THEORY OF SPIRAL STRUCTURE*

C. C. LIN
Massachusetts Institute of Technology, Cambridge, Mass., U.S.A.

It is an honor for me to continue the discussion of galactic spirals following Professor Bok, who has contributed to the subject for many years. My experience has been relatively short. My first encounter with the study of galactic spirals occurred in 1961, when I was invited by Professor Bengt Strömgren to attend a conference on interstellar matter at Princeton, N.J. There I first learned about the winding dilemma from Professor Jan Oort. After the conference, Professor Lodewijk Woltjer, who edited the Proceedings, visited me for about a month. Thus, I began slowly to learn about spiral galaxies and to work on the subject. It turned out to be an extremely rewarding experience, for the observational data were already ripe for theoretical analysis, and the hydromagnetic theory of spiral arms was clearly encountering great difficulties. Since I was not educated as an astronomer, I owe my gratitude to Professors Strömgren, Woltjer, and Kevin Prendergast, on whom I depended for correct astronomical facts as I started my work. Without their help, I would not be standing here today. Later on, I was to receive help from many other distinguished astronomers, including Professor Bok from whom you just heard. I am greatly impressed with the community of astronomers as a dedicated group of scientists.

I wish to dedicate my talk this evening to the memory of a former President of the IAU – who was a good friend of Professor Strömgren's and of many of you present here – the late Bertil Lindblad. As is well known, he first suggested the concept of density waves as a basis for the spiral structure in disk-shaped galaxies. Over a number of years, he and his collaborators attempted to establish this concept by showing that stars in epicyclic motion *tend* to aggregate into a spiral gravitational well, which is then maintained in turn by the excess of stars gathered there. Unfortunately, the mathematical method he used did not enable him to calculate the *collective* behavior of the stars in a convenient manner, and he could not produce the necessary *quantitative* conclusions for comparison with observations in order to substantiate his reasoning. His ideas were therefore not widely accepted.

Modern computing machinery provides one approach to the quantitative treatment of stellar systems. This method was first adopted by Lindblad (1960, 1962) for the study of spiral structure. More recently, other investigators (Miller and Prendergast, 1968; Hohl and Hockney, 1969; Hohl, 1970) have carried out more extensive 'numerical experiments' along similar lines. The most extensive of these, just carried out by Miller *et al.* (1970) involves the consideration of both a gaseous component and a

* The preparation of this paper and much of the work reported here were done with the partial support of the National Aeronautics and Space Administration and the National Science Foundation.

De Jager (ed.), Highlights of Astronomy, 88–121.

stellar component, the latter consisting of approximately 10^5 stars. Their general conclusions support the concept of density waves. The numerical method has the advantage of providing at least a qualitative description of the process of evolution. However, it has not, as yet, yielded any quantitative results for specific comparison with observations. (A short section of their movie was shown and briefly discussed during the discourse.)

Quantitative results can be more readily obtained by analytical methods more suitable for the study of *collective modes*. In order to stay close to comparison with observations, Frank Shu and I formulated, a few years ago, (Lin and Shu, 1964, 1966, 1967) the *hypothesis of quasi-stationary spiral structure* (QSSS hypothesis); that is, we adopt as a working hypothesis the statement that a density wave pattern of spiral form, however it was originated, does exist in a galaxy, simply because it is directly observable. We then work in two different directions from this central position. On the one hand, we examine its consequences and compare them with observational data. On the other hand, we examine the basic dynamical mechanisms to see how such patterns can be initiated and maintained in an almost permanent manner.

The study of basic mechanisms turns out to be – as one would expect – close to the study of *inhomogeneous* electromagnetic plasmas, with magnetic field replaced by rotation. Various aspects of these problems have by now been studied by a number of investigators in the gravitational case. (See Section 3A for references.) Most of my presentation will be devoted to the discussion of the *consequences* of the QSSS hypothesis, but I shall also briefly refer to the problem of the origin and permanence of galactic spirals.

In order to maintain continuity with the lecture by Dr. Bok, let it be mentioned now that one of the first theoretical results obtained is the pattern of the Milky Way System presented by Frank Shu and myself at the *IAU Symposium No. 31* held at Noordwijk in 1966. This is the first theoretical spiral pattern ever worked out on the basis of dynamical principles. We believe that it still remains essentially correct, although minor refinements of the model have since been introduced. Frank Shu has since worked out the spiral pattern of three other galaxies. The significance of his results will be discussed later.

Many other theoretical predictions have by now been worked out and found to be in satisfactory agreement with observations by myself and my collaborators Frank Shu, William Roberts, and Chi Yuan. These I shall report briefly this evening in the following four sections:

1. Nature of the problem: material arms or density waves?
2. General spiral features observed
3. Theory of density waves, including applications to external galaxies
4. Application to the Milky Way system

Toward the end of this paper, the reader will find a discussion of future prospects and some remarks on other observable features and physical processes that we might examine in view of recent developments.

Table of Contents

1. Material Arms or Density Waves?

As is well known, the first problem that faces us is the winding dilemma; i.e., whether the spiral arms observed each contain the same material over many revolutions of the stellar system. Superficially, the answer appears to be a definite 'yes', for the brilliant young stars marking the spiral arms are definitely material objects. However, the spiral pattern for many galaxies must then change appreciably over a period of time of the order of two or three revolutions of the system; for the inner parts of a galaxy are generally rotating at a rate several times that of the outer parts, as exemplified by the Milky Way system. It is most unlikely that such a rapid change of appearance is actually taking place, for the classification of spiral galaxies into Sa, Sb, and Sc types is not only based on geometry but also on other physical characteristics; e.g., the gas content and the mass concentration in the nuclear region. A more subtle difficulty is the implication that the galactic magnetic field must steadily increase in the course of time, if indeed the material arms wind up more tightly. Clearly, both difficulties would be avoided if the spiral structure were associated with a *wave pattern*.

As it turns out, if we adopt the concept of density waves (or density waves modified by hydromagnetic effects), we can explain not only the 'winding dilemma' just discussed, but also a number of other observational features (Section 2). But before we turn to this discussion, let us take up another issue: *the nature of the intergalactic bridge*, such as that connecting M 51 (NGC 5194) and its companion (NGC 5195). Many such interconnected galaxies have been examined by Arp (1969). Such a bridge is doubtless a material arm, and yet it usually joins into one of the principal spiral arms in the wave pattern. The question again arises: are spiral arms material objects or wave patterns?

Obviously, this difficulty disappears if the wave pattern *co-rotates* with the material somewhere in the outerparts of the galaxy; i.e., if they have the same angular velocity. Within the co-rotating radius, the observed spiral structure is a wave pattern. Beyond this radial distance, the spiral arm may well consist of essentially the same material. In fact, it is difficult to expect wave propagation in these outer parts. Notice that there is no observational evidence against a change of the geometrical shape of the intergalactic bridge*. Indeed, as long as a material bridge persists, its shape is expected to change, as the galaxies move relative to each other.

As we shall see later, there are other pieces of supporting evidence for the co-rotation of the spiral pattern with the material objects in the outer parts of a galaxy. These will be discussed below in connection with the process of star formation and the origin of galactic spirals. (Section 3E.)

