211	0.225	0.226	0.245
$E_t[\text{AD\_PRICE}_{t+1}]/\text{AD\_PRICE}_t - 1$	−6.04%	−4.28%	−4.32%	−3.42%	−2.04%	−4.55%	2.89%	6.84%	1.36%	2.24%

volatility estimates. As for low left-tail risk portfolios, they are overpriced compared with our statistical forecasts as one would expect. The underpricing of bear spread prices for high left-tail risk portfolios could come from the underestimation of IV spreads and/or ATM implied volatility. Although IV spreads are overestimated (Panel B), the underestimation of ATM implied volatility (Panel C) dominates, causing bear spread prices to be underpriced in deciles 7–10. Statistical forecasts of ATM implied volatility are higher than actual ATM volatilities for high left-tail risk portfolios. In summary, we conclude that the positive relation between left-tail risk and bear spread returns can potentially be explained by volatility underreaction for high left-tail risk portfolios.

B. Underreaction to Left-Tail Return Momentum

A behavioral explanation for the positive relation between firms' left-tail risk and future bear spread returns is that option traders underestimate the persistence of losses. In Table 7, we examine the transition matrix of the average 12-month-ahead left-tail risk portfolios. We compute the average probability that a firm in decile i (rows) in month t will be in decile j (columns) in month $t+12$.[6] If VAR5 were random across firms, all probabilities should be around 10%. However, left-tail risk

[6]We use a 12-month gap to avoid overlapping observations given that we compute value-at-risk using the previous 12 months of daily data.

TABLE 7

Transition Matrix

Table 7 presents transition probabilities (in %) for VAR5 at a lag of 12 months. At each month *t*, all stocks are sorted into deciles based on an ascending ordering of VAR5. The procedure is repeated in month *t* + 12. Decile 1 is the portfolio of stocks with the lowest value-at-risk, and decile 10 is the portfolio of stocks with the highest value-at-risk. For each VAR5 decile in month *t*, the percentage of stocks that fall into each of the month *t* + 12 VAR5 decile is calculated. Time-series averages of these transition probabilities are reported. Each row corresponds to a different month *t* VAR5 portfolio, and each column corresponds to a different month *t* + 12 VAR5 portfolio. The sample period is from Jan. 1996 to Dec. 2017 for stocks in the OptionMetrics database.

Statistics	1	2	3	4	5	6	7	8	9	10
Decile 1	57	23	10	5	3	1	1	0	0	0
2	25	30	21	13	6	3	1	1	0	0
3	11	23	24	18	11	6	3	2	1	0
4	5	14	21	20	17	11	7	4	2	1
5	2	6	13	19	20	16	11	7	4	2
6	0	3	6	13	19	20	16	11	7	4
7	0	1	3	7	13	18	21	17	13	7
8	0	0	1	3	7	14	19	23	19	12
9	0	0	0	1	3	8	14	22	27	23
Decile 10	0	0	0	0	1	3	6	13	26	51

is very persistent for deciles 1 and 10: 57% (51%) of the firms in decile 1 (10) remain in that decile 12 months later. Moreover, firms in decile 10 (1) have a 77% (80%) chance of being in deciles 9 and 10 (1 and 2) after 12 months. We conclude that left-tail risk is an extremely persistent characteristic in our option sample.

This persistence of left-tail risk combined with the anomalous relation between left-tail risk and bear spread returns indicates that investors may underestimate the persistence, which is documented in the transition matrix of Table 7. We analyze the impact of two effects to gauge whether investors underestimate the left-tail risk persistence: the left-tail momentum effect and the anchoring effect.

Atilgan et al. (2020) show that investors underestimate loss persistence and such underestimation contributes to the left-tail return momentum, a negative relation between firms' left-tail risk and expected equity returns. George and Hwang (2004) show that anchoring behavior helps explain loss momentum around the 52-week low, albeit with the anchoring effect being weaker than the 52-week high. Driessen, Lin, and Van Hemert (2013) show that option implied volatilities decrease when stock prices approach their 52-week low, suggesting that investors underestimate persistence in risk due to the anchoring bias. The anchoring effect alone cannot directly explain our main finding. We examine the combined impact of the anchoring effect and the left-tail momentum effect on bear spread returns to uncover potential investor underestimation of left-tail persistence.

To analyze the impact of these two effects, we construct two measures: i) ΔVAR5, change of VAR5, is the difference in VAR5 from month $t-1$ to month t. A positive ΔVAR5 indicates an increase in left-tail risk. ii) NL, nearness to the 52-week low, is the current stock price divided by the lowest stock price in the previous year. A lower NL indicates that the stock price is closer to its 52-week low. We expect that the positive relation between firms' left-tail risk and future bear spread returns is stronger when ΔVAR5 is positive or when NL is low.

