Proceedings of the Edinburgh Mathematical Society (2025) **68**, 44–50
doi:10.1017/S0013091524000695

CENTERS OF ARTIN GROUPS DEFINED ON CONES

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(Received 11 June 2024)

Abstract We prove that the Center Conjecture passes to the Artin groups whose defining graphs are cones, if the conjecture holds for the Artin group defined on the set of the cone points. In particular, it holds for every Artin group whose defining graph has exactly one cone point.

Keywords: Artin groups; the Center Conjecture

2010 Mathematics subject classification: 20F36; 20F65

1. Introduction

An Artin group A is given by the presentation

$$A = \langle s_1, \dots, s_n \,|\, \underbrace{s_i s_j s_i \cdots}_{m_{ij}\ terms} = \underbrace{s_j s_i s_j \cdots}_{m_{ij}\ terms} \rangle$$

where $m_{ij} = m_{ji} \geq 2$. The data of an Artin group can be encoded by its *defining graph* Γ whose vertex set is $V(\Gamma) = \{s_1, \dots, s_n\}$, and each relation involving s_i, s_j with terms of length m_{ij} corresponds to an edge (s_i, s_j) with label $m_{ij} \leq \infty$. The Artin group with defining graph Γ will be denoted by A_Γ. Every Artin group has a naturally associated Coxeter group quotient $A_\Gamma \to W_\Gamma$ obtained by adding relations $s_i^2 = 1$ for all $i = 1, \dots, n$.

An Artin group A is *spherical* if the corresponding Coxeter group is finite, and otherwise A is *infinite type*. A *special subgroup* of A is a subgroup generated by some subset of $K \subseteq V(\Gamma)$, denoted A_K. Each special subgroup A_K is itself isomorphic to the Artin group with defining graph Γ_K, where Γ_K is the subgraph of Γ induced by K [13].

A *2-labelled join* of labelled graphs Γ_1, Γ_2 is a graph join $\Gamma_1 * \Gamma_2$ where every edge (s_1, s_2), with $s_1 \in V(\Gamma_1)$ and $s_2 \in V(\Gamma_2)$, has label 2, and the labels of edges contained in factors Γ_1, Γ_2 remain the same. We denote a 2-labelled join by $\Gamma_1 *_2 \Gamma_2$. If $\Gamma = \Gamma_1 *_2 \Gamma_2$, then $A_\Gamma = A_{\Gamma_1} \times A_{\Gamma_2}$. An Artin group A is *irreducible* if its defining graph Γ does not split as a 2-labelled join, in which case we also call Γ irreducible.

Each Artin group A_Γ admits (a possibly trivial) *decomposition into irreducible factors* $A_\Gamma = A_{\Gamma_1} \times \cdots \times A_{\Gamma_n}$, where each $\Gamma_i \subseteq \Gamma$ is irreducible. Equivalently, this corresponds the maximal decomposition of Γ as an n-fold 2-labelled join of subgraphs $\Gamma_1, \ldots, \Gamma_n$.

Every irreducible spherical Artin group A has an infinite cyclic centre, and furthermore if $\{s_1, \ldots, s_n\}$ is the standard generating set of A then $Z(A)$ is generated by a power of $s_1 s_2 \cdots s_n$ [3, 6]. If z is a central element in a spherical factor A_{Γ_i} of A_Γ, then $z \in Z(A_\Gamma)$. Conjecturally, all the central elements of A_Γ arise from irreducible factors that are spherical.

The Center Conjecture.

Let A_Γ be an Artin group with irreducible factor decomposition

$$A_\Gamma = A_{\Gamma_1} \times \cdots \times A_{\Gamma_n}.$$

Then $Z(A_\Gamma) = \mathbb{Z}^k$ where k is the number of spherical factors A_{Γ_i}.

When $A = B \times C$ then $Z(A) = Z(B) \times Z(C)$ so, equivalently, the Center Conjecture states that every irreducible Artin group of infinite type has trivial centre.

Apart from the spherical Artin groups, the Center Conjecture also holds for FC-type Artin groups, two-dimensional Artin groups [8] and Euclidean Artin groups [12]. More generally, every Artin group that satisfies the $K(\pi, 1)$-conjecture also satisfies the Center Conjecture [10].