2. General Spiral Features Observed

We shall adopt a semi-empirical approach in presenting the concept of density waves, beginning with the consideration of a number of general features observed in galactic spirals. In the study of these features, there is one important theme to be kept in mind: *coexistence***. The complicated spiral structure of the galaxies indicates the coexistence of material arms and density waves – and indeed of the possible coexistence of several wave patterns. These features influence but do not destroy one another. When apparently conflicting conclusions are indicated by observations, the truth might indeed lie in the coexistence of material arms and several wave patterns. To be sure, before taking this 'easy way out', one should examine each interpretation of observational data as critically as possible.

Furthermore, as is well known, the different categories of optical objects defining essentially the same spiral arm usually appear displaced relative to one another (cf. Morgan, 1970). The radio features again do not necessarily coincide with the optical features.

Eleven prominent spiral features, including those briefly discussed above, will first be described. It will then be shown that these features can all be explained in terms of the concept of density waves. (See Table I for a brief outline including the theoretical interpretations.) Other support for the density wave theory may be found from detailed observations in the Milky Way system. These data will be discussed in Section 4.

* A. Toomre and J. Toomre have recently demonstrated (in a paper presented at the June 1970 meeting of the American Astronomical Society) that an intergalactic bridge can be produced by the close encounter of a galaxy with its satellite at a suitable relative velocity.

** Zwicky (1957) used the term 'coexistence' to describe the 'blue' and the 'red' components of M 51. We are using it in a broader context.

TABLE I

Observed features in spiral galaxies and their theoretical explanation in terms of density waves

Observation	Theory (QSSS hypothesis)
(1) existence of a grand design	(1) wave pattern
(2) persistence of spiral pattern	(2) quasi-stationary spiral structure
(3) intergalactic bridge joining smoothly into spiral pattern	(3) co-rotation of wave pattern and material in outer regions
(4) spiral pattern usually two-armed	(4) $\Omega - \kappa/2$ nearly constant (Lindblad)
(5) multiple-armed in outer regions	(5) $\Omega - \kappa/m$ nearly constant in outer regions
(6) ring structure	(6) $\lambda \approx 0$ at resonance
(7) H II regions arranged like a string of beads	(7) galactic shock triggering star formation
(8) dust lanes on inner side of bright spiral arms	(8) gas compressed at galactic shock before stars form
(9) abundance distribution of ionized hydrogen varies greatly over the disk; none inside ring, except at center	(9) wave amplitude varies (according to definite laws); becomes tightly wound as resonance is approached
(10) peak of abundance distribution of neutral hydrogen outside of H II distribution	(10) shock mechanism needed for star formation; not available in outer regions
(11) magnetic field generally weak	(11) absence of perennial stretching by differential rotation

(1) *Grand Design*

The first impressive feature of disk-shaped galaxies (already mentioned above) is the existence of a grand design in the form of a spiral pattern extending over the whole galactic disk, usually with other fragmentary spiral features superimposed*.

(2) *Persistence*

The persistence of spiral structure against differential rotation may be inferred from the fact that the spacing between spiral arms, which is used to classify normal spirals into Sa, Sb, Sc types, is correlated with other physical characteristics of the galaxy, such as the total mass, gas content, and concentration of nuclear mass. In particular, a smaller nucleus is generally associated with wider spacing (Sc galaxies).

(3) *Intergalactic Bridge*

In many pairs of galaxies, there is observable an intergalactic bridge, which joins smoothly with a major spiral arm in the main pattern.

(4) *Two-armed Trailing Pattern*

The spiral pattern is generally two-armed, especially in the central regions. As far as is known, the spiral arms are always trailing. (See also footnote to (1).)

* Sometimes these extra spiral features suggest the existence of a secondary spiral pattern, e.g., in M 51. The blue objects in M 51 clearly delineate one two-armed pattern. When this pattern is subtracted out from the ordinary photograph, there appears to remain another two-armed pattern displaced relative to the first by a quarter of a turn. Cf. NGC 6946, often labelled a four-armed spiral (Lynds, 1967).

(5) *Multiple-armed structure*

Multiple-armed structures are often observed in the outer regions of many galaxies. A well-known multiple-armed Sc spriral is M 101 (NGC 5457). These galaxies still have a two-armed structure in the central regions.

(6) *Ring Structure*

A ring structure is often found in the central regions of many normal galaxies (and also in the outer regions of barred galaxies). This inner ring structure is very clearly seen in NGC 5364, the Sc galaxy described by Sandage (1961) in the *Hubble Atlas* as 'one of the most regular galaxies in the sky'.

(7) *Narrowness of Spiral Arms*

The newly formed stars and young H II regions are neatly arranged like beads on a string, forming spiral arms much narrower than the spacing between the arms. This indicates that the process of star formation goes on in restricted regions, but simultaneously over a wide front.

(8) *Dust Lanes*

The principal dust lanes always lie on the inner side of the bright optical arm, although secondary dust features may appear in other forms. Indeed, the dust lanes often delineate the spiral pattern better than the bright stars or H II regions (Lynds, 1970).

(9) *Abundance Distribution of Ionized Hydrogen*

Both the continuum survey of Westerhout (1958) and the recombination line observations of Burke, Mezger and their collaborators indicate a marked deficiency in the abundance of H II regions inside of the '3 kpc circle' in the Milky Way. The abundance increases rather abruptly outside of this circle, and then declines with increasing distance from the center, to negligible amounts at about 13 kpc from the center. This may be compared with the absence of H II regions within the ring of NGC 5364. Extensive data on H II regions are now available from the work of Courtès, Hodge and their collaborators.

(10) *Abundance Distribution of Neutral Hydrogen*

Roberts (1966, 1968) found, by studying a number of external galaxies, that the peak of the distribution of neutral hydrogen usually appears to lie well *beyond* the bright optical structure. A prominent example is the galaxy M 33 (Figure 1). It is also well known that our galaxy conforms to this description. This fact suggests that the formation of stars requires more than the mere availability of hydrogen gas. Some triggering mechanism appears to be essential.

(11) *Magnetic Field*

The magnetic field in the Milky Way appears to be relatively weak, generally of the

