The book starts by providing an introduction to the geometry of self-similar sets. This involves defining what is meant by a self-similar set, describing their topological features and how we can define an appropriate measure on such sets. The analysis then develops by recognizing the close analogy of these graphs and the effective resistance of their electrical network counterparts. This leads to the study of the convergence of Dirichlet forms on a sequence of finite sets and finally to the definition of the Laplacian. This then naturally leads to the study of eigenvalues and eigenfunctions of both Dirichlet and Neumann type Laplacians. It can be seen even at this stage that their nature is quite distinct from those obtained on bounded real domains: for instance, the existence of localized eigenfunctions. It is also possible by defining an eigenvalue-counting function to study the asymptotic behaviour and derive a result of Weyl's theorem type. The last chapter in the book studies solutions to the heat equation and develops useful tools in this regard, such as a maximum principle.

In truth, we are only at the beginning of what this analysis can accomplish. A wide vista of opportunities has opened up for the study of linear field equations in fractal media. Some direction has already come from the physics literature where, for instance, the wave equation and reaction–diffusion equations have been studied. Anyone with a background in the analysis of linear field equations, with an interest in heterogeneous media, or who is looking to breathe new life into their research, should read this book.

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ROBINSON, J. C. *Infinite-dimensional dynamical systems* (Cambridge University Press, 2001) 461pp., 0 521 63564 0 (paperback), £24.95, 0 521 63204 8 (hardback), £70.

This impressive book offers an excellent, self-contained introduction to many important aspects of infinite-dimensional dynamical systems. The title of the book is accompanied by two different subtitles, namely 'From Basic Concepts to Actual Calculations', which appears on the front cover, and then, several pages later, 'An Introduction to Dissipative Parabolic PDEs and the Theory of Global Attractors'. Each gives a clear indication of the author's intention not only to develop the basic theory of infinite-dimensional dynamical systems but also to highlight applications to partial differential equations. The end result is a highly readable and beautifully explained account of a variety of topics ranging from fundamental concepts in functional analysis to the application of compactness methods for establishing the existence and uniqueness of solutions to partial differential equations, and concluding with the importance of the global attractor, and its fractal dimension, in determining the asymptotic dynamics of a dissipative system. The style of presentation is particularly appealing, due in no small part to the author's success in providing an informative insight into the thought processes behind the mathematics.

Following the introduction, where a simple example involving the one-dimensional heat equation nicely motivates the need for an infinite-dimensional theory of dynamical systems, the book is divided into four main parts. These are entitled 'Functional Analysis', 'Existence and Uniqueness Theory', 'Finite-Dimensional Global Attractors' and 'Finite-Dimensional Dynamics'.

The author devotes Part I to presenting a rigorous treatment of topics and results from functional analysis and operator theory that are essential for a full understanding of the later material. Chapters on Banach and Hilbert spaces, ordinary differential equations, linear operators, dual spaces and Sobolev spaces are included. This prerequisite material is developed in a user-friendly manner which should make it accessible to most readers. For example, in the section on spectral theory for unbounded symmetric operators, the desired result on the existence of an orthonormal basis of eigenfunctions is derived using a clever approach that avoids

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the usual technicalities associated with unbounded self-adjoint operators and, remarkably, the definition of the spectrum itself! Also, for those wishing for a gentle route into the theory of Sobolev spaces, it is pointed out that proofs of the key results are greatly simplified in the case of periodic functions, with details supplied in an appendix. The only prior knowledge that is required in Part I is of the Lebesgue integral, which the book covers rather briefly in less than three pages.

Existence, uniqueness and regularity of solutions to various partial differential equations are discussed in Part II. As one would expect from the book's title, the emphasis is on time-dependent equations. However, the basic strategy of using Sobolev-space methods to analyse a weak formulation of an equation is first applied to Poisson's equation involving either periodic or Dirichlet boundary conditions. The Galerkin method is then introduced and applied in succession to a linear parabolic equation, scalar reaction–diffusion equations, and the two-dimensional Navier–Stokes equations with periodic boundary conditions. In each case, the existence of both weak and strong solutions is discussed.

Part III begins with a discussion on limit sets and attractors for dissipative semigroups on infinite-dimensional phase spaces. The existence of the global attractor for any dissipative semigroup is proved, and a description of its structure is provided for the case when the semigroup has a Lyapunov function. A brief account of both the fractal and Hausdorff dimensions is also given. To illustrate this general theory, the reaction–diffusion and Navier–Stokes equations considered earlier are shown to have global attractors, and a method is described that enables bounds on the fractal dimensions of these attractors to be obtained.

The aim of the last part of the book is to explain how the finite fractal dimension of the global attractor for an infinite-dimensional dynamical system leads to finite-dimensional asymptotic dynamics. Both the squeezing property and its stronger counterpart are introduced, and important consequences highlighted. In particular, it is shown that the squeezing property implies that only a finite number of 'determining modes' is required to describe the flow on the global attractor, while the strong squeezing property results in the existence of an inertial manifold. This is a smooth invariant manifold containing the attractor on which the dynamics are described by a finite-dimensional system of ordinary differential equations. The fact that the attractor can be parametrized using a finite number of coordinates is also demonstrated and it is proved that there is a finite-dimensional system that reproduces the dynamics on the attractor. Once again, the general theory is applied to the reaction–diffusion and Navier–Stokes equations.

At the end of each chapter is a set of exercises, for which full solutions can be found on the World Wide Web, and also a section containing additional notes. These notes provide interesting information on the material covered in the chapter, suggest useful sources for further reading, and give a clear indication of the author's expertise and breadth of knowledge in this field. A further test of the reader's understanding of the techniques developed in the book is provided by the final chapter, which consists of a series of exercises on the Kuramoto–Sivashinsky equation.

There are relatively few aspects of the book that disappoint. A number of typographical and mathematical errors occur, but a list of errata is provided on the World Wide Web. Also, there are several places, particularly in the technically demanding treatment of the dimension of the global attractor, where considerable effort is required from the reader. However, these do not detract significantly from a text that the reviewer found a pleasure to read.

At the outset, the author states that his aim was to produce a didactic text suitable for first-year graduate students. Unquestionably he has achieved his goal. This book should prove invaluable to mathematicians wishing to gain some knowledge of the dynamical-systems approach to dissipative partial differential equations that has been developed during the past 20 years, and should be essential reading for any graduate student starting out on a PhD in this area.

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