A *cone point* in a graph Γ is a vertex that is adjacent to every other vertex of Γ. Charney and Morris-Wright have shown the Center Conjecture holds for Artin groups whose defining graphs are not cones [4]. In other words, this is the case where the set of cone points of Γ is empty. The proof of Charney and Morris-Wright relies on the geometry of the clique-cube complex, which is a CAT(0) cube complex associated with Γ. Their result can also be deduced from the following proposition, whose proof is brief and follows directly from the presentation of A_Γ.

Proposition 1.1. *Let A_Γ be an Artin group, where $T \subseteq V(\Gamma)$ is the set of cone points of Γ. Then $Z(A_\Gamma) \subseteq Z(A_T)$.*

In particular, if in the above theorem $Z(A_T) = \{1\}$, then $Z(A_\Gamma) = \{1\}$, i.e. A_Γ satisfies the Center Conjecture.

Godelle and Paris showed that if all Artin groups whose defining graph is a clique satisfy the Center Conjecture, then all Artin groups satisfy the Center Conjecture [8]. Our second result gives alternative proofs of this fact and the Center Conjecture for FC-type Artin groups, as well as the Center Conjecture for many new Artin groups (see Figure 1 for some examples).

Theorem 1.2. *Let A_Γ be an Artin group, where $T \subseteq V(\Gamma)$ is the set of cone points of Γ. If A_T satisfies the Center Conjecture, then A_Γ satisfies the Center Conjecture.*

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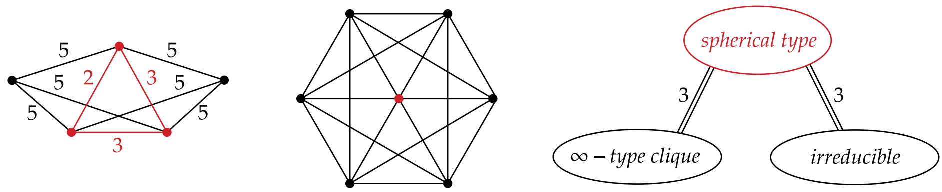

Figure 1. New examples of graphs whose associated Artin groups satisfy the Center Conjecture. The red subgraphs are spanned by the set of cone points. The middle graph can have any labels. The diagram on the right represents an infinite class of such graphs; the double edges represent joins, with all labels equal to 3.

To prove Proposition 1.1, we consider splittings of non-clique Artin groups. This generalizes the result of [4] using a different method. Then to deduce Theorem 1.2 from Proposition 1.1 we use a retraction map (first used in [5] and recently discussed more explicitly in [1]) to prove some combinatorial restrictions on central elements of irreducible Artin groups; namely that any spelling of a central element which does not use every generator cannot be strictly positive or strictly negative.

While preparing this manuscript, Kato and Oguni announced a proof that all Artin groups whose defining graphs are not cliques are acylindrically hyperbolic and obtained our results as corollaries [11]. Their methods are a generalization of the techniques used in [4] and are therefore distinct from the techniques used here.

2. Central elements are generated by cone points

Let Γ be a graph and let $x, y \in V(\Gamma)$ so that $m_{x,y} = \infty$. A direct consequence of the presentation of A_Γ is that it splits as the following amalgamated product:

$$A_\Gamma = A_{\Gamma-\{x\}} *_{A_{\Gamma-\{x,y\}}} A_{\Gamma-\{y\}}.$$

The following lemma can be proven by considering normal forms in amalgamated free products and was utilized in [8].

Lemma 2.1. *Let $G = H_1 *_K H_2$ be the free product of H_1, H_2 amalgamated over K. Then $Z(G) = Z(H_1) \cap Z(H_2) \subseteq K$.*

Using these two results, we can prove our first theorem.

Proposition 1.1. *Let A_Γ be an Artin group, where $T \subseteq V(\Gamma)$ is the set of cone points of Γ. Then $Z(A_\Gamma) \subseteq Z(A_T)$.*

