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GLACIER VOLUME ESTIMATION ON CASCADE VOLCANOES: AN ANALYSIS AND COMPARISON WITH OTHER METHODS

by

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ABSTRACT

During the 1980 eruption of Mount St. Helens, the occurrence of floods and mudflows made apparent a need to assess mudflow hazards on other Cascade volcanoes. A basic requirement for such analysis is information about the volume and distribution of snow and ice on these volcanoes.

An analysis was made of the volume-estimation methods developed by previous authors and a volume-estimation method was developed for use in the Cascade Range. A radio echo-sounder, carried in a backpack, was used to make point measurements of ice thickness on major glaciers of four Cascade volcanoes (Mount Rainier, Washington; Mount Hood and the Three Sisters, Oregon; and Mount Shasta, California). These data were used to generate ice-thickness maps and bedrock topographic maps for developing and testing volume-estimation methods. Subsequently, the methods were applied to the unmeasured glaciers on those mountains and, as a test of the geographical extent of applicability, to glaciers beyond the Cascades having measured volumes.

Two empirical relationships were required in order to predict volumes for all the glaciers. Generally, for glaciers less than 2.6 km in length, volume was found to be estimated best by using glacier area, raised to a power. For longer glaciers, volume was found to be estimated best by using a power law relationship, including slope and shear stress. The necessary variables can be estimated from topographic maps and aerial photographs.

INTRODUCTION

During 1980, the devastating eruption and mudflows of Mount St. Helens, in Washington State, made apparent the need for mudflow-hazard analysis on other Cascade volcanoes. Data on the volume and distribution of snow and ice was required for this analysis. These data were obtained by using the USGS mono-pulse radar, operating at about 5 MHz, and carried in backpacks on these steep volcanic cones. About 200 mono-pulse radar measurements were made on 23 glaciers on Mounts Rainier, Hood, and Shasta and on the Three Sisters. Bed topography was developed on maps on scales of 1:10 000 and 1:24 000 on Mount Rainier; ice volumes were calculated for each glacier (Kennard, 1983).

Because of logistical constraints, not all glaciers of interest could be measured. Therefore, a scheme based on surface characteristics (slope and area) was sought, to account for the measured volumes.

Although numerous volume-estimation methods have been developed, the bedrock topography derived from measurements on the Cascade glaciers permitted establishment of equations particular to the glaciers of this region. In this paper, these methods are tested on other glaciers of known volume, in various environments in order to determine the extent of their applicability. Only a very small fraction of the world's glaciers have been sounded, so a way to estimate thickness and volume would be useful for studies of glacier flow, for volcanic hazard analysis, and for determining the spatial distribution of glacial ice.

DEVELOPMENT OF THE ESTIMATION METHODS

Glacier area is likely to correlate well with glacier volume or thickness. Others have investigated this statistical relationship, using measured glaciers for calibration, such as Brückl (1970), Paterson (1970), Müller (1976), and Macheret and Zhuravlev (1982).

Figure 1 shows the relationship between areas and volumes of measured glaciers on the Cascade volcanoes. From this graph, it can be seen that a correlation exists between area and volume, although volume can be estimated only with considerable error.

Paterson (1970), building upon the work of Nye (1952), suggested that it was possible to estimate mean glacier thickness by using glacier slope and an assumed constant shear for an infinitely-wide glacier, with laminar flow. From the relation

$$\tau = \rho g h \sin\alpha \qquad (1)$$

where τ is the basal shear stress, ρ is the density of ice, g is the acceleration of gravity, h is ice thickness, and α is the surface slope, the approximate relation

$$\bar{h} = k/\bar{\alpha} \qquad (2)$$

follows, where $\bar{h}$ is the ice thickness, averaged over the area, $\bar{\alpha}$ is the average surface slope, and k includes an assumed shear stress, ice density, and geometric variables. To test if the constant shear stress assumption was confirmed with the Cascade data, a basal shear stress was calculated for each glacier. This was done by subdividing each glacier into parts and calculating the interval basal shear stress (τ_i) for each part, by an equation similar to (1):

$$\tau_i = \rho g \frac{V_i}{A_i} \sin\alpha_i \qquad (3)$$

where V is volume, A is surface area, and the subscript, i, indicates an interval value. The interval values are found for discrete parts of a glacier defined by arbitrary altitude limits. An interval formulation is used, because variables that strongly affect depth vary locally and these variations would be missed if the glacier were only considered as a whole.