Previous results from the multivariate Fama–MacBeth (1973) regression in Table 5 show that firm size is a significant predictor of bear spread returns in the presence of VAR5 and additional control variables. In addition, there is a highly

TABLE 8

Underreaction to Left-Tail Return Momentum

Table 8 presents the dollar-open-interest-weighted returns (in %) of delta-hedged bear spread decile portfolios dependently sorted on firm size, the monthly change in VAR5 (ΔVAR) in Panel A or the nearness to 52-week low (NL) in Panel B, and VAR5. Delta-hedged bear spread portfolios are first sorted into terciles by size, and then into ΔVAR > 0 and ΔVAR ≤ 0 groups or high/low NL groups. Then decile portfolios are formed based on VAR5 within each subsample. VAR5 is the 5% value-at-risk that corresponds to −1 times the 5th percentile of daily returns in the past year. ΔVAR is defined as the difference between VAR5 in month t and month $t-1$. NL is calculated as the previous month-end stock price divided by the minimum price in the previous year. The returns for decile 10 and decile 1 portfolios, the return spread ("10–1"), and its associated 5-factor alpha (5F_ALPHA) within each subsample are reported. 5F_ALPHA is calculated after adjusting for Fama–French three factors, Carhart (1997) momentum factor, and Coval and Shumway (2001) systematic volatility factor. Newey–West (1987) adjusted t-statistics are presented in parentheses. The sample period is from Jan. 1996 to Dec. 2017 for stocks in the OptionMetrics database.

Statistics	Small Size		Mid Size		Large Size	
Panel A. Sorts Based on ΔVAR						
	ΔVAR > 0	ΔVAR ≤ 0	ΔVAR > 0	ΔVAR ≤ 0	ΔVAR > 0	ΔVAR ≤ 0
1	−0.70	−0.27	−1.26	−0.50	−0.80	−0.33
10	1.35	0.26	0.85	0.15	−0.08	0.46
10–1	2.05	0.53	2.09	0.64	0.78	0.79
(t-Stat)	(2.09)	(0.88)	(3.69)	(1.47)	(1.98)	(1.85)
F alpha	1.81	0.57	2.52	0.86	0.83	0.47
(t-Stat)	(1.83)	(0.87)	(4.12)	(1.56)	(1.99)	(1.26)
Panel B. Sorts Based on Nearest to 52-Week Low (NL)						
	Low NL	High NL	Low NL	High NL	Low NL	High NL
1	−0.61	−0.45	−0.81	−0.58	−0.56	−0.62
10	2.51	−0.10	0.41	−0.01	0.40	0.04
10–1	3.13	0.35	1.22	0.57	0.96	0.66
(t-Stat)	(2.89)	(0.57)	(2.38)	(1.22)	(2.51)	(1.39)
F alpha	3.06	0.08	1.29	0.84	1.03	0.62
(t-Stat)	(2.83)	(0.10)	(2.22)	(1.50)	(2.50)	(1.39)

negative correlation between firm size and VAR5 of −51%. For this reason, we control for firm size in our subsequent analyses. To test the impact of the two effects, we first sort portfolios into terciles by size and then by ΔVAR5 (ΔVAR5 > 0 and ΔVAR5 ≤ 0) or by NL (Low NL and high NL). Then, within each subsample, we further sort bear spreads into deciles based on the left-tail risk measure VAR5.

Table 8 reports the DOI-weighted monthly returns for the highest and lowest VAR5 decile portfolios in each subsample, together with the return spreads ("10–1") and their corresponding 5-factor alphas. Newey–West (1987) adjusted t-statistics are reported in parentheses. Panel A presents results when sorting by ΔVAR5. When ΔVAR5 > 0, the "10–1" return spreads and the alphas are positive and statistically significant for all size levels. However, for ΔVAR5 ≤ 0, the "10–1" return spreads and alphas remain positive but are statistically insignificant in almost all cases. We conclude that the positive relation between VAR5 and future bear spread returns is only significant when ΔVAR5 > 0.

Panel A of Table 8 reports a strong and consistent positive relation between VAR5 and future delta-hedged bear spread returns when ΔVAR5 is positive. Since stocks that have experienced recent large losses are more likely to experience similar large losses in the near future (stock price's left-tail risk momentum), the protection provided by bear spreads on these stocks should be more valuable. However, option traders seem to underestimate the left-tail risk persistence and underprice bear spreads on stocks with high recent extreme losses, showing a left-tail risk momentum effect.

Panel B of Table 8 presents results across size terciles for two groups of NL, the nearest to the 52-week low price. Only the low NL subsamples report a positive and significant "10–1" returns and alphas across all size terciles. In addition, the "10–1" bear spread return is the largest for the small-size subgroup. In the high NL subsample, the "10–1" return spread and alphas are positive but insignificant.

Panel B of Table 8 reports that the underestimation of left-tail risk in the options market is stronger when the stock price is nearer to its 52-week low. Option traders anchor their loss expectation around the 52-week low and underestimate the persistence of stock price declines, leading to a stronger positive relation between firms' left-tail risk and future bear spread returns. These results are consistent with Driessen et al. (2013) because option traders' underestimation of the chance of downward breakthroughs leads to a stronger underpricing of bear spreads when stock prices approach their 52-week low, thus showing an anchoring effect.

Next, we provide evidence that the "10–1" significant alphas within subgroups are also economically significant. In our main results in Table 2, the "10–1" portfolio represents 23.1% of the total option dollar open interest percentage (% DOI). In Table 8, only two subgroups report significant "10–1" alphas: ΔVAR5 > 0 and Low NL. These two subgroups represent 51% and 48% of that 23.1% %DOI (Panel B of Table A4 in the Supplementary Material). We conclude that the bear spread mispricing is both statistically and economically significant across subgroups.