Proof. Let $R = V(\Gamma) - T$ and $r \in R$. There exists some $q_r \in V(\Gamma) - T$ so that $m_{r,q_r} = \infty$. Thus

$$A_\Gamma = A_{\Gamma-\{r\}} *_{A_{\Gamma-\{r,q_r\}}} A_{\Gamma-\{q_r\}}.$$

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By Lemma 2.1, this implies that for every $r \in R$ we have $Z(A_\Gamma) \subseteq A_{\Gamma-\{r,q_r\}} \subseteq A_{\Gamma-\{r\}}$. In particular, we have

$$Z(A_\Gamma) \subseteq \bigcap_{r\in R} A_{\Gamma-\{r\}}.$$

In [13], van der Lek showed that for any $S, Q \subseteq V(\Gamma)$ we have $A_S \cap A_Q = A_{S\cap Q}$. In particular, $\bigcap_{r\in R} A_{\Gamma-\{r\}} = A_{\Gamma-R} = A_T$. Since $z \in Z(A_\Gamma)$, z is also central in A_T. $\square$

We obtain the following corollary immediately.

Corollary 2.2. *Suppose that Γ is a graph with cones points T, and suppose A_T has trivial centre. Then A_Γ has trivial centre.*

In particular, if $T = \emptyset$ then $Z(A_\Gamma) = \{1\}$ and we recover [4, Theorem 3.3]. The condition that $Z(A_T) = \{1\}$ is not a necessary condition: if $T = \{t\}$ then $A_T \cong \mathbb{Z}$, so A_T does not have trivial centre. On the other hand, if there is any vertex of Γ which does not commute with t then $Z(A_\Gamma)$ is trivial.

Corollary 2.3. *Suppose Γ is a graph with a single cone point t. Then $A(\Gamma)$ satisfies the Center Conjecture, i.e. $Z(A_\Gamma) = \{1\}$ if and only if there exists $s \in S$ so that $m_{st} \neq 2$.*

Proof. Assume there exists $s \in S$ with $m_{s,t} \neq 2$. By Proposition 1.1, if $z \in Z(A_\Gamma)$ then $z \in A_{\{t\}} = \langle t\rangle$. Let $z = t^i$. Since $A_{\{t,s\}}$ is spherical, the centre of $A_{\{t,s\}}$ is generated by a power of ts. Hence z is central in $A_{\{t,s\}}$ only if $i = 0$.
If $m_{st} = 2$ for all $s \in S - \{t\}$, then $A_\Gamma \cong \langle t\rangle \times A_S$, so $\langle t\rangle \leq Z(A_\Gamma)$. $\square$

In the following section, we generalize this argument to larger sets of cone points.

3. Cone sets satisfying the Center Conjecture

3.1. Retraction map

The main goal of this subsection is Theorem 3.4. Its proof relies on the retraction map $\pi_X : A_\Gamma \to A_X$ described in [1, 5]. The definition of the map π_X relies on passing to the Coxeter group W_Γ. We re-establish notation here so that we can differentiate easily between elements in the Coxeter group and elements in the Artin group.

Fix a graph Γ. Let $\Sigma = \{\sigma_v | v \in V(\Gamma)\}$ and $S = \{s_v | v \in V(\Gamma)\}$ be generating sets for the Artin and Coxeter groups A_Γ and W_Γ, respectively. Let $\theta : A_\Gamma \to W_\Gamma$ the natural surjection sending $\sigma_v \mapsto s_v$. The kernel of θ is the *pure Artin group* PA_Γ. Let $X \subseteq V(\Gamma)$. We denote the subgroup of W_Γ generated by X by W_X and similarly for the corresponding subgroup of PA_Γ and subsets of Σ, S.

Let X, Y be two subsets of S. An element $w \in W_\Gamma$ is *(X, Y)-reduced* if it is of minimal length amongst the elements of the double coset $W_X w W_Y$. For any element $w \in W_\Gamma$, the *length* of w, denoted $|w|$, is the shortest length of a word in S needed to express w.

Lemma 3.1. (Lemma 2.3 [9]) *Let $X, Y \subseteq V(\Gamma)$ and let $w \in W_\Gamma$. There exists a unique (X, Y)-reduced element in $W_X w W_Y$. Furthermore, the following are equivalent:*

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- *an element $w \in W$ is $(X, \emptyset)$-reduced,*
- *$|sw| > |w|$ for all $s \in X$ and*
- *$|v \cdot w| = |v| + |w|$ for all $v \in W_X$.*

The map is defined as follows.