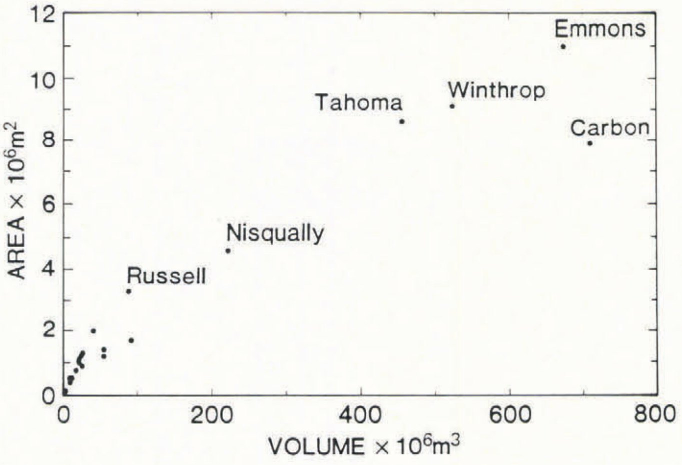

Fig.1. The volumes of measured Cascade glaciers are plotted against area. Smaller glaciers cluster near the origin while the distribution of larger glaciers suggests a power law relationship.

The average basal shear stress ($\bar{\tau}$) is found by:

$$\bar{\tau} = \frac{\sum_{i=1}^{n} W_i \, \tau_i}{\sum_{i=1}^{n} W_i} \qquad (4)$$

where W_i is a size-weighting factor and the summations are over the entire glacier. If $W_i = 1$, equation 4 calculates the mean, basal shear stress.

In Figure 2, where the mean basal shear stresses for the measured glaciers are plotted against glacier areas, the interval (τ_i) values were found for parts of glaciers, defined by 1000 ft (300 m) elevation intervals. As can be seen, the glaciers appear to be divided into two distinct groups: those glaciers with basal shear stresses above one bar (here termed group B glaciers), and those with a shear stress appreciably less than 1 bar (group A glaciers).

This suggests that certain glaciers (group B) reach sufficient thickness to obtain a critical shear stress. The remaining (group A) glaciers do not obtain this threshold shear stress and their value can be anything less.

For purposes of estimating ice volumes from variables derived from surface maps, we applied a basal shear stress approach to group B glaciers and an area approach to those in group A.

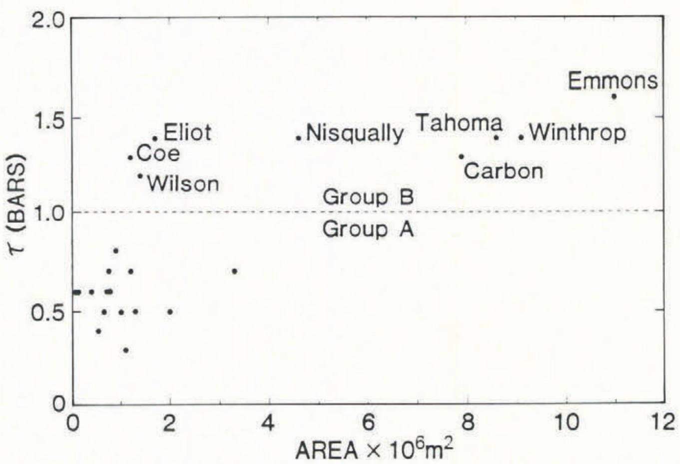

Fig.2. The graph illustrates a calculated basal shear stress versus area for glaciers in the Cascade volcano study.

Table I lists glaciers measured on the Cascade volcanoes, along with their measured volume, length, area, and calculated basal shear stress. While Figure 2 shows that the groups are not uniquely partitioned by area, from Table I it is seen that those glaciers with larger areas and greater length have basal shear stresses greater than 1 bar and smaller glaciers have lower values of basal shear stress.