Overall, the results in Table 8 suggest that both the left-tail momentum effect and the anchoring effect have a strong impact on the underpricing of bear spreads. The existence of both effects indicates that one of the driving forces of the underreaction to firms' left-tail risk in the options market is option traders' underestimation of left-tail return momentum. Next, we study the impact of information uncertainty and investor sentiment on the relation between left-tail risk and bear spread returns. Zhang (2006a), (2006b) shows that information uncertainty exacerbates investors' underreaction. Baker and Wurgler (2006), Stambaugh et al. (2012), and Byun and Kim (2016) document that investor sentiment results in mispricing of risky assets.

1. Information Uncertainty

Prior literature (Hong et al. (2000), Jiang, Lee, and Zhang (2005), Zhang (2006a), (2006b), and Kumar (2009)) shows that information uncertainty amplifies investor behavioral biases. In particular, high information uncertainty may lead to investors reacting slowly to news (especially bad news), causing predictable price drift or momentum.

Following the literature, we construct five information uncertainty proxies: i) SIZE is the market capitalization; ii) AC is analyst coverage; iii) DISP, analysts' forecast dispersion, is the standard deviation of the analysts' forecasts scaled by the stock price in the previous quarter; iv) TURN is the stock return turnover; and v) AGE, firm age, denotes the number of years that a firm is listed on Compustat at the previous year-end. Zhang (2006b) uses all five proxies to measure information uncertainty. Hirshleifer and Teoh (2003) use firm size and analyst coverage as proxies for investor inattention. Kumar (2009) uses firm age to measure valuation uncertainty. Taking into account conceptual overlap and mixed interpretation of proxies between information uncertainty, investor inattention, and valuation uncertainty, we use small firm size, young firm age, low analyst coverage, and high dispersion in analyst forecasts as proxies of high information uncertainty. We expect that high information uncertainty amplifies the positive relation between left-tail risk and future bear spread returns.

Panel A of Table 9 reports the returns and alphas for deciles 1 and 10, and for the "10–1" portfolio across size quintiles. The results show that the return spreads

TABLE 9

Information Uncertainty and Disagreement

Table 9 presents the dollar-open-interest-weighted return (in %) comparisons between bear spread decile portfolios in subsamples sorted based on quintiles by size in Panel A, and in Panel B, information uncertainty and disagreement proxies: analyst coverage residual (AC_{RES}), firm age (AGE), analysts' forecast dispersion residual ($DISP_{RES}$), turnover residual ($TURN_{RES}$), and idiosyncratic volatility residual ($IVOL_{RES}$). AC_{RES}, $DISP_{RES}$, $TURN_{RES}$, and $IVOL_{RES}$ are calculated as the residuals from the cross-sectional regression of the corresponding values on the logarithm of firm's market capitalization in the previous quarter. Decile portfolios are formed based on the underlying stocks' VAR5 within each subsample. VAR5 is the 5% value-at-risk that corresponds to −1 times the 5th percentile of daily returns in the past year. Portfolio returns, the "10–1" return spread, and their associated 5-factor alpha are reported. 5F_ALPHA is calculated after adjusting for Fama–French three factors, Carhart (1997) momentum factor, and Coval and Shumway (2001) systematic volatility factor. Newey–West (1987) adjusted *t*-statistics are presented in parentheses. The sample period is from Jan. 1996 to Dec. 2017 for stocks in the OptionMetrics database.

Panel A. Size Versus VAR5

	Raw Return			5F_ALPHA		
Size Quintiles	1	10	10–1	1	10	10–1
Low 1	−0.68	1.10	1.78	−0.47	1.16	1.63
	(−2.84)	(1.26)	(1.98)	(−1.99)	(1.85)	(2.21)
2	−0.73	0.69	1.42	−0.16	1.30	1.46
	(−2.84)	(1.26)	(2.97)	(−0.56)	(2.08)	(2.37)
3	−0.64	0.56	1.21	−0.28	1.13	1.41
	(−3.28)	(1.18)	(2.21)	(−1.15)	(2.17)	(2.35)
4	−0.68	0.03	0.71	−0.31	0.40	0.71
	(−3.54)	(0.07)	(1.76)	(−1.33)	(1.03)	(1.73)
High 5	−0.54	0.00	0.54	−0.25	0.45	0.70
	(−2.71)	(0.00)	(1.46)	(−1.13)	(1.15)	(1.45)

