slow convergence in powers of ν/n, it appears desirable to avoid the binomial expansions, even at the cost of considerable complication of the development.

The part S_2 of S is subsequently obtained from the equation

$$n\frac{\partial S_2}{\partial y_1} - \nu\frac{\partial S_2}{\partial y_2} = \left\{\frac{3}{2}\frac{n}{x_1}\left(\frac{\partial S_1}{\partial y_1}\right)^2 + \frac{\partial F_1}{\partial x_1}\frac{\partial S_1}{\partial y_1} + \frac{\partial F_1}{\partial x_2}\frac{\partial S_1}{\partial y_2} - \frac{\partial F_1^*}{\partial y_2}\frac{\partial S_1}{\partial x_2}\right\}_b$$

the subscript b indicating the part having y_1 in the arguments. The first part in the right-hand member contributes terms with new arguments of the form $jy_1 + 4y_2$ and with factors

$$(1 - 2\nu/jn)^{-1}(1 - 2\nu/kn)^{-1}$$

For $j \neq k$ it is easily seen that this product may be written as a sum

$$\frac{(1 - 2\nu/jn)^{-1}}{1 - j/k} + \frac{(1 - 2\nu/kn)^{-1}}{1 - k/j}$$

with

$$\tfrac{1}{2}(1 - 2\nu/jn)^{-1} + \tfrac{1}{2}(1 + 2\nu/jn)^{-1}$$

as special case for $k = -j$. For $j = k$ no such reduction is available.

In S_2, therefore, factors of this form as well as factors of the form

$$(1 - 2\nu/jn)^{-1}(1 - 2\nu/kn)^{-1}(1 - 4\nu/pn)^{-1}$$

will appear.

Obviously, the complexity of these factors increases with each order. On the IBM 650 and IBM 7090 the calculation of S_2 has been completed with the assistance of Mrs Lois Frampton and Dr G. Hori. The calculations for the higher approximations are in progress.

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4. PRESENT AND FUTURE REQUIREMENTS FOR PLANETARY EPHEMERIDES

R. L. Duncombe

The traditional uses of the planetary ephemerides have been twofold:

(*a*) to predict the places of the planets to facilitate their observation; and
(*b*) to form a standard against which to compare observations.

The long series of published national ephemerides have fulfilled these traditional requirements.

The introduction in astronomy of automatic computing techniques, and more recently of electronic calculators, has provided impetus in the evaluation of planetary co-ordinates over extended periods of time, the analysis of deficiencies in existing planetary theories, and the construction of new theories.

Examples are: the evaluation of Newcomb's tables of Venus, and of the Sun, from 1800–2000; the comparison of observations of Mercury and Venus over the past 200 years with Newcomb's tables of their motion; the simultaneous numerical integration of the equations of motion of the 5 outer planets, from 1653–2060; the comparison of Newcomb's theory of

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the Earth with a numerical integration of the equations of motion; and the formation of a new general theory of the motion of Mars.

A requirement made evident by this research is that henceforth planetary ephemerides must be available in machine-readable form.

In America, the U.S. Naval Observatory has been the repository for planetary data in this form. During the past two and one half years, we have filled 44 individual requests from government, university and commercial laboratories for machine-readable planetary ephemerides comprising over 600 000 punched cards. In addition, the Space Technology Laboratory has placed a large part of these planetary data on magnetic tape and has made copies available to numerous other users.

This increasing demand for planetary-co-ordinate data reflects the renewed interest in celestial mechanics engendered by the introduction of electronic calculators, as well as the requirements of space technology for planetary data for the planning of space-probe trajectories, etc.

The comparison of observations with the planetary ephemerides has revealed many minor defects, inconsistencies and some errors. The first step in meeting the requirement for more accurate ephemerides has been the recent introduction of the simultaneous numerical integration of the equations of motion of the 5 outer planets, 1653–2060, by Eckert, Brouwer and Clemence as the basis for the published co-ordinates. This has replaced the previous defective theories of Jupiter, Saturn, Uranus and Neptune, which were all diverging from the observed positions. In addition to improving the basic accuracy, this has also placed the ephemerides of the 5 outer planets on a system of planetary masses consistent with the tables of the inner planets.

The next step in improving the accuracy of the planetary ephemerides will be to replace Newcomb's tables of Mars by Clemence's new theory of the planet's motion. This awaits only the determination of the definitive values of the constants from a comparison of the theory with observations over the past 200 years; a program for this is now in progress at the U.S. Naval Observatory.

Analysis of observations of the Sun, Moon and planets by H. R. Morgan, of Mercury by Clemence and of Venus by Duncombe, have indicated an error in Newcomb's centennial variation of the obliquity. A comparison of Newcomb's tables of the Sun with Herget's numerical integration of the equations of motion of the Earth-Moon system, 1920–2000, confirms the presence of an error in the secular change of the obliquity. Deficiencies in Newcomb's representation of the Earth's latitude were also indicated by this comparison, the largest identified being due to the omission of a periodic perturbation by Saturn with a coefficient of $0''\cdot034$. A comparison of Leverrier's tables of the Sun with the numerical integration indicated his expression for the latitude was considerably more accurate than Newcomb's. The observed error in the variation of the obliquity appears to stem from the neglect of a contribution by the squares and products of the disturbing forces, which were calculated on the assumption that all the planets move in exact ellipses.

At the present time, a new theory of the motion of the Earth is being formed by Clemence, which it is hoped, in the future, will replace the defective tables of Newcomb as the basis for the solar ephemeris.