Definition 3.2. ([1]) *Let $(\Sigma \cup \Sigma^{-1})^*$ denote the free monoid on $\Sigma \cup \Sigma^{-1}$. Let $X \subseteq V(\Gamma)$, and let $\widehat{\alpha} = \sigma_{v_1}^{\varepsilon_1}\sigma_{v_2}^{\varepsilon_2}\cdots\sigma_{v_p}^{\varepsilon_p} \in (\Sigma \cup \Sigma^{-1})^*$.*

Set $u_0 = 1 \in W_\Gamma$, and for $i \in \{1, \dots, p\}$ set $u_i = s_{v_1}s_{v_2}\cdots s_{v_i} \in W_\Gamma$. We can write each u_i as $u_i = v_i w_i$ where $v_i \in W_X$ and w_i is $(X, \emptyset)$-reduced.

Now define

$$t_i = \begin{cases} w_{i-1}s_{v_i}w_{i-1}^{-1} & \text{if } \varepsilon = 1, \\ w_i s_{v_i} w_i^{-1} & \text{if } \varepsilon = -1. \end{cases}$$

If $t_i \notin S_X$ we set $\tau_i = 1$. Otherwise, we have $t_i \in S_X$. Thus $t_i = s_{x_i}$ for some $x_i \in X$, and we set $\tau_i = \sigma_{x_i}^{\varepsilon_i} \in \Sigma_X \cup \Sigma_X^{-1}$.

Finally, we define

$$\widehat{\pi}_X(\widehat{\alpha}) = \tau_1 \dots \tau_p \in (\Sigma_X \cup \Sigma_X^{-1})^*.$$

We collect in the following proposition several properties of the map $\widehat{\pi}_X$ and the induced map π_X (see Proposition 3.3(1)) that will be of use in later results.

Proposition 3.3. *Let $X, Y \subseteq V(\Gamma)$, and $\alpha \in (\Sigma \cup \Sigma^{-1})^*$.*

(i) *[1, Prop 2.3(1)] The map $\widehat{\pi}_X : (\Sigma \cup \Sigma^{-1})^* \to (\Sigma_X \cup \Sigma_X^{-1})^*$ induces a set-map $\pi_X : A_\Gamma \to A_X$.*
(ii) *[1, Prop 2.3(2)] For all $\alpha \in A_X$ we have $\pi_X(\alpha) = \alpha$.*
(iii) *[1, Prop 2.3(3)] The restriction of π_X to PA_Γ is a homomorphism $\pi_X : PA_\Gamma \to PA_X$.*
(iv) *[7, Prop 0.2(vi)] Let A_Γ^+, A_X^+ denote the Artin monoids on $V(\Gamma), X$, respectively. Then $\pi_X : A_\Gamma^+ \to A_X^+$.*
(v) *[7, Prop 0.2(ix)] If $\omega \in A_Y$ then $\pi_X(\omega) \in A_{Y\cap X}$.*
(vi) *[7, Prop 0.3(ii)] For any $\omega \in A_X$ we have $\pi_X(\omega\alpha) = \omega\pi_X(\alpha)$.*

Theorem 3.4. *Let A_Γ be an irreducible Artin group and let z be either a central element of A_Γ of infinite order, or a non-trivial central element of PA_Γ, such that z admits a positive spelling in $V(\Gamma)$. There does not exist any $K \subsetneq V(\Gamma)$ so that $z \in A_K$.*

Proof. Suppose z is central in A_Γ and has infinite order, and suppose that $z \in A_K$ for some $K \subsetneq V(\Gamma)$. Suppose further that z is a positive word on K; that is, there exists an $\widehat{\alpha} \in K^*$ so that $z = \widehat{\alpha}$. Since z is central, $\theta(z)$ lies in the centre of W_Γ. In particular, it has finite order [2]. By possibly passing to a finite positive power of z, we can assume that $z \in Z(PA_\Gamma)$.