Panel B. Other Measures of Information Uncertainty and Disagreement

	Raw Return			5F_ALPHA		
Subsample	1	10	10–1	1	10	10–1
LOW_AC_{RES}	−0.62	0.74	1.36	−0.33	1.19	1.52
	(−3.89)	(1.19)	(2.22)	(−2.16)	(1.97)	(2.67)
HIGH_AC_{RES}	−0.37	0.49	0.86	−0.14	1.00	1.14
	(−1.94)	(1.26)	(2.11)	(−0.82)	(2.47)	(2.69)
LOW_AGE	−0.43	1.50	1.93	−0.29	1.72	2.00
	(−2.04)	(3.41)	(4.40)	(−1.47)	(4.11)	(4.66)
HIGH_AGE	−0.73	−0.12	0.61	−0.49	0.46	0.95
	(−4.29)	(−0.09)	(1.26)	(−2.95)	(0.96)	(2.03)
LOW_$DISP_{RES}$	−0.49	−0.53	−0.04	−0.20	−0.10	0.10
	(−2.72)	(−1.52)	(−0.11)	(−1.03)	(−0.26)	(0.26)
HIGH_$DISP_{RES}$	−0.61	0.83	1.44	−0.49	1.40	1.89
	(−2.98)	(1.83)	(2.81)	(−2.13)	(2.78)	(3.23)
LOW_$TURN_{RES}$	−0.68	−0.65	0.02	−0.44	−0.24	0.21
	(−4.21)	(−1.88)	(0.06)	(−2.52)	(−0.78)	(0.56)
HIGH_$TURN_{RES}$	−0.30	0.86	1.17	−0.02	1.42	1.45
	(−1.48)	(1.81)	(2.55)	(−0.12)	(2.69)	(2.76)
LOW_$IVOL_{RES}$	−0.74	0.24	0.98	−0.51	0.45	0.95
	(−4.09)	(0.64)	(2.62)	(−2.64)	(1.26)	(2.52)
HIGH_$IVOL_{RES}$	−0.41	0.57	0.98	−0.21	1.11	1.32
	(−2.11)	(1.09)	(1.91)	(−1.10)	(1.94)	(2.44)

are more pronounced for the low size quintiles compared to the high size quintiles. More importantly, the results show that the "10–1" spread returns are positive and significant for all size quintiles except for the largest one. In addition, note that the P1 portfolio consistently earns average negative returns across size quintiles when buying crash insurance, as one would expect. However, P10 bear spread returns are positive for all size quintiles. In addition, the average positive return on P10 shrinks when we move from size quintile 1 to size quintile 5. Size quintile 1, a proxy of high information uncertainty, reports the largest bear spread returns for P10 portfolios. The underreaction in high left-tail risk is larger for small firms than for large firms. Since high information uncertainty (i.e., small size) amplifies behavioral biases, the stronger positive return among small-sized firms with high left-tail risk confirms that our results are driven by underreaction to left-tail risk.

Next, we use the methodology of Hong et al. (2000) to compute the residual value of three information uncertainty variables after orthogonalizing to firm size. The three information variables are analyst coverage (AC), analysts' forecast dispersion (DISP), and stock return turnover (TURN). We denote their residual values by $\mathrm{AC}_{\mathrm{RES}}$, $\mathrm{DISP}_{\mathrm{RES}}$, and $\mathrm{TURN}_{\mathrm{RES}}$.[7] We compute the residual from the cross-sectional regression of the logarithm of each variable on the logarithm of the firm's market capitalization in the previous quarter.

Panel B of Table 9 reports returns and alphas for deciles 1 and 10, and for the "10–1" portfolio for the four information uncertainty measures. The "10–1" portfolio returns remain positive and statistically significant. In addition, the positive relation between bear spread returns and left-tail risk is larger when information uncertainty is high. That is, when analyst coverage is low, firm age is low, analyst forecast dispersion is high, and stock return turnover is high. Panel C of Table A4 in the Supplementary Material also reports that the economic impact of the long-short portfolios with low age, high analyst forecast dispersion, and high return turnover is high given that they account for more than 50% of the option dollar open interest percentage of the "10–1" portfolio from Table 2.

Overall, the positive relation between firms' left-tail risk and future bear spread returns is stronger in a high information uncertainty environment. As information uncertainty usually amplifies investors' behavioral biases, such as investor underreaction to bad news, our findings suggest that the options market underreacts to firms' left-tail risk.

2. Investor Sentiment

Sentiment is a biased investor belief conditional on available information (Barberis et al. (1998)). Asset mispricing and risk underestimation are more likely to happen during high investor sentiment periods (Baker and Wurgler (2006), Yu and Yuan (2011), Stambaugh et al. (2012), Lemmon and Ni (2014), and Byun and Kim (2016)). Stambaugh et al. (2012) show that high investor sentiment contributes to the significant profits from the short legs of long-short strategies building upon a large set of anomalies. Byun and Kim (2016) document that the overvaluation of lottery-like options is attributable to high investor sentiment. While prior literature

[7]We do not take the residual of AGE because firm age is only available since 1972 and it could bias the age measure for firms created before 1972 downward.

focuses more on the overpricing of risky assets, the underpricing of safer, protective assets could also occur when investor sentiment is high.

Complementary to the prior literature, we analyze potential underpricing for crash insurance in high-sentiment periods. We use the monthly market-based sentiment index (BW sentiment index) constructed by Baker and Wurgler (2006) to classify high and low investor sentiment months. A high (low) sentiment month occurs when the value of the BW sentiment index in the previous month is above (below) the median value for the sample period. Within the subsample with high (low) sentiment months, we form decile portfolios of delta-hedged bear spreads based on VAR5 and calculate the time-series average monthly returns for decile portfolios.