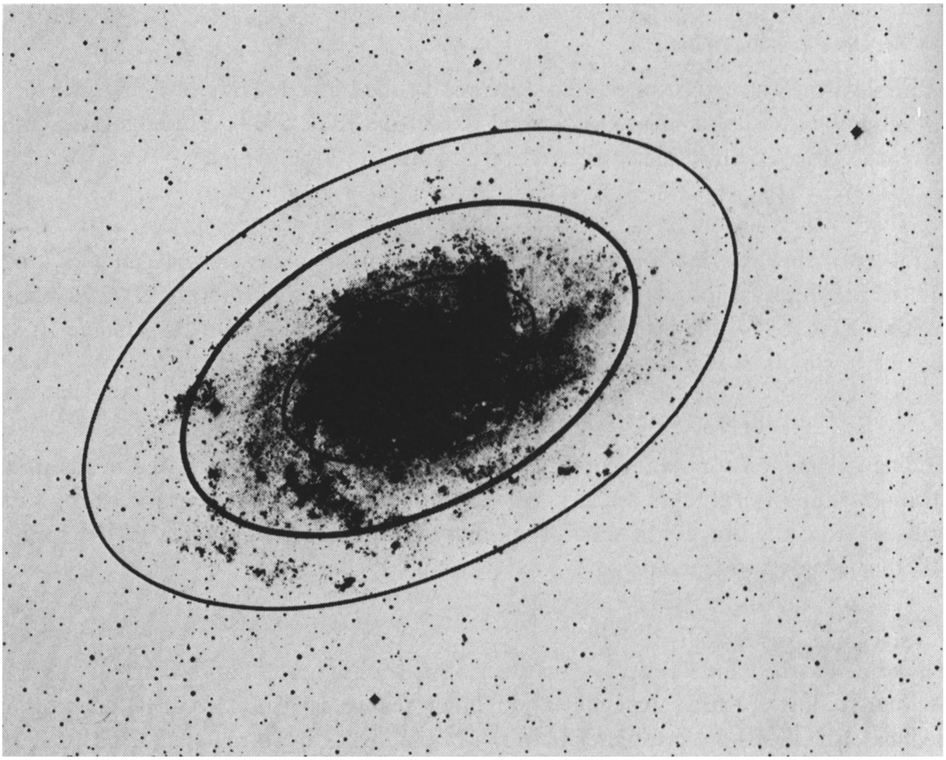


Fig. 1. The galaxy M 33, with region of maximum abundance of neutral hydrogen marked by two solid lines (after Morton Roberts). Notice that this region is entirely exterior to the optical spiral structure.

order of 1–5 μG. The structure of the magnetic field, however, remains largely unknown.

Because of the weakness of the magnetic field, we must look toward gravitational forces for a theory of spiral structure.

3. Theory of Density Waves

A. A SURVEY OF THEORETICAL DEVELOPMENTS BY ANALYTICAL METHODS

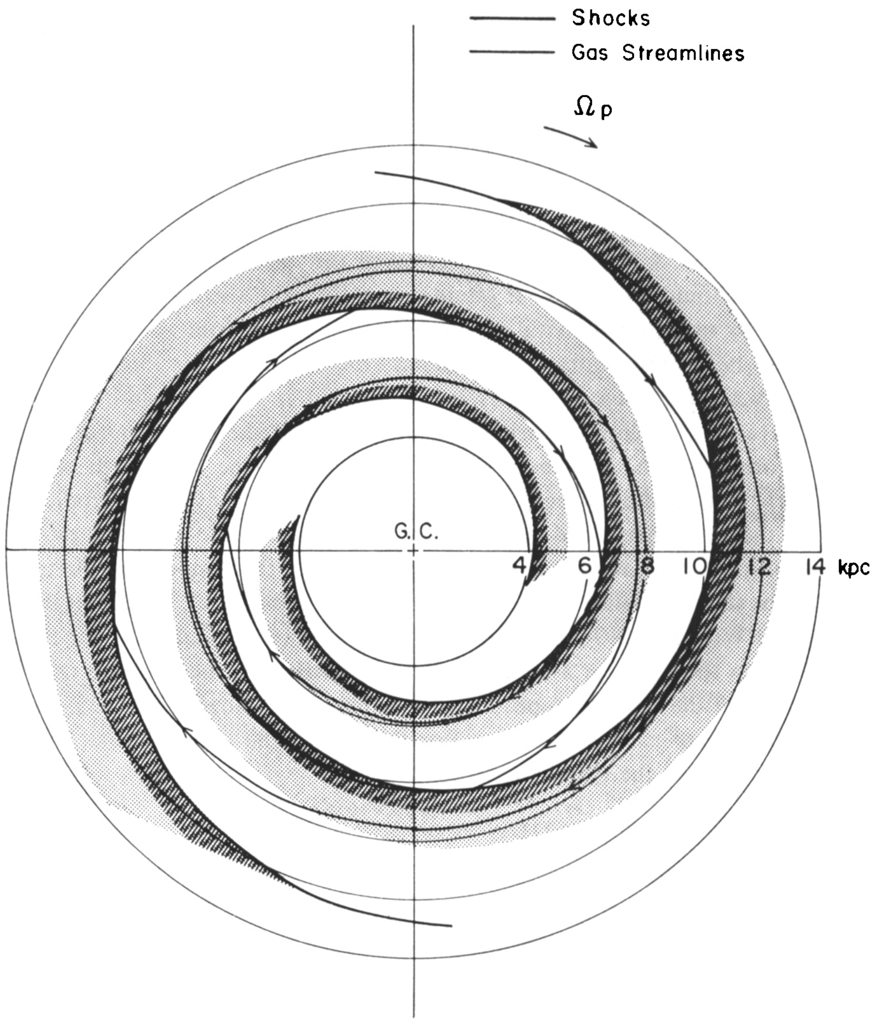
As mentioned above, it is desirable to develop analytical methods to obtain useful results from the density wave theory for comparison with observations. In particular, it is found convenient to adopt the *hypothesis of quasi-stationary spiral structure*. The first theoretical calculations are naturally based on a linearized theory, assuming small density variations in a spiral pattern extending over a large portion of the galactic disk. But it is reassuring to find that, in the case of the Milky Way, one can obtain a spiral pattern that is in general agreement with that observed if one adopts a pattern speed of about one-half of the circular speed in the solar vicinity. To account for the process of star formation, it is necessary to show that the gas increases in density by a large factor (of the order of ten) as it goes into the spiral arm. Such calculations were made by Roberts (1969, 1970) for a two-armed spiral pattern determined by Lin *et al.* (1969) from an extensive study of observational data in our own Galaxy. These results will be discussed in some detail below.

The analytic approach is also more flexible for the exploration of the dynamical processes in order to reach a deeper understanding. Various aspects of these problems have been studied by Contopoulos (1970a, b, c), Goldreich and Lynden-Bell (1965), Julian and Toomre (1966), Kalnajs (1965, 1970), Lynden-Bell (1970), Lynden-Bell and Ostriker (1967), Marochnik and Suchkov (1969), Shu (1968, 1970a, b, c), and Toomre (1964, 1969, 1970). Subjects such as the antispiral theorem, the origin and permanence of galactic spirals are considered in these papers. They are very challenging and not yet completely solved. I regret that I shall not be able to do justice to all these works in this review, but I shall have occasion to refer to some of them in the subsequent discussions. At this point, I only wish to refer you to the survey paper by Contopoulos (1970a) and Shu's papers (1970a, b) for the latest status of the theory. A complete formulation of the linear theory may be found in these papers. I should also call your attention to studies of the dynamics of the whole galactic disk by using self-gravitating models either in the 'cold' case (Hunter, 1963, 1965, 1969a, 1970; Miyamoto, 1969; Rehm, 1965; Yabushita, 1968, 1969a, b) or in the case with dispersive velocities by the use of moment equations for stellar dynamics (Hunter, 1969b). In particular, Hunter noted the tendency for a galactic disk to deform into an oval shape (as did Kalnajs for the case of the uniformly rotating stellar disk, unpublished). Nonlinear spiral waves have been studied in the asymptotic theory by Berry and Vandervoort (1970) in the 'cold' case and by Vandervoort (preprint) for a gaseous disk with pressure. Another important direction for research is the study of stellar orbits by Contopoulos (1970a, b, c), Barbanis (1968a, b), Barbanis and Woltjer (1969), and others in a spiral gravitational field; for these will eventually form the basis for a deeper theory. In particular, they may be used in the future to connect the original approach of Lindblad and Langebartel (1953) with the modern approach of *collective* modes. For the rest of the paper, I shall follow the approach centered on the QSSS hypothesis.