When glaciers with lengths exceeding 2.6 km are considered to be in group B, and the others group A, all the Cascade glaciers are assigned to their correct group.

GLACIERS WITH VOLUMES THAT CORRELATED WITH BASAL SHEAR STRESS

A volume-estimation method was sought for the group B glaciers, based on their basal shear stress (τ^*). The unknown volume (V^*) of a glacier, presumed to be in group B, can be found by

$$V^* = \frac{\tau^*}{\rho g} \sum_{i=1}^{n} \frac{A_i}{\sin\alpha_i} \qquad (5)$$

once an appropriate τ^* can be found. The data from the measured glaciers were used to find this τ^* value.

When calculating a basal shear stress for a measured glacier ($\bar{\tau}$), it was found useful to apply a size-weighting factor to each altitude interval:

$$W_i = \left(\frac{A_i}{\cos\alpha_i}\right) \quad \left(\frac{1}{\sin\alpha_i}\right) \qquad (6)$$

where $\left(\frac{A_i}{\cos\alpha_i}\right)$ is the actual surface area and A_i is the map area, the actual surface area projected onto a horizontal plane.

This result was relatively insensitive to the value used for W_i; similar results were obtained for $W_i = 1$, $W_i = A_i$, $W_i = V_i$, or W_i as defined by equation (6); and for various measures of glacier size

$$\left(V,\ A,\ \Sigma \frac{A_i}{\cos\alpha_i \sin\alpha_i} \right).$$

The optimum fit suggests that τ^* can be calculated from area and slope by the empirical relation:

$$\tau^* = 2.7\times10^4 \sum_{i=1}^{n} \left(\frac{A_i}{\cos\alpha_i}\right)^{0.106} \qquad (7)$$

where area is in m^2 and τ^* in Pascals (1 Pascal = 10^{-5} bars). The best interval value of several examined, over which A_i and $\cos\alpha_i$ are calculated, was found to be that defined by 1000 ft (300 m) elevation intervals.

Once τ^* is found for an unmeasured glacier by equation (7), volume is found by equation (5), where A_i and $\sin\alpha_i$ are also defined by 1000 ft intervals.

AREA-CORRELATED GLACIERS

For the smaller (group A) glaciers, a regression analysis was done on the area-volume relationship. The final relation that minimizes the individual glacier percentage and magnitude differences is

$$V^* = 3.93A^{1.124} \qquad (8)$$

where A is the total area in m^2 and the estimated volume V^* is in m^3 (Kennard, 1983), and is shown in Figure 3.

TABLE I RADAR MEASURED GLACIERS ON THE CASCADE VOLCANOES, INDICATING LOCATION AND ASSOCIATED BASAL SHEAR STRESS, AREA, MEASURED VOLUME, AND GLACIER LENGTH. THE LOCATIONS ARE INDICATED BY (R) MOUNT RAINIER, 46°50'N, 121°45'W; (H) MOUNT HOOD, 45°20'N, 121°30'W; (T) THREE SISTERS, 44°10'N, 121°25'W.

Measured glacier	Location	Group	Basal shear stress, in bars	Area km²	Volume x10⁶m³	Map length of glacier km
Emmons	R	B	1.6	11.0	67.	7.2
Winthrop	R	B	1.4	9.1	52.	8.1
Tahoma	R	B	1.4	8.6	46.	7.4
Carbon	R	B	1.3	7.9	71.	9.7
Nisqually	R	B	1.4	4.6	22.	6.6
Russell	R	A	0.7	3.3	88.	2.4
Newton Clark	H	A	0.5	2.0	40.	2.0
Eliot	H	B	1.4	1.7	91.	4.0
Wilson	R	B	1.2	1.4	54.	2.6
Coe	H	B	1.3	1.2	54.	3.3
Sandy	H	A	0.7	1.2	23.	1.9
Collier	T	A	0.3	1.1	20.	2.1
Prouty	T	A	0.5	1.0	20.	1.6
Ladd	H	A	0.8	.9	25.	2.0
ZigZag	H	A	0.6	.78	17.	2.4
Reid	H	A	0.7	.75	17.	1.9
Hayden	T	A	0.6	.72	17.	1.3
Diller	T	A	0.5	.66	14.	1.2
White River	H	A	0.4	.54	8.5	1.9
Lost Creek	T	A	0.4	.54	11.	1.4
Langille	H	A	0.6	.4	8.5	1.5
Palmer	H	A	0.6	.13	2.0	.4
Coalman	H	A	0.6	.08	1.1	.5