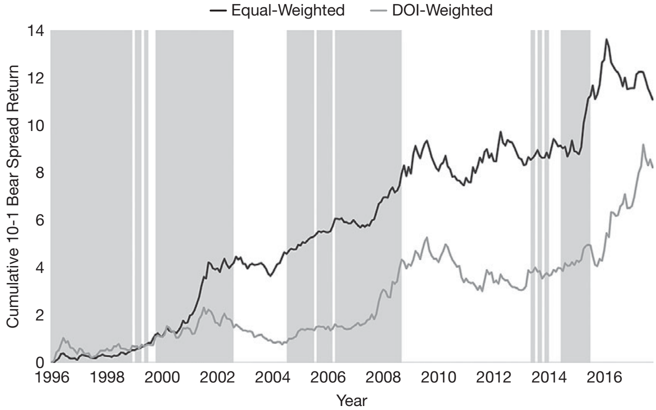
Table A9 in the Supplementary Material reports the returns across VAR5 decile portfolios, together with the return spreads ("10–1"). Newey–West (1987) *t*-statistics are reported in parentheses. The results show that the "10–1" bear spread returns are positive and significant only during high-sentiment periods. During low-sentiment periods, all deciles report negative returns, as one would expect. However, during high-sentiment periods, most deciles report positive bear spread returns with decile 10 reporting the largest positive return. When market sentiment is high, option traders might overlook downside risk and underprice the downside protection provided by bear spreads.

To further understand the relation between the "10–1" bear spread return during high-sentiment periods, we plot the cumulative "10–1" DOI- and equal-weighted bear spread returns in Figure 1 and highlight the periods of high sentiment. We observe that in periods of high sentiment, the "10–1" cumulative returns increase as previously reported.

FIGURE 1

Sentiment and Cumulative Bear Spread Strategy Return

Figure 1 reports the time series of the cumulative "10–1" DOI- and equal-weighted bear spread returns. Each month, decile portfolios of delta-hedged bear spreads are formed and held to maturity by sorting underlying stocks by VAR5. VAR5 is the 5% value-at-risk that corresponds to −1 times the 5th percentile of daily returns in the past year. The "10–1" portfolio is the spread between decile 10 portfolio (with the highest left-tail risk metric) and decile 1 portfolio (with the lowest left-tail risk metric). The shaded areas capture periods of high sentiment. Sentiment is the market-wide sentiment constructed by Baker and Wurgler (2006). A high-sentiment period is selected when the sentiment level is above its sample median. The sample period is from Jan. 1996 to Dec. 2017 for stocks in the OptionMetrics database.



Our finding, together with prior research on risky asset overpricing in high-sentiment periods, supports the economic intuition that overvaluation in risky assets and undervaluation in safer, protective assets may happen at the same time (Acharya and Naqvi (2019)). Han (2008) shows that when market sentiment is high, the index option volatility smile is flatter and the risk-neutral skewness of index returns extracted from index option prices is less negative, suggesting decreased risk-hedging demand. Our results are consistent with Han (2008) as low hedging demand leads to stronger underpricing of bear spreads during high market sentiment periods.

C. Disagreement

A possible explanation of our results is that left-tail risk is a measure of disagreement. We consider three measures of disagreement: i) dispersion of analysts' forecasts used in Diether, Malloy, and Scherbina (2002), ii) stock turnover used in Yu (2011), and iii) idiosyncratic volatility used in Boehme, Danielsen, and Sorescu (2006) and Chatterjee, John, and Yan (2012). Panel B of Table 1 reports that idiosyncratic volatility is increasing in decile portfolios sorted by VAR5. We use the methodology of Hong et al. (2000) to compute the residual value of these three variables after orthogonalizing to firm size and obtain DISP_{RES}, TURN_{RES}, and IVOL_{RES}.

Panel B of Table 9 reports the "10–1" portfolio returns for two subgroups, low and high, of the three disagreement measures. High disagreement is proxied by high dispersion, high turnover, and high idiosyncratic volatility. We find that, in two cases, the long-short returns and alphas remain positive and significant only when disagreement is high, that is, with high dispersion and high turnover. For low dispersion and low turnover, the long-short raw returns and alphas are insignificant and even turn negative. However, for idiosyncratic volatility, the long-short returns and alphas are of similar magnitude for the two subgroups. We conclude that disagreement can only partially explain our results.

V. Transaction Costs and Robustness

In this section, we analyze the profitability of the "10–1" bear spread alpha when accounting for transaction costs. We also examine the robustness of our results across different subsamples such as earnings announcement periods, business cycles, and high/low volatility periods, among others. Finally, we consider various weighting methods to calculate the bear spread return and perform modified Fama–Macbeth regressions and WLS. Below we report our results during business cycles and different volatility periods. The remaining robustness results are reported in the Supplementary Material.

A. Transaction Costs

Transaction costs play an important role in option profitability. In this section, we analyze whether the "10–1" delta-hedged bear spread monthly CAPM alpha of 1.28% with a *t*-statistic of 3.02 reported in Table 3 remains profitable after accounting for real bid–ask spreads and margin costs.

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Table 1 reports that the average quoted half bid–ask spread of bear spreads is 22.5%. This cost is comparable with the raw bear spread returns without delta-hedging reported in Table A10 in the Supplementary Material. When sorting by VAR5, the "10–1" bear spread return without delta-hedging is 9.64% with a t-statistic of 2.44. Therefore, a profitable strategy would require an effective bid–ask spread below 50% of the quoted spread for the "10–1" strategy to remain profitable. Below we perform a rigorous analysis of the impact of transaction costs on delta-hedged bear spread alphas.