B. IMPLICATIONS OF A SPIRAL PATTERN OF DENSITY WAVES

The QSSS hypothesis is suggested by the existence and the persistence of a grand design. To explain the correlation of the spacing between spiral arms with the concentration of nuclear mass, however, requires a quantitative calculation related to the mechanism for sustaining the density waves. Similarly, a detailed analysis is needed to explain why a two-armed structure is preferred, why a multiple-armed structure often occurs on the outer parts of certain spiral galaxies, and why the spiral arms often wind up tightly around a ring structure, which is at a substantial distance from the center of a galaxy. These calculations will be presented in the written text in the next section. I regret that, because of limitation of time, I shall have to omit this part from my oral presentation. On the other hand, it is possible to give at least a qualitative explanation of the other five observational features [(7)–(11)] described above, provided that we are willing to accept certain simple physical pictures obtained from calculated results (Roberts, 1969; see also earlier work of Fujimoto discussed in Roberts' paper).

Consider an observer in a coordinate system co-rotating with the wave pattern. To this observer, the pattern is fixed , and material objects are moving through its gravitational field. We may visualize them as moving along imaginary stream tubes.



Spiral Pattern in the Galaxy

Fig. 2. Theoretical diagram showing the spiral structure of a galaxy (approximately the Milky Way) and the associated kinematical behavior of the gas and the stars (after W. W. Roberts). Notice the large scale galactic shocks that precede regions of star formation.

https://doi.org/10.1017/S1539299600000071 Published online by Cambridge University Press

Calculations show that the gas is suddenly compressed near the minimum of the gravitational potential. We shall call this compression along a wide front of many kiloparsecs a *galactic shock* (Figure 2). At a shock front, a streamline suddenly changes its direction, and eventually closes on itself after another similar turn at the other shock. The sudden compression of the gas collects the existing dust particles into a prominent *dust lane*, whose strength may be further enhanced by the formation of more molecules and dust particles, induced by increased gas density. This dust lane is very *narrow*, since the calculated results show that the sudden compression of the gas at the galactic shock is followed by a rather rapid decompression.

This same compression process also provides a triggering mechanism for star formation, by bringing the individual gaseous clouds into a state of continuing gravitational collapse. Once begun, this process will continue for each cloud complex and for the individual clouds of the complex, even after decompression sets in on the scale of the spiral arm. Brilliant stars are therefore formed almost simultaneously *over a wide front* of many kiloparsecs. These stars and the associated H II regions are aligned as a narrow spiral arm, like beads on a string, for their lifetime is on the order of 10^7 yr, and there is very little radial displacement over such a short period of time. *The observed spiral arm is therefore the brilliant manifestation of very young objects whose location and arrangement are controlled by the invisible gravitational field determined by the older stars*, – an establishment behind the scenes.

The formation of these bright young objects is triggered off by compression on a large scale. However, this compression does not guarantee the formation of stars, since the intial conditions of the gas also play an important role. Thus, the distribution of H II regions is often found to be *patchy*, and the spiral arms are often better delineated by the dust lanes (Lynds, 1970).

The bright stellar arm is expected to be somewhat separated from the dust lane where the compression occurs, for there is a time of the order of $30–50 \times 10^6$ yr required for the collapse of the gaseous clouds into stars. It may be shown that this separation is generally of the order of several hundred parsecs. The exact extent of the separation depends, among other things, upon the component of the velocity at which the gas rushes through the dusty region in the direction normal to the shock. As it turns out, this relative velocity decreases with increasing distance from the galactic center, and hence the separation is minimal towards the end of the spiral pattern. As we move inwards, greater separation is expected. Such a change can indeed be seen in M 51. At the inner resonance ring, where the spiral features are tightly wrapped together, there may again be very little separation. However, at least in M 51, up to the point where a dust lane may be traced, the winding is not yet so tight that a substantial separation is still noticeable.

The process of star formation described above also explains why the *abundance* distribution of neutral hydrogen does not always match that of ionized hydrogen, for the latter can be readily produced only by density waves, and the strength of such waves varies greatly over the galactic disk. However, the theory enables us to calculate these variations. In particular, a spiral density wave may terminate by winding into a ring, and hence there are no H II regions inside*. In the outer parts of a galaxy, even though neutral hydrogen may be present beyond the tip of the observed spiral arm, there are no strong density waves, and hence star formation on a large scale cannot take place and no spiral structure is observed. In this picture, the tip of the spiral arm is roughly the location where the material objects co-rotate with the wave pattern.

In the coordinate system co-rotating with the spiral pattern, the gaseous motion is essentially steady. The magnetic field lines are expected to coincide with the stream line, and run in a direction nearly parallel to the spiral arm (except where the field is very weak). There is no perennial stretching of the lines of the magnetic field to increase its strength. The field remains weak (if it is initially so) and plays only a secondary role in the dynamical processes (e.g., in determining the rolling motion inside of a spiral arm; cf. Fujimoto and Miyamoto, 1970).

One may also speculate, from our dynamical picture, about the magnetic lines associated with a local spur of material produced by differential rotation of a clump of gas. The field lines would lie roughly in the direction of the material arm, but could point in opposite directions, perhaps above and below the galactic plane (cf. Roberts and Yuan, 1970). This is compatible with certain observed features in the Orion arm.

Recently, Mathewson and his collaborator (Mathewson, 1968, 1969; Mathewson and Nicholls, 1968) suggested the existence of a local helical magnetic field in addition to a field on a larger scale. I do not feel competent to comment on these suggestions, but would refer you to the papers by Woltjer (1970) and Fujimoto and Miyamoto (1970) at the Basel Symposium.

C. SPIRALS WITH MODERATELY SMALL PITCH ANGLE**

Most of the spirals, – even the Sc's – have their principal spiral arms inclined at moderately small pitch angles. For such spirals, the asymptotic method of WKBJ (see Morse and Feshbach, 1953) may be applied, and simple relationships can be obtained. We shall discuss some of the general conclusions obtained from such an approach in the present section. In the next section, we shall apply the results to a general discussion of the spiral patterns in the Milky Way and three other external galaxies. The detail treatment of observational data in the Milky Way will be presented in Section 4.

1. The theory predicts the possible existence of a density wave pattern propagating around the galaxy with a pattern speed Ω_p. The pattern can only extend over the part of the galactic disk for which

$$\Omega - \kappa/m < \Omega_p < \Omega + \kappa/m,$$

where $\Omega(\varpi)$ is the circular velocity (in angular measure) of the material around the galactic center, at a distance ϖ therefrom, $\kappa(\varpi)$ is the epicyclic frequency, and m is an integer giving the number of arms.

* Except possibly at the center, where the H II regions may be formed by different mechanisms.
** Omitted from oral presentation.