An analysis of observations extending over the past 200 years compared with Newcomb's tables of Venus, while removing the discrepancy between the observed and theoretical motion of the node of Venus, has indicated corrections to the elements of the Earth and Venus. In order to meet the requirements for an accurate ephemeris of Venus, it would seem necessary in the future to formulate an improved theory of its motion.

With the introduction of radar ranging of Venus, a new dimension has been added to our observations of the planets. Observations extending for several months either side of the April inferior conjunction have been made by the Jet Propulsion Laboratory of N.A.S.A., and the Lincoln Laboratory of M.I.T. These observations, compared with theory, show some systematic discrepancies. Provisional application to the Jet Propulsion Laboratory results of the corrections to the elements of Venus and the Earth summarized in the investigation of the motion of Venus (1) indicate a significant improvement in the agreement of observation with theory. The corrections applicable to the published co-ordinates of Venus and the Earth for the period February to July 1961, resulting from these changes in the elements, are now being evaluated at the Naval Observatory for inclusion in the radar observation reductions.

Continuation of these radar range observations of Venus and also of Mars will provide a valuable source of data which, in combination with the usual angle measurements, may permit a more precise determination of the definitive constants of the theories of their motions.

Requirements have arisen in recent years for accurate computed velocities for the analysis and interpretation of Doppler observations of the planets. Since the published co-ordinates of Mercury, Venus, Earth and Mars are derived from tables as the sums of numerous terms, the accumulation of rounding errors introduces an unevenness in their end figures which make it difficult to derive consistent values of the velocities by numerical differentiation of the co-ordinates. If the requirement continues with the increasing accumulation of radio Doppler observations, it may become necessary to form precise numerical integrations of the equations of motion of these planets, carrying sufficient additional figures to provide velocities having numerical consistency even though the constants of the theory cannot be determined from observation with an equivalent accuracy. The users of such ephemerides would of course be warned that the additional figures were physically fictitious.

The mere storage of, and access to, the extensive tables of planetary-co-ordinate data often overtaxes the memory and data-moving capability of even the larger electronic calculators. Several methods to allow storage of planetary data in a more accessible form have been proposed or are being used. One is the semi-analytical approach where the Keplerian or undisturbed motion is represented by analytical expressions to be evaluated by the computer while the portion due to the perturbations is stored in tabular form. This allows the tabulation of the contribution due to the perturbations at a much larger interval than would have been possible for a tabulation of the co-ordinates themselves, with a resulting economy of storage space.

Another approach is the use of economized polynomial representation of planetary positions over restricted arcs. Here, only the coefficients of the polynomial are required to be stored.

The ultimate and most flexible solution may be the storage and evaluation of the planetary theory itself. Two examples of this come to mind. Clemence's new theory of Mars was evaluated at a four day interval from 1950–2000 on an IBM 650 (a relatively slow electronic calculator); the computation required only one minute per date. Recently scientists at the Jet Propulsion Laboratory have programmed the IBM 7090 to evaluate positions of the Earth directly from Newcomb's theory. For the comparison of Clemence's new theory of Mars with observations it has been necessary to form an ephemeris from 1750–1960. For this purpose co-ordinates of the Earth, consistent with those published in *Astr. Pap. Wash.* **12,** were required for the period 1750–1800. These co-ordinates were evaluated directly from Newcomb's theory for 630 dates on the IBM 7090 in slightly less than 12 minutes.

It is almost certain that no more planetary tables will ever be constructed. Even tables to a reduced accuracy for use in obtaining approximate positions at remote dates will probably be replaced by ephemerides to a low order of precision extending over the whole of historic time.

Although printed co-ordinates, to a reduced accuracy, will be required for the positioning of telescopes, the fundamental planetary ephemerides, accumulating year after year on library shelves, as the standard for comparison with observations seem certain to be replaced by compilations of data in machine-readable form, since undoubtedly, all future analyses of observation residuals will be performed with the aid of electronic calculating equipment.

While the fundamental ephemerides may no longer be printed in the annual volumes, they should continue to be published, to full accuracy, in the same form as the co-ordinates of the Sun (2) and Venus (3) for 1800–2000. As in these volumes, the rectangular co-ordinates as well as the spherical co-ordinates will undoubtedly be required.

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5. THEORY OF THE MOTIONS OF MARS AND THE EARTH

G. M. Clemence

My first-order theory of the heliocentric motion of Mars was published in 1949 in *Astronomical Papers of the American Ephemeris*, Vol. XI, Part II. The theory has recently been completed by the addition of the perturbations of the second and third orders, which will be published late in 1961 or early in 1962 in Vol. XVI, Part II of the same series. My aim has been to calculate all of the inequalities in the motion having coefficients as large as 0".0001 in the case of the periodic terms, and 0".0001 per century in the case of the secular and mixed terms.

The following tabulation shows for the complete theory the number of inequalities in the mean longitude arising from various sources, the mass-factors being denoted by the initial letters of the planets concerned. Thus, S denotes terms factored by the mass of Saturn, V^2 terms factored by the square of the mass of Venus, and J^2ST terms factored by the square of the mass of Jupiter, the mass of Saturn, and the time.

First order		Second order		Third order	
Source	No.	Source	No.	Source	No.
Me	11	V^2 and VM	5	J^2S and JS^2	5
V	45	E^2 and EM	30	J^2ST and JS^2T	37
E	157	J^2 and JM	22	J^2ET	1
J	73	S^2 and SM	5	J^2E	56
S	43	V^2T and VMT	12	Total	99
U	16	E^2T and EMT	47		
N	10	J^2T and JMT	25		
Total	355	S^2T and SMT	14		
		EV	31		
		JE	96		
		SJ	261		
		JV	10		
		SE	13		
		US	7		
		Total	578		

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