To reduce the impact of transaction costs, we follow previous studies such as Goyal and Saretto (2009), Bali and Murray (2013), and Zhan et al. (2022) and hold the position for 1 month without rebalancing the delta hedge. We also hold the position until maturity to avoid paying the option bid–ask spreads and receive the underlying stock at maturity when the option is in the money. To account for transaction costs, we buy at the ask and sell at the bid, and we measure option transaction costs by the effective bid–ask spread when selling and buying the put options. Thus far, we assume that the effective spread is equal to 0 (i.e., option returns are computed with a price equal to the midpoint of the bid and ask quotes). Since OptionMetrics does not provide effective bid–ask spread, we assume that the effective to quoted spreads are equal to 0%, 10%, 29.6%, 50%, and 58.4%. We use spreads of 29.6% and 58.4% given that Muravyev and Pearson (2020) document that the average effective spread paid by algorithmic traders and all traders is 29.6% and 58.4% of the quoted half spread.

We also account for the margin requirement necessary when writing options. For a long bear spread position, the option strategy involves holding delta-shares of the underlying stock for one short unit of a DOTM put option and a long unit of an OTM put option. We follow the CBOE initial margin requirement for a delta-hedged bear spread position, which is "for the same underlying instrument and, as applicable, the same index multiplier; the amount by which the long put (short call) aggregate exercise price is below the short put (long call) aggregate exercise price. Long side must be paid for in full. Proceeds from short option sale may be applied," and "50% requirement on long stock position." For a short bear spread position, there is no requirement on the short put, only the short sale proceeds plus 50% requirement on the short sale in the underlying. We assume that the margin cost is the cost of borrowing the additional capital to meet the margin requirement over the holding period, which is 1 month (Weinbaum, Fodor, Muravyev, and Cremers (2023)). We compute an adjusted return to account for the margin requirements of the delta-hedged bear spread as follows:

$$\text{RETURN} = \frac{(\Delta_{2,t} - \Delta_{1,t})S_T + \max(K_1 - S_T, 0) - \max(K_2 - S_T, 0) - \frac{r}{12}M}{(\Delta_{2,t} - \Delta_{1,t})S_t + \text{PUT1} - \text{PUT2}} - 1,$$

where PUT1 (PUT2), $\Delta_{1,t}$ ($\Delta_{2,t}$), and K_1 (K_2) are the price, delta, and strike price of the OTM (DOTM) put at time t, S_t (S_T) is the price of the underlying stock at time t (T, maturity), r is the 1-month Libor rate, and M is the CBOE required margin. Section F of the Supplementary Material contains a practical example of the computation of returns and transaction costs of delta-hedged bear spreads.

Since bear spreads may contain OTM put options with high bid–ask spreads and low prices, the bid–ask spread will be large relative to the price, and thus the returns are measured with potentially large errors. To reduce transaction costs, we present results for different subsamples that include options with low bid–ask spreads. Heston et al. (2023) restrict their sample to options with quoted half bid–ask spreads of 0.05. We present results for different levels of the quoted half bid–ask spreads: below 0.1, 0.2, 0.3, and for the full sample. Note that the 25th, 50th (median), and 75th percentiles of the quoted half bid–ask spreads are 10.4%, 17.3%, and 27.7%. Our subsamples capture about the same quartiles of the sample data. Starting from May 2003, we obtain observed effective bid–ask spreads from the Options Price Reporting Authority (OPRA) as in Muravyev and Pearson (2020).[8]

Table 10 examines the impact of transaction costs and margin requirements on the profitability of the bear spread trading strategy and reports the DOI-weighted bear spread CAPM alphas of portfolios Short, Long, and Long + Short. The Short and Long portfolios report the CAPM alphas with transaction costs of short-selling decile 1 and buying decile 10. The column "Full sample" with 0% effective-to-quoted spread replicates the results from Table 3 when sorting by VAR5. When using the full sample, the effective-to-quoted spread must be below 10% for the bear spread strategy to be profitable. A similar result is observed for bear spreads with quoted half bid–ask spread lower than 0.3. In both cases, more than 50% of the long-short alpha is generated from portfolio 10, the Long leg, confirming the underreaction to left-tail risk.

For a quoted half bid–ask spread below 0.2, a positive and significant long-short alpha is obtained when the effective-to-quoted spread is equal to or lower than 29.6%, the effective spread paid by algorithmic traders according to Muravyev and Pearson (2020). Including an additional transaction cost, margin requirements, does not change our conclusions. For a quoted half bid–ask spread below 0.1, the sample period is from 2003 to 2017 so that portfolios are well populated with an average of 190 firms per month. For this subsample, the long-short alphas are positive and statistically significant even for an effective-to-quoted spread of 58.4%, the effective spread paid by all traders according to Muravyev and Pearson (2020). Including margin costs when paying an average effective-to-quoted spread of 29.6% does not change our conclusions. Finally, about 70% of the long-short alpha is generated by the Long portfolio. In the OPRA subsample, the long-short bear spread alphas are only significant when the quoted half spread is lower than 0.1. For the other subgroups of the quoted half spread, the "10–1" strategy is positive but insignificant.

In practice, investors would trade bear spreads without delta-hedging. Table A11 in the Supplementary Material reports naked bear spread CAPM alphas with transaction costs. Overall, naked bear-spread alphas are larger than delta-hedged ones. Alphas are positive and significant for all effective-to-quoted spreads with a quoted half spread below 0.1. In the OPRA subsample, the long-short alphas are significant for a quoted half spread below 0.3. Most of the long-short alphas are generated by the Long leg.

[8]We thank the author for sharing the effective spreads data.