This relationship implies that only two-armed spirals are expected to occur in prevalence while multiple-armed structure can be expected only in the outer parts of a galaxy. This can be easily seen if we examine the trend of the curves for $\Omega(\varpi) \pm \kappa(\varpi)/m$ in the Galaxy (m=integer). The flatness of the curve $\Omega - \kappa/2$ (Figure 3) is at the root of the conclusion. We may expect all other galaxies to have a similar behavior simply because $\Omega - \kappa/2 = 0$ for a galactic disk in uniform rotation. In the outer parts, patterns with other values of m are permitted. Thus a composite structure with two arms inside and more arms outside is expected. The Sc galaxy M 101 is an excellent example for a composite structure of this kind.

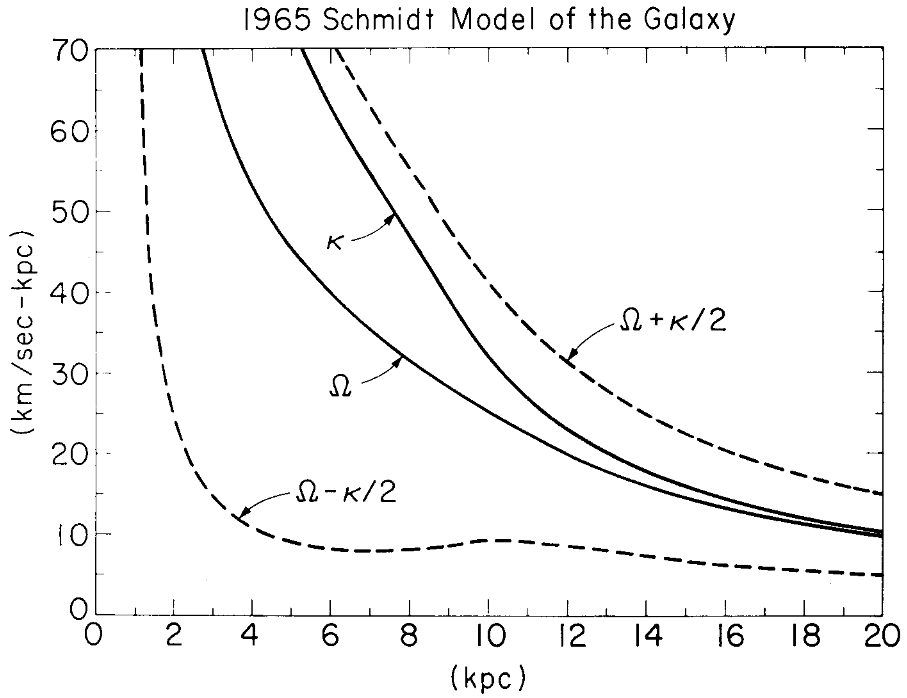


Fig. 3. Rotation curve, etc., of our own Galaxy according to the Schmidt model of 1965. (Symbols defined in Figure 4). Notice the flatness of the curve of $\Omega - \kappa/2$ for a substantial part of the Galaxy, as noted by Bertil Lindblad.

2. A dispersion relationship can be obtained connecting the spacing (or wavelength) between the spiral arms with the frequency at which the material moves through the wave pattern. In dimensionless form, we have the curve shown in Figure 4 (after Lin and Shu, 1967). The typical length scale is $\lambda_* = 4\pi^2 G\sigma_*/\kappa^2$, where G is the gravitational constant, σ_* is the mass surface density, and κ is the epicyclic frequency. The parameter ν is the frequency in dimensionless form: $\nu = m(\Omega_p - \Omega)/\kappa$, where $\Omega(\varpi)$ and Ω_p are defined above.

3. Note that the spacing λ approaches zero as the points of resonance (if they exist)

are approached and the spiral pattern should terminate as a ring both inside and outside. The inner ring is observed in many disk-shaped galaxies but not the outer ring, since it is often too far out for the theory to be applicable*. In these outer regions, gaseous clumps are formed by gravitational collapse, and regular density waves are not favored. Within the inner resonance ring, there are no strong density waves, and hence no H II regions. A possible exception is the very center of the galactic nucleus, where H II regions can be formed by other means of condensation.

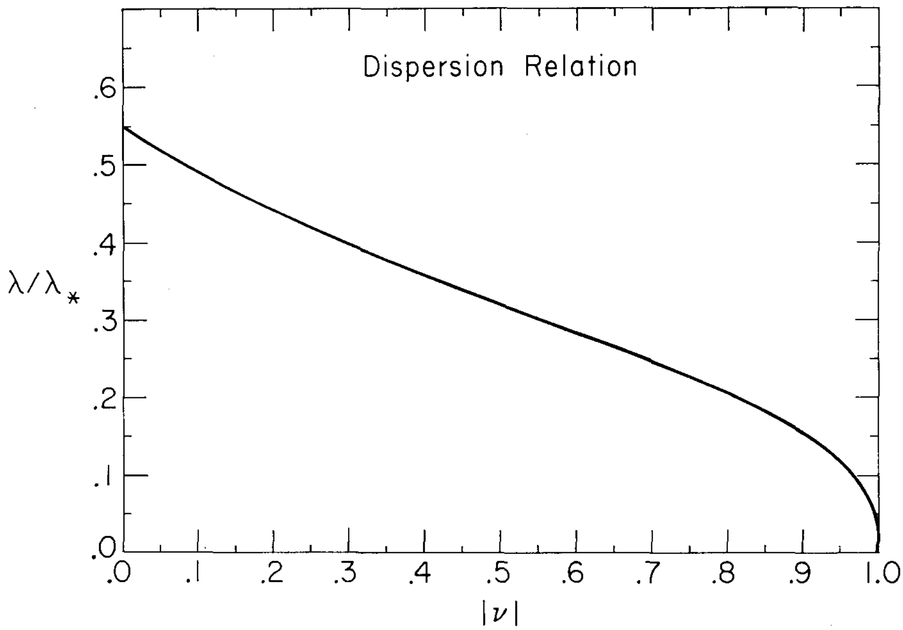


Fig. 4. The Dispersion relationship. Symbols are defined as follows: $\nu = m(\Omega - \Omega_p)/\kappa$, where $m = 2$ is the number of arms, $\Omega(\varpi)$ is the circular velocity in angular measure at galactocentric distance ϖ, $\kappa(\varpi)$ is the epicyclic frequency, λ is the spacing between two neighboring arms, and λ_* is a typical length scale defined by $\lambda_* = 4\pi^2 G\sigma_*/\kappa^2$, where σ_* is the projected surface density of the stars, and G is the constant of universal gravitation. (After Lin and Shu, 1967). Note that a ring structure ($\lambda = 0$) is associated with resonance ($|\nu| = 1$).

4. The ratio of the scale λ_* to the diameter of the galactic disk can be shown to be roughly proportional to the ratio of the mass of the galactic disk to the total mass, including the galactic nucleus. Thus, Sc galaxies with smaller nuclei tend to be loosely wound.

We have thus explained why a two-armed structure is preferred, why a multiple-armed structure can only occur in the outerparts of certain spiral galaxies, and why the spiral arms often wind up tightly around a ring structure which is at a substantial distance from the galactic center. We have also given one reason for the loose structure

* The situation may be different for barred spirals.

of Sc spirals. Another reason will be found from a discussion of the pattern speed at which quasi-stationary spirals are maintained.