* (A) indicates that a volume-to-area relation was used to estimate volume; (B) indicates a relation with basal shear stress

APPLICATION OF THE METHOD TO OTHER GLACIERS

In order to test the geographical applicability of these estimation methods, they were applied to some glaciers with well-measured basal topography. The glaciers' locations and dimensions are listed in Table II. The estimation methods gave satisfactory results when the glaciers were assigned to the correct group, yielding a standard deviation error for volume of less than 8 per cent.

However, three of the nine glaciers (Isfalls, Whitney, Grinnell) were assigned to the wrong group, when the length criterion developed for the measured glaciers in the Cascades was used. An incorrect assignment of an unmeasured glacier introduces significant error, discussed later.

The estimation methods were applied to the whole glacier, in all but two cases. The Athabasca Glacier, of British Columbia, is an outlet glacier of the much larger Columbia Icefield, where only the ablation area was measured and later estimated. The Dinwoody Glacier, in the Wind River Range of Wyoming (USA), is composed of a compound cirque accumulation area. The volume was established only in the two southern cirques, through the terminus, because radar measurements were completed in this region only.

COMPARISON WITH OTHER METHODS

A comparison was made of measured and estimated volumes, as calculated with the methods of Driedger and Kennard, 1984; Post and others, 1971; Müller, 1976; and Macheret and Zhuravlev, 1982.

Paterson (1970) and others built upon the work of Nye (1952) and others to make theoretical calculations of $\bar{h}$. But it was not until there was an accumulation of published, sub-glacial, topographic data, that volume estimation methods could be developed.

One of the earlier estimation procedures was suggested in Post and others (1971), where glaciers were assigned volumes according to five size classes. A mean thickness was established for each area category (20 m for areas between 0 and 0·5 km²; 40 m for areas between 0·5 and 1 km²; 65 m for areas between 1 and 2 km²; 90 m for areas betwen 2 and 5 km²; and 120 m for areas between 5

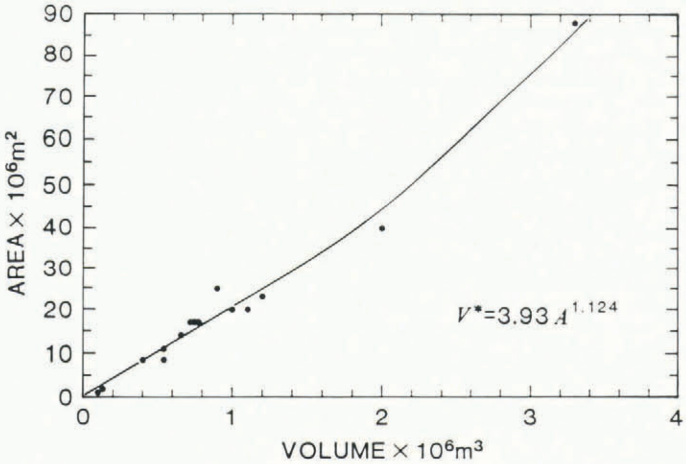

Fig.3. The graph illustrates the power curve $V = 3.93A^{1.124}$. derived when area is plotted against measured volume for smaller (group A) glaciers in the Cascades.