Without loss of generality, by possibly passing to a smaller subset of K we can assume that all the letters of K appear in $\widehat{\alpha}$. We claim that there exists $t \in K$ and $s \in V(\Gamma) - K$ so that $m_{st} \neq 2$. Indeed, if there is no such $t \in K$, then $\Gamma = K *_2 (\Gamma - K)$, which contradicts the assumption that A_Γ is irreducible. Let $\widehat{\alpha}'$ denote a cyclic permutation of $\widehat{\alpha}$ beginning with t. Note that every cyclic permutation of $\widehat{\alpha}$ is a conjugate of $\widehat{\alpha}$, and therefore $\widehat{\alpha}$ represents z, since z is central. Consider the map $\pi_{\{s,t\}} : A_\Gamma \to A_{\{s,t\}}$. By Proposition 3.3(6), we know that $\widehat{\pi}_{\{s,t\}}(\widehat{\alpha}')$ starts with the letter t, and by Proposition 3.3(4) we know that $\widehat{\pi}_{\{s,t\}}(\widehat{\alpha}')$ is a positive word. In particular, $\pi_{\{s,t\}}(z)$ is non-trivial. By Proposition 3.3(5), $\pi_{\{s,t\}}(z) \in A_{K \cap \{s,t\}} = A_{\{t\}}$, and therefore $\pi_{\{s,t\}}(z)$ is a positive power of t.

By Proposition 3.3(3), the map $\pi_{\{s,t\}} : PA_\Gamma \to PA_{\{s,t\}}$ is a surjective homomorphism, so $\pi_{\{s,t\}}(z)$ is central in $PA_{\{s,t\}}$. But since $A_{\{s,t\}}$ is spherical, its centre is generated by a power of st. In particular, no non-trivial power of t commutes with any non-trivial power of s in $A_{\{s,t\}}$. Therefore $z = 1$. $\square$

In the case that Γ is not a clique, together with Proposition 1.1 this implies that no central element of A_Γ admits a positive spelling.

3.2. Proof of Theorem 1.2

We are now ready to prove the main theorem of this note.

Theorem 1.2. *Let A_Γ be an Artin group, where $T \subseteq V(\Gamma)$ is the set of cone points of Γ. If A_T satisfies the Center Conjecture, then A_Γ satisfies the Center Conjecture.*

Proof. If Γ is a clique, then $V(\Gamma) = T$, so by assumption A_Γ satisfies the Center Conjecture. Suppose instead that Γ is not a clique.

It suffices to prove the theorem for irreducible Γ. Indeed, if $\Gamma = \Gamma_1 *_2 \cdots *_2 \Gamma_n$ then $Z(A_\Gamma) = Z(A_{\Gamma_1}) \times \cdots \times Z(A_{\Gamma_n})$ so the Center Conjecture holds for Γ if and only if it holds for each irreducible factor Γ_i. The set of cone-points T of Γ is the union of the sets $T_i = T \cap V(\Gamma_i)$ of cone points in each Γ_i. Furthermore, if T splits as $T = T_1' *_2 \cdots *_2 T_n'$ then each T_i is a union of sets $T_{i_1}', \ldots, T_{i_j}'$. Hence A_{T_i} satisfies the Center Conjecture for each i because A_T satisfies the Center Conjecture.

Suppose that Γ is irreducible. By Proposition 1.1, $Z(A_\Gamma) \subseteq Z(A_T)$. Let $A_T = A_{T_1} \times \cdots \times A_{T_m} \times \cdots \times A_{T_n}$ be the irreducible factor decomposition of A_T, where the factor A_{T_i} is spherical if and only if $i \leq m$. Since A_T satisfies the Center Conjecture, $Z(A_T) = \langle z_1, \ldots, z_m \rangle \simeq \mathbb{Z}^m$ where z_i is a positive element of $A(T_i)$ generating $Z(A_{T_i})$ such that any spelling of z_i uses every letter of T_i at least once. Let z be an arbitrary non-trivial element $Z(A_\Gamma)$. Then z can be expressed as $z = z_1^{k_1} \cdots z_m^{k_m}$ for some $k_i \in \mathbb{Z}$ for $i = 1, \ldots, m$. Since z is non-trivial, we know that at least one of the k_i is non-zero. Up to renumbering the factors and replacing z with z^{-1}, we may assume that $k_1 > 0$. There is some $k > 0$ so that $z^k \in PA_\Gamma$. Let $x = z_1^{kk_1}$ and $y = z_2^{kk_{j+1}} \cdots z_m^{kk_m}$ so that $z^k = xy$. Let $\widehat{\alpha}_x, \widehat{\alpha}_y$ denote minimal spellings of x, y, respectively. Note that $\widehat{\alpha}_x$ is positive.