All the eleven general observational features have been accounted for. We shall now turn toward more specific discussions.

D. SPIRAL PATTERNS OF THE MILKY WAY, M 33, M 51 AND M 81: DETERMINATION OF PATTERN SPEED

For spirals with moderately small pitch angles, the theory permits us to determine the spiral pattern in a simple manner once the pattern speed is known. In addition to the Milky Way, three other galaxies, M 33, M 51 and M 81, have been studied by Shu *et al.* (1970). In each case, after the mass model is known*, there remains only one adjustable parameter; namely, the pattern speed Ω_p. However, if we wish to make the calculated spiral pattern agree with observations, there is practically no freedom in its choice, for the spiral structure depends very sensitively on the pattern speed. In particular, whenever there is an inner resonance ring where the spiral structure terminates, a convenient determination presents itself; for the theory asserts that, near such a ring, the stars are encountering the spiral gravitational field at the local epicyclic frequency. Such a ring appears to exist in the cases of the Milky Way, M 51, and M 81.

The results are shown in Figures 5, 6, 7, 8; there is good agreement between theory and observations. The case of the Galaxy was briefly mentioned above, and will be treated in greater detail below. In the case of M 51, there is an inner resonance ring, within which giant H II regions are deficient. Good agreement is obtained with the primary spiral pattern. There also appears to exist a long-wave pattern and a secondary two-armed pattern which is displaced by an angle of 90° from the first. Their significance will be discussed in Section 3E.

The general features of M 81 are similar to those in M 51. In the case of M 33, there is no inner resonance ring, and the spiral pattern terminates at the nucleus where there is a short bar. In both cases, there are also indications of secondary spiral patterns, although they are not as clear as in M 51.

In all three cases, Frank Shu found that the pattern is *co-rotating* with the material in the outer parts of the galaxy. This is a very encouraging indication that the mechanism proposed earlier (Lin, 1970) to explain the origin and permanence of spiral patterns is essentially correct. We shall consider this presently. The result is also compatible with the processes of star formation considered in Section 3B, and our description of the intergalactic bridge in Section 1.

E. ORIGIN AND PERMANENCE OF GALACTIC SPIRALS

So far, we have adopted a semi-empirical approach. The QSSS hypothesis has led to a number of deductions in agreement with the major features observed in galactic spirals. Just how the observed spiral patterns originated is not yet fully explained. They

* To be sure, there is uncertainty in the observed rotation curve, and hence of the mass model. But these variations are not large enough to alter the following general conclusions.

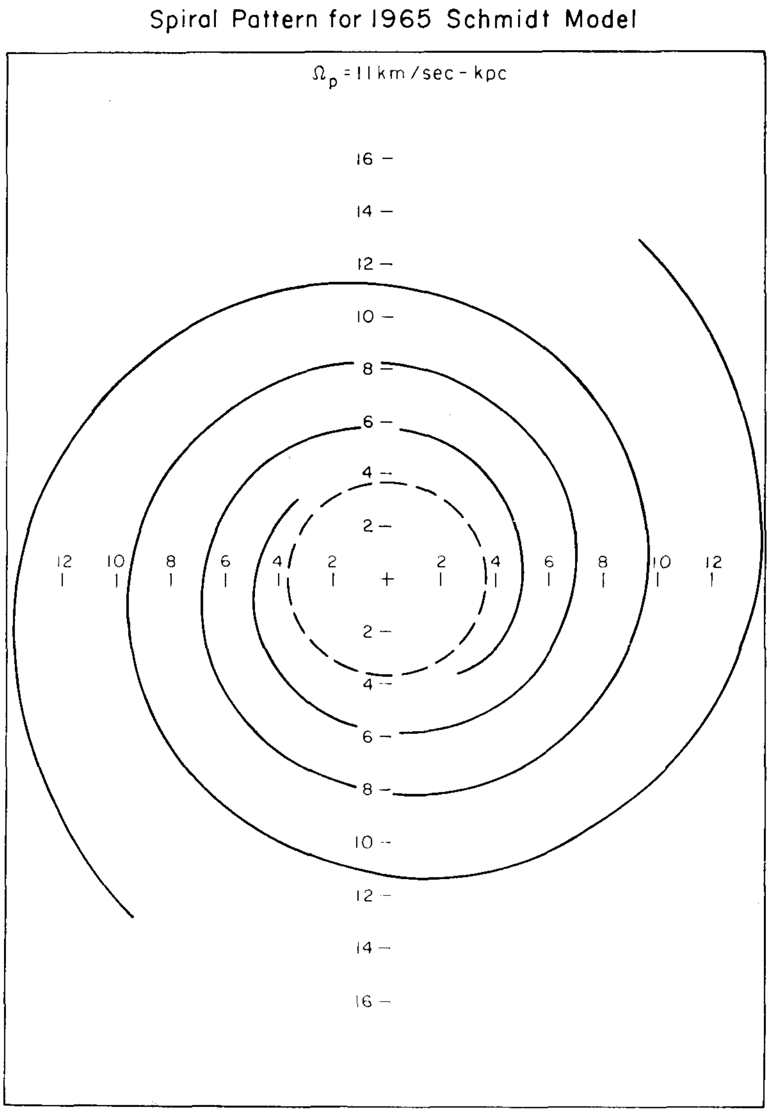


Fig. 5. Spiral pattern of the Milky Way.

might be caused simultaneously by several mechanisms. Both excitation external to the galaxy and instability internal to the system may be present. However, from the prevalence of spiral galaxies without nearby companions, it appears that *there must be a mechanism inherent to the galactic system itself.*

Search for an unstable spiral normal mode via the solution of the boundary-value problem (Hunter, 1965, 1969, 1970; Kalnajs, 1965, 1970; Miyamoto, 1969; Shu, 1970b) is not yet wholly successful. Indeed, the problem of setting realistic boundary condi-

Fig. 6. Spiral pattern of M 33, comparison of theory and observations (after Shu).

tions in the outer parts of the galaxy may be very complicated, because of the clumpy structure of the gaseous component.

One of the mechanisms proposed (Lin, 1970) for the initiation and the maintenance of density waves is the Jeans instability in the outer regions of the galaxy*. Once the gaseous condensations are produced by the process of gravitational clumping and stretched into material arms (Goldreich and Lynden-Bell, 1965) – aided by the excitation of density 'wakes' in the stellar sheet (Julian and Toomre, 1966) – short trailing waves may be initiated that propagate both inwards and outwards as a group (Toomre, 1969). The outward-propagating waves cannot be strong, for the difference is small between the wave speed and the material speed. The clumpy and irregular distribution of gas in this area is also expected to prevent the waves from being well organized. For the inward-propagating waves, the energy density tends to pile up in the interior where the 'group velocity' of such waves is small (Toomre, 1969; Shu, 1970a). If there is no inner resonance ring, the waves would reach the center and be 'reflected' via the resultant bar-like structure (e.g., in the case of M 33). If there is a resonance ring, the

* Jeans (1929, p. 349) specifically stated "that condensations cannot form in elliptical nebulae – or in the central masses of spiral nebulae, but they must inevitably form in the equatorial extensions of the spiral nebulae". However, in the work of Jeans, the effect of differential rotation of the galactic disk was not included. Toomre (1964) did the first work including such an effect for a stellar system. The simpler case of the gaseous system was then given by Lynden-Bell (1967).