TABLE II OTHER GLACIERS INCLUDED IN TEST OF THE METHODS' GEOGRAPHICAL APPLICABILITY. GROUP DESIGNATION REFERS TO THE USE OF AN AREA (A) OR BASAL SHEAR STRESS (B) RELATION TO VOLUME. THE ESTIMATED VOLUMES WERE OBTAINED, USING THE METHODS DESCRIBED IN THIS PAPER

Glacier	Location	Country	Lat., Long.	Area (km^2)	Length (km)	$\bar{h}$ (m)	Volume $x10^{6} m^{3}$ meas.	est.	Group	Reference
Rabots	Kebnekaise Massif	Sweden	67°55'N;11°30'E	4.1	3.6	84	346	372	B	Björnsson 1981
Athabasca	Rocky Mountains	British Col., Canada	51°40'N;116°50'W	3.8	3.6	150	574	600	B	Trembley written com. 1985
Stor	Kebnekaise Massif	Sweden	67°64'N;18°36'E	3.1	3.5	99	306	303	B	Björnsson 1981
S. Cascade	North Cascades Range	Washington, USA	48°122'N;121°3'W	2.0	3.3	99	196	198	B	Hodge, 1979
Dinwoody*	Wind River Range	Wyoming, USA	43°10'N;109°40'W	1.5	2.6	55	80	100	B	
Isfalls	Kebnekaise Massif	Sweden	67°55'N;18°35'E	1.3	2.0	72	92	82	B	Björnsson 1981
Whitney	Mount Shasta	California, USA	41°30'N;121°50'W	1.3	2.8	20	26	29	A	Driedger and Kennard, 1984
Grinnell	Rocky Mountains	Montana, USA	48°50'N;113°50'W	1.0	1.0	64	62	59	B	
Maclure	Sierra Nevada Mountains	California, USA	37°40'N;119°10'W	.2	.8	17	4	4.2	A	

* Only the two southern-most cirques and the terminus region of the glacier were estimated in this study because they are the only portions measured by ice radar.

and 10 km^2). These relations were based upon gravity and drilling measurements on Pacific North-west glaciers, as well as measurements of some small Russian and Canadian glaciers, as described by Ommanney, 1969.

Müller (1976) developed a scheme whereby glaciers are divided into size categories and a mean thickness is assumed for each category, according to

$$A < 0.5\ km^2,\ \bar{h} = 5\ m;$$

$$0.5 < A \leq 23\ km^2,\ \bar{h} = 5.2 + 15.4\ A^{\frac{1}{2}}$$

and, for $A > 23\ km^2$ (where A is in km^2 and $\bar{h}$ is in meters), volume determination is done on an individual basis. The relations were developed using area and mean thickness values for 16 mountain and valley glaciers in the Alps, as derived from the methods of Brückl (1970), whose method alone can only be applied to glaciers for which some thickness information is known beforehand. Similar volume estimates were made by Østrem et al (1973) in the determination of glacier volumes in Scandinavia.

Macheret and Zhuravlev (1982) made parabolic approximations of radar-sounded cross-sections on 59 glaciers of Svalbard. They determined that volume was related to area according to the relation:

$$V = 11.4\ A^{1.120}$$

where A is in m^2, and V is in m^3. The exponent above is the same, to three significant figures, as in the Cascades estimation method for group A glaciers (equation 8); the coefficient, however, is approximately three times that of the Cascades method.

Usable maps of Svalbard sub-glacial topography were not readily available. Also, additional study by Dowdeswell and others (1984) shows that more recent radar interpretations may yield greater volumes than currently published.

Therefore, no other methods were tested on Svalbard glaciers, although their method was applied to glaciers elsewhere.

Table III illustrates the relationship of the measured volumes to those found with three estimation methods.

CONCLUSIONS

Based on application to the original measured Cascade glaciers, the root mean square for the individual glacier estimated volume percentage errors was 13% for group A and B glaciers together. With an estimated uncertainty of ±20 per cent, for the Cascade measured volumes (Kennard, 1983), the error in determining the unknown volume of an unknown glacier is estimated to be 25%.

This does not include the potential error associated with incorrectly designating the glacier group, prior to estimating volume. For larger glaciers, the volume difference, caused by improper designation, can exceed 100 per cent.

The empirical criterion for assignment to Group A or B was shown to be reliable only in the Cascades. There is an interval of uncertainty for glaciers of lengths from about 2.8 to 1.0 km. Until a more fundamental, differentiation method is found, it is suggested that photographs of glaciers in this intermediate zone be compared with known glaciers of both groups. Relative size and shape and evidence of relative thickness, as expressed by surface morphology, should help in more positive identification.