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TABLE 10

Transaction Costs

Table 10 examines the impact of transaction costs (bid–ask spreads and margin requirements) on the profitability of the DOI-weighted "10–1" delta-hedged bear spread CAPM alphas (in %) sorted on VAR5. CAPM alphas are calculated after adjusting "10–1" delta-hedged bear spread returns for the CAPM market risk factor. We report "10–1" delta-hedged bear spread CAPM alphas for different ratios of the effective to quoted bid–ask spread: 0% (no cost), 10%, 29.6%, 50%, 58.4%, and the actual effective option bid–ask spread obtained from intraday options from OPRA data. We also examine subsamples of different half quoted bid–ask spreads: lower than 0.1, lower than 0.2, lower than 0.3, and for the full sample. The margin requirement adjusted return is computed using the initial option margin requirements of the CBOE. Following Weinbaum et al. (2023), the margin cost equals the cost of borrowing the additional capital to meet the margin requirement. Each month t, decile portfolios of delta-hedged bear spreads are formed and held to maturity by sorting underlying stocks on the VAR5 left-tail measure. VAR5 is the 5% value-at-risk that corresponds to −1 times the 5th percentile of daily returns in the past year. Newey–West (1987) adjusted t-statistics are presented in parentheses. The sample period is from 2003 to 2017 for the subsample with half quoted bid–ask spread lower than 0.1 and for the OPRA subsample. We report the average number of stocks per month for each subsample. The sample period is from 1996 to 2017 for stocks in the OptionMetrics database for the other subsamples.

Subsample	Statistics	0% (No Cost)	10%	29.6%	50%	58.4%	29.6% + Margin	OPRA (2003–2017)
Quoted half spread < 0.1	Short	0.45	0.43	0.39	0.36	0.34	0.39	0.37
Period: 2003–2017		(2.48)	(2.40)	(2.23)	(2.06)	(1.98)	(2.20)	(2.12)
	Long	1.09	1.03	0.90	0.77	0.72	0.90	0.75
		(2.33)	(2.20)	(1.93)	(1.65)	(1.53)	(1.92)	(1.61)
	Long + Short	1.54	1.46	1.30	1.13	1.06	1.29	1.12
		(3.48)	(3.30)	(2.94)	(2.56)	(2.41)	(2.91)	(2.54)
Avg. stocks per month					190			190
Quoted half spread < 0.2	Short	0.56	0.52	0.45	0.38	0.35	0.44	0.28
Period: 1996–2017		(3.26)	(3.06)	(2.68)	(2.27)	(2.09)	(2.60)	(1.71)
	Long	0.85	0.74	0.51	0.29	0.20	0.51	0.39
		(1.85)	(1.60)	(1.12)	(0.63)	(0.43)	(1.10)	(0.87)
	Long + Short	1.40	1.26	0.97	0.67	0.55	0.94	0.67
		(3.08)	(2.76)	(2.13)	(1.47)	(1.20)	(2.08)	(1.61)
Avg. stocks per month					348			415
Quoted half spread < 0.3	Short	0.51	0.47	0.39	0.31	0.27	0.38	0.28
Period: 1996–2017		(3.30)	(3.05)	(2.56)	(2.04)	(1.79)	(2.47)	(1.86)
	Long	0.74	0.61	0.35	0.08	−0.06	0.34	0.20
		(1.68)	(1.37)	(0.78)	(0.18)	(−0.14)	(0.76)	(0.49)
	Long + Short	1.25	1.07	0.74	0.39	0.21	0.71	0.49
		(2.83)	(2.44)	(1.68)	(0.89)	(0.49)	(1.63)	(1.24)
Avg. stocks per month					451			537
Full sample	Short	0.47	0.43	0.34	0.24	0.20	0.34	0.26
Period: 1996–2017		(3.09)	(2.81)	(2.25)	(1.60)	(1.33)	(2.23)	(1.83)
	Long	0.80	0.63	0.33	−0.04	−0.18	0.30	−0.06
		(1.90)	(1.51)	(0.77)	(−0.10)	(−0.44)	(0.73)	(−0.15)
	Long + Short	1.28	1.06	0.67	0.20	0.02	0.64	0.20
		(3.02)	(2.53)	(1.59)	(0.48)	(0.04)	(1.54)	(0.51)
Avg. stocks per month					587			708

We conclude that the delta-hedged bear spread trading strategy is profitable for algorithmic traders and all traders that can trade at low quoted half spreads below 0.1. Naked bear spreads are profitable for algorithmic traders. The impact of margin requirements on our trading strategy is negligible.