There must exist some $r \in V(\Gamma) - T, t \in T_1$ so that $m_{rt} \neq 2$. Indeed, if this is not the case then $\Gamma = (\Gamma - T_1) *_2 T_1$, contradicting the irreducibility of Γ. Since all cyclic permutations of $\widehat{\alpha}_x$ represent x, we may take t to be the initial letter of $\widehat{\alpha}_x \widehat{\alpha}_y$. Now consider the map $\pi_{T_1 \cup \{r\}} : A_\Gamma \to A_{T_1 \cup \{r\}}$. We have $\pi_{T_1 \cup \{r\}}(z^k) \in Z(PA_{T_1 \cup \{r\}})$.

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By Proposition 3.3(6), $\pi_{T_1\cup\{r\}}(xy) = x\pi_{T_1\cup\{r\}}(y)$, since $x \in A_{T_1\cup\{r\}}$. Since $y \in A_{T-\{T_1\cup\{r\}\}}$, by Proposition 3.3(5), we have $\pi_{T_1\cup\{r\}}(y) = 1$. So $\pi_{T_1\cup\{r\}}(z^k) = x$. Thus, x is a positive central element of $PA_{T_1\cup\{r\}}$ which does not use every letter of $T_1 \cup \{r\}$. By Theorem 3.4, $x = 1$, contradicting the choice of k_1. □

Acknowledgements. The first author was partially supported by NSF grants DMS-2203307 and DMS-2238198. The second author was partially supported by NSF grant number DMS-2317001. We thank the referee for their comments.

References

(1) M. A. Blufstein and L. Paris, Parabolic subgroups inside parabolic subgroups of Artin groups, *Proc. Amer. Math. Soc.* **151**(4) (2023), 1519–1526. doi:10.1090/proc/16289.

(2) N. Bourbaki, *Lie Groups and Lie Algebras. Chapters 4–6.* Elements of Mathematics (Berlin) (Springer-Verlag, Berlin, 2002) Translated from the 1968 French original by Andrew Pressley. doi:10.1007/978-3-540-89394-3

(3) E. Brieskorn and K. Saito, Artin-Gruppen und Coxeter-Gruppen, *Invent. Math.* **17** (1972), 245–271. doi:10.1007/BF01406235.

(4) R. Charney and R. Morris-Wright, Artin groups of infinite type: trivial centers and acylindrical hyperbolicity, *Proc. Amer. Math. Soc.* **147**(9) (2019), 3675–3689. doi:10.1090/proc/14503

(5) R. Charney and L. Paris, Convexity of parabolic subgroups in Artin groups, *Bull. Lond. Math. Soc.* **46**(6) (2014), 1248–1255. doi:10.1112/blms/bdu077

(6) P. Deligne, Les immeubles des groupes de tresses généralisés, *Invent. Math.* **17** (1972), 273–302. doi:10.1007/BF01406236.

(7) E. Godelle, On parabolic subgroups of Artin-Tits groups, *J. Algebra* **632** (2023), 520–534. doi:10.1016/j.jalgebra.2023.06.004

(8) E. Godelle and L. Paris, Basic questions on Artin-Tits groups. In *Configuration Spaces*, Volume 14, pp. 299–311 (Ed. Norm, Pisa, 2012) of CRM Series. doi:10.1007/978-88-7642-431-1_13.

(9) E. Godelle and L. Paris, $K(\pi, 1)$ and word problems for infinite type Artin-Tits groups, and applications to virtual braid groups, *Math. Z.* **272**(3-4) (2012), 1339–1364. doi:10.1007/s00209-012-0989-9.

(10) K. Jankiewicz and K. Schreve, The $K(\pi, 1)$-conjecture implies the center conjecture for Artin groups, *J. Algebra* **615** (2023), 455–463. doi:10.1016/j.jalgebra.2022.10.024

(11) M. Kato and S. ichi Oguni, Acylindrical hyperbolicity and the centers of Artin groups that are not free of infinity (2024), 1–26. arXiv: 2406.09432 [math.GR]. https://arxiv.org/abs/2406.09432.

(12) J. McCammond and R. Sulway, Artin groups of Euclidean type, *Invent. Math.* **210**(1) (2017), 231–282. doi:10.1007/s00222-017-0728-2.

(13) H. van der Lek. Phd thesis, University of Nijmegen, Nijmegen, Netherlands, *The homotopy type of complex hyperplane complements* (1983).

https://doi.org/10.1017/S0013091524000695 Published online by Cambridge University Press