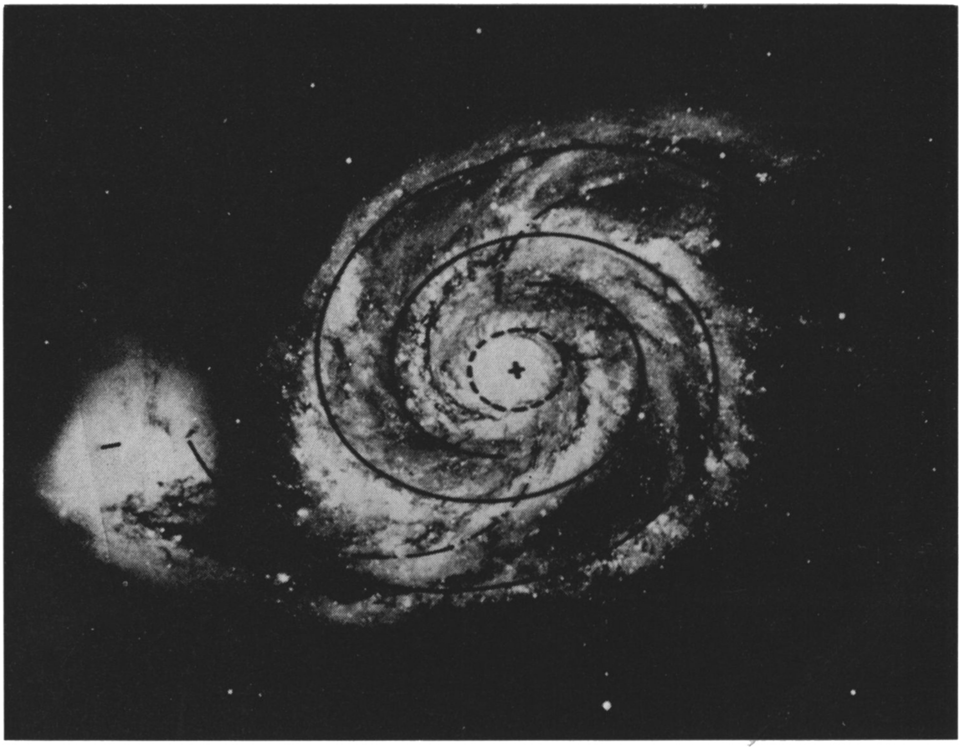


Fig. 7. Spiral pattern of M 51, comparison of theory and observations (after Shu). The dotted curve shows the 'long-wave' pattern.

waves would be 'reflected' at the ring by a two-step process (e.g., in the cases of the Milky Way, M 51, and M 81). First, the waves would be absorbed there by Landau damping according to the linear theory (James Mark, private communication). This damping process would be 'saturated' and cease after about a billion years (Contopoulos, 1970c) and the ring would be deformed into two oval structures with their major axes at right angles to each other (Contopoulos, 1970a, 1970c). The gravitational influence of these oval structures is *in step* with the waves at the outer ring of initiation, and thereby constitutes an *effective* feedback mechanism, which maintains the permanence of the wave pattern. It has been suggested (Lin, 1970) that this 'reflection' process may take the form of a 'long-wave pattern'. Indications of such a pattern have been found by Shu *et al.* in M 51 (cf. Figure 7).

The mechanism just discussed can be subjected to two observational tests. First, a correct spiral pattern must be obtained if the wave pattern is assumed to be co-rotating with the material in the outer parts of the galactic disk. Secondly, the 'piling up' of the wave energy would lead to an increase in the number density of H II regions towards the galactic center even when there is little increase of H I. As mentioned above, the first test is satisfied, as shown by Shu *et al.* in their investigations of the Milky Way and of the external galaxies M 33, M 51 and M 81. The second (partial)

https://doi.org/10.1017/S1539299600000071 Published online by Cambridge University Press

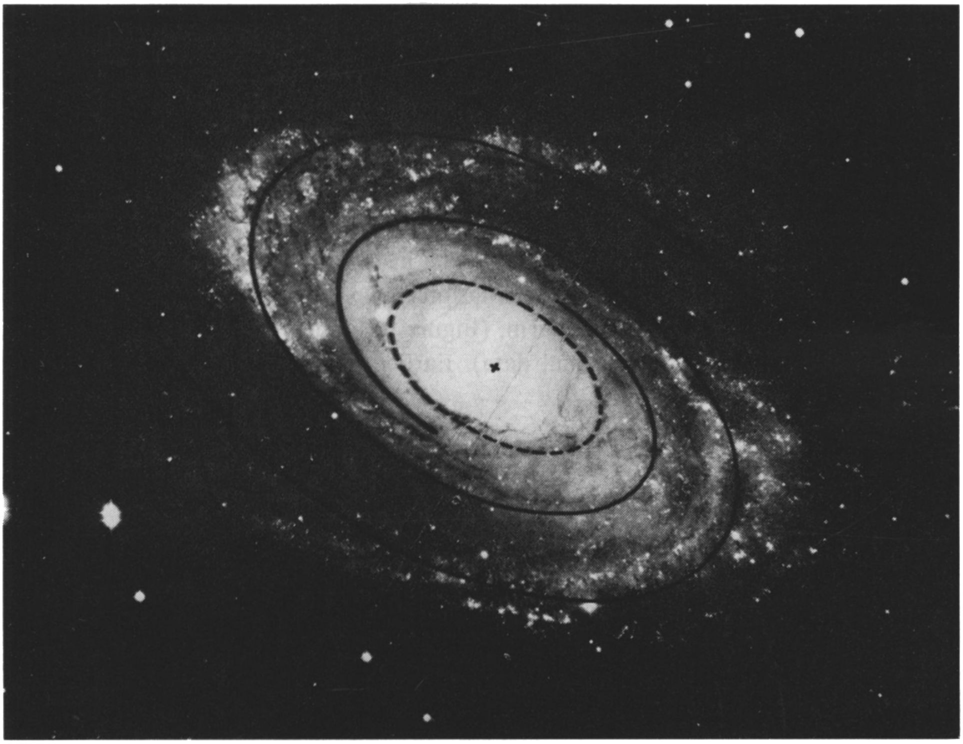


Fig. 8. Spiral pattern of M 81, comparison of theory and observations (after Shu).

test will be taken up (Section 4A) as part of the comparison of the theory with observations in the Milky Way system. This system has the fortunate feature that its H I abundance changes very little over a distance in which its H II abundance changes by about one order of magnitude. For other galaxies such as those studied by Hodge (1969), a more detailed knowledge of their H I distribution is required for a complete analysis.

4. The Milky Way

The theory discussed above can be tested against more detailed observations in the Milky Way system. Indeed, many such data clearly suggest the existence of density waves. However, it should be made clear at the very outset that the determination of the spiral structure of the Milky Way is intrinsically a very difficult task. The observer, being inside of the system, can be easily distracted by local features. Under such circumstances, it is difficult to distinguish a double two-arm spiral pattern, such as that seen in M 51, from a single two-arm spiral pattern which is doubly as tight. What I shall present is therefore based on a self-consistent dynamical model which is in reasonable agreement with a number of observations of various kinds. Any other suggestion should also be checked against the observational data discussed below.