B. Business Cycles and Volatility

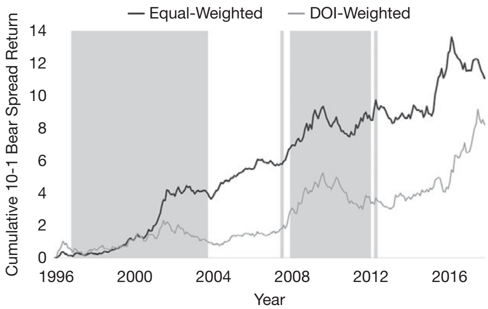
The impact of business cycles or periods with high volatility could potentially affect our findings. We now study bear spread returns for subperiods with high/low volatility and high/low economic growth. We choose these subsamples using cutoff points of the median value of the volatility index from the CBOE (VIX) and the zero value for Chicago Fed National Activity Index (CFNAI). CFNAI is a monthly index designed to gauge overall economic activity and related inflationary pressure. CFNAI is used as a market-wide indicator to examine the performance of trading

FIGURE 2

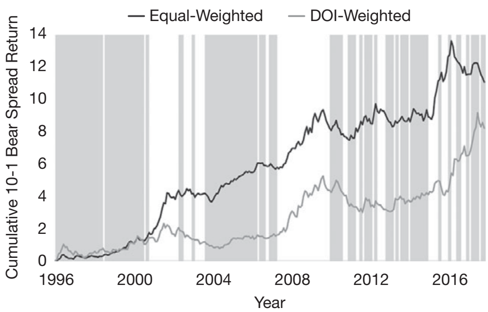
Volatility, Business Cycles, and Cumulative Bear Spread Returns

Graphs A and B of Figure 2 show the time series of the cumulative "10–1" DOI- and equal-weighted bear spread returns. Each month, decile portfolios of delta-hedged bear spreads are formed and held to maturity by sorting underlying stocks by VAR5. VAR5 is the 5% value-at-risk that corresponds to −1 times the 5th percentile of daily returns in the past year. The "10–1" portfolio is the spread between decile 10 portfolio (with the highest left-tail risk metric) and decile 1 portfolio (with the lowest left-tail risk metric). The shaded areas capture periods of high volatility in Graph A and high economic growth in Graph B. VIX is the volatility index from the CBOE, and Chicago Fed National Activity Index (CFNAI) is a monthly index designed to gauge overall economic activity and related inflationary pressure. A high volatility period (high economic growth period) is selected when the VIX level (CFNAI level) is above its sample median (0). The sample period is from Jan. 1996 to Dec. 2017 for stocks in the OptionMetrics database.

Graph A. High Volatility (VIX) versus Low VIX



Graph B. High Economic-Growth (CFNAI) versus Low CFNAI



strategies during high/low economic growth periods (Bali and Murray (2013), Atilgan et al. (2020)).

In Figure 2, we visualize the relation of the cumulative "10–1" DOI- and equal-weighted bear spread returns with levels of volatility (Graph A) and with business cycles (Graph B). We produce two time-series graphs where we highlight in gray the periods of high volatility and high economic growth. In Graph A, when the VIX level is above the sample median, we label the current month as a "high-VIX period" and in Graph B, when CFNAI is above 0, we label the current month as a "high-economic growth period."

Graphs A and B of Figure 2 show the results for the cumulative "10–1" DOI- and equal-weighted bear spread returns under different levels of VIX and CFNAI. From 1997 to 2004, a period of high volatility, the cumulative bear spread return increased. However, in the two subperiods of low volatility (2004–2007 and 2012–2017), the cumulative bear spread return also increased. A visual analysis of economic growth does not show a clear pattern.

To further understand the impact of high/low volatility and high/low economic growth, we perform univariate sorts of bear spread returns on VAR5 for these subperiods. Table A9 in the Supplementary Material reports decile portfolio returns and the "10–1" DOI-weighted bear spread returns. We show that the "10–1" DOI-weighted bear spread returns remain positive and significant across all subsamples, and returns are larger in periods of low volatility and low economic growth. We conclude that bear spread returns hold across business cycles and volatility periods. Equal-weighted returns confirm these findings and are reported in Table A15 in the Supplementary Material.

VI. Conclusion

Understanding the hedging and pricing of tail risk is vital in asset pricing. Investors are averse to portfolio losses and downside moves. Adequately pricing firms' left-tail risk is important for equity investors, option traders, and well-functioning financial markets in general.

In this article, we study the profitability of crash risk insurance. We show that firms' left-tail risk is a strong positive predictor of future crash insurance returns. We conduct comprehensive tests to show that risk-based explanations cannot explain our findings.

Our results are mainly explained by two types of underreaction: volatility underreaction for high left-tail risk portfolios and underreaction to left-tail return momentum. Using statistical forecasts of crash insurance premiums, we show that observed crash insurance premiums in high left-tail deciles are lower than their forecasts. Since our forecasts do not include variance risk premia, this is evidence of volatility underreaction. We also analyze whether investors underestimate the persistence in left-tail risk. We show that the positive relation between crash insurance and left-tail risk is stronger for stocks with larger recent losses or that are trading closer to their 52-week lowest price, suggesting that investors do not adequately factor in the persistence of losses. Additionally, higher information uncertainty amplifies investor underreaction to bad news, leading to stronger crash insurance underpricing. Investor sentiment also has a significant impact on left-tail risk underreaction; we show that the underreaction mainly happens during high market sentiment periods.

We proxy crash insurance with bear put spreads, an option strategy that buys an OTM put and sells a deeper OTM put option. Our findings suggest that, although the loss aversion against left-tail risk plays an important role in financial markets, option traders fail to adequately price bear spreads to compensate for firms' left-tail risk.

Our study contributes to the literature by using an option trading strategy, bear spread, to isolate and analyze crash insurance risk and showing that merely recognizing the importance of left-tail risk is not enough, investors need to overcome behavioral biases to adequately price crash risk insurance.

Supplementary Material

To view supplementary material for this article, please visit http://doi.org/10.1017/S0022109024000309.

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