TABLE III GLACIERS INCLUDED IN COMPARISON OF ESTIMATION METHODS. ALL VOLUMES ARE x $10^6 m^3$. ROOT SQUARE VALUES ARE LISTED FOR AREA CLASSES.

Glacier	Area (km^2)	Measured volume	$\bar{h}$ (m)	Estimated volumes according to: This paper	% dif *	Post et al 1971	% dif *	Muller et al 1976	% dif *	Macheret & Zhuravlev 1982	% dif *
1 Emmons	11.	672.	60	653.	-3†	1340.	99	633.	-6	891.	32
2 Winthrop	9.2	525.	57	573.	9†	1102.	110	476.	-9	715.	36
3 Tahoma	8.7	457.	52	473.	3†	1046.	129	442.	-3	675.	48
4 Carbon	7.9	710.	90	669.	-7†	950.	34	384.	-46	606.	-15
rms				6.1		100		24		35	
5 Nisqually	4.6	220.	48	214.	-3†	415.	89	176.	-20	331.	50
6 Rabots	4.1	346.	84	372.	8	369.	7	149.	-57	290.	-16
7 Athabasca	3.8	574.	150	600.	5	344.	-40	135.	-77	268.	-53
8 Russell	3.3	86.	26	83.	-4†	297.	244	110.	27	227.	163
9 Stor	3.1	306.	99	303.	-1	279.	-9	100.	-67	212.	-31
10 Newton-Clark	2.0	39.	20	4.7	2†	129.	229	54.	36	129.	228
11 S. Cascade	2.0	196.	99	198.	1	128.	-35†	53.	-73	128.	-35
rms				4.1		132		55		110	
12 Eliot	1.7	91.	54	93.	2†	109.	20	42.	-54	107.	17
13 Dinwoody	1.4	80.	55	100.	25	95.	18	34.8	-57	9.12	14
14 Wilson	1.4	54.	38	55.	1†	94.	73	34.	-37	89.	66
15 Isfalls	1.3	92.	72	82.	-11	84.	-10	29.	-68	79.	-14
16 Whitney	1.3	26.	20	29.	13	83.	226	29.	14	79.	209
17 Coe	1.2	53.	43	52.	-2†	81.	52	28.	-48	76.	43
18 Sandy	1.2	25.	21	26.	4†	77.	204	26.	3	72.	185
19 Collier	1.1	21.	19	24.	14†	71.	239	23.	11	66.	215
rms				11		141		43		128	
20 Grinnell	0.9	62.	64	59.	- 5	39.	-37	20.	-68	58.	6
21 Prouty	0.9	16.	17	21.	28†	39.	137	19.	21	58.	-252
22 Ladd	0.9	24.	33	19.	-21†	36.	47	18.	-27	53.	117
23 Zigzag	0.8	17.	22	16.	- 6†	31.	79	15.	-16	45.	160
24 Reid	0.8	18.	24	16.	-12†	30.	68	14.	-22	44.	143
25 Hayden	0.7	19.	26	15.	-20†	29.	53	14.	-30	42.	120
26 Diller	0.7	13.	20	14.	5†	26.	102	12.	-11	37.	188
27 Lost Creek	0.5	14.	21	11.	-22†	22.	56	9.0	-35	30.	117
28 White River	0.5	89.	16	11.	23†	22.	144	8.9	1	30.	239
rms				17		88		31		30	
29 Langille	0.4	8.0	20	7.6	-4†	7.8	-1	2.0	-75	21.	165
30 Maclure	0.2	4.0	17	4.2	5	4.6	15	1.2	-71	12.	188
31 Palmer	0.1	2.0	16	2.1	5†	2.5	25	0.6	-69	5.8	190
32 Coalman	0.1	1.0	14	1.3	18†	1.6	45	0.4	-64	3.5	218
rms				10		27		70		191	

* percentage difference from measured volume.
† signifies glaciers from which methods were developed.

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