These include the distribution and motion of (1) neutral hydrogen, (2) ionized hydrogen, and (3) stars.

A. NEUTRAL HYDROGEN

On the basis of the density wave concept, it is expected that, together with associated radial motions, there exists a higher circular speed of gaseous motion on the outer side of a concentration of hydrogen gas and a lower speed on the inner side of such a concentration, than would have been observed in its absence. Lin *et al.* (1969) used this to correlate the oscillations in the rotation curve with the location of the Sagittarius arm and the Norma-Scutum arm. (Figure 9, Figure 10. See Kerr, 1969, for a survey of the relevant observational data). Earlier, Shane and Bieger-Smith (1966)

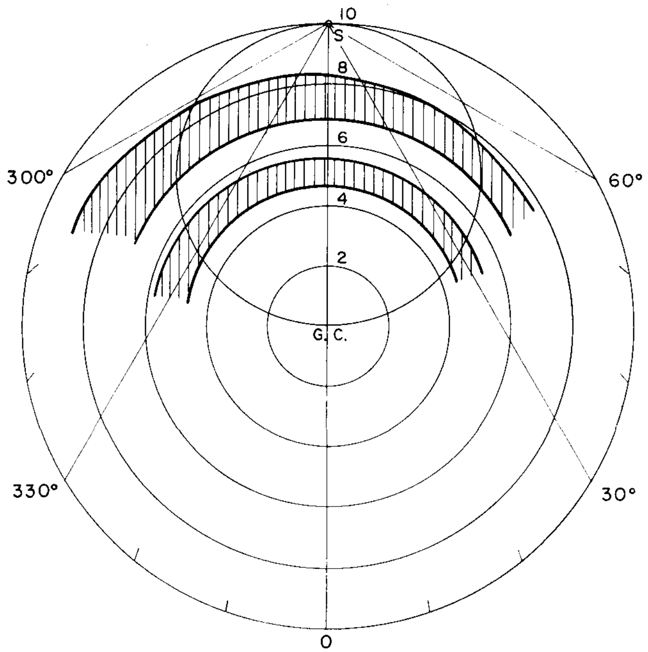


Fig. 9. Locating gaseous arms for variations in the rotation curves shown in Figure 10.

represented apparent irregularities in the measured rotation curve by attributing them to non-circular motions. A high stream of gas outside of the Sagittarius arm was noted by Burton (1966). More recently, Burton (1970), Burton and Shane (1970) found that much of their data could be satisfactorily interpreted only by invoking streaming motions similar to those implied by the concept of density waves.

Once noncircular motions are admitted, the analysis of the observational data becomes very complicated, and a process of successive approximation (or trial-and-error) has to be adopted. From a theoretical standpoint, one would then begin by calculating a spiral pattern and then attempt to reproduce the observational data with

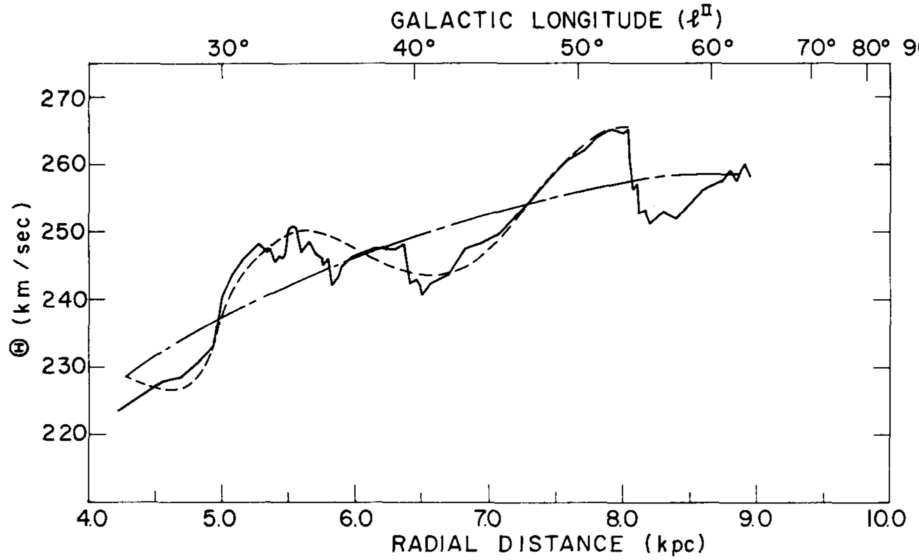


Fig. 10a.

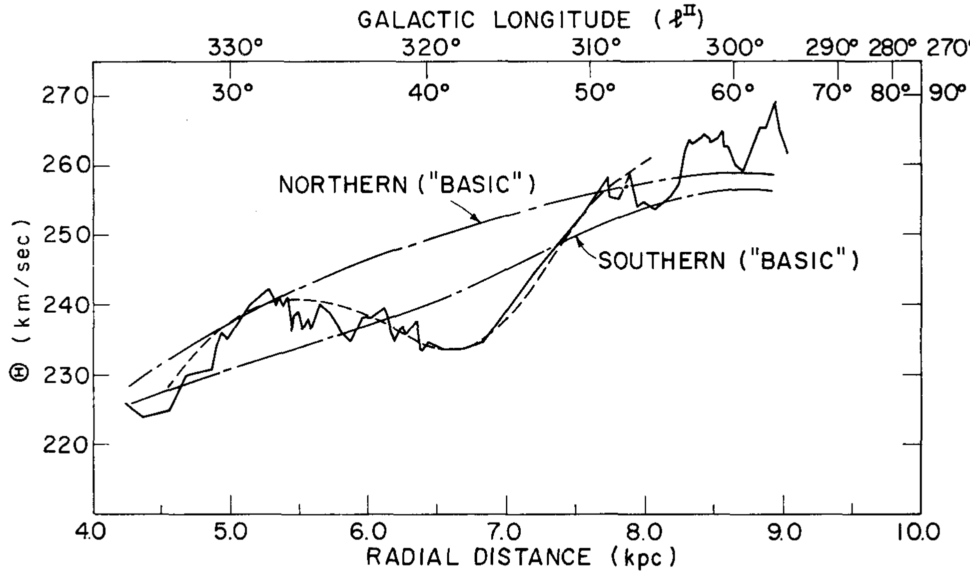


Fig. 10b.

Figs. 10a, b. Rotation curves: (a) 'Northern' Hemisphere; (b) 'Southern' Hemisphere. Observation (Kerr, 1964) ———; Theory — — — —; Mean motion (theory) ·—·—·

theoretical profiles. Such calculations were started by Yuan about two years ago and a preliminary report was given by him at Basel (Yuan, 1970a). His current results are reproduced in Figure 11. The overall comparison between theory and observations for all galactic longitudes is reasonably satisfactory – at least to the extent that the grand design may be said to be properly represented – but refinement of the theoretical model is certainly needed in order to account for detailed features*, especially in the central regions, ($|l^{\mathrm{II}}|<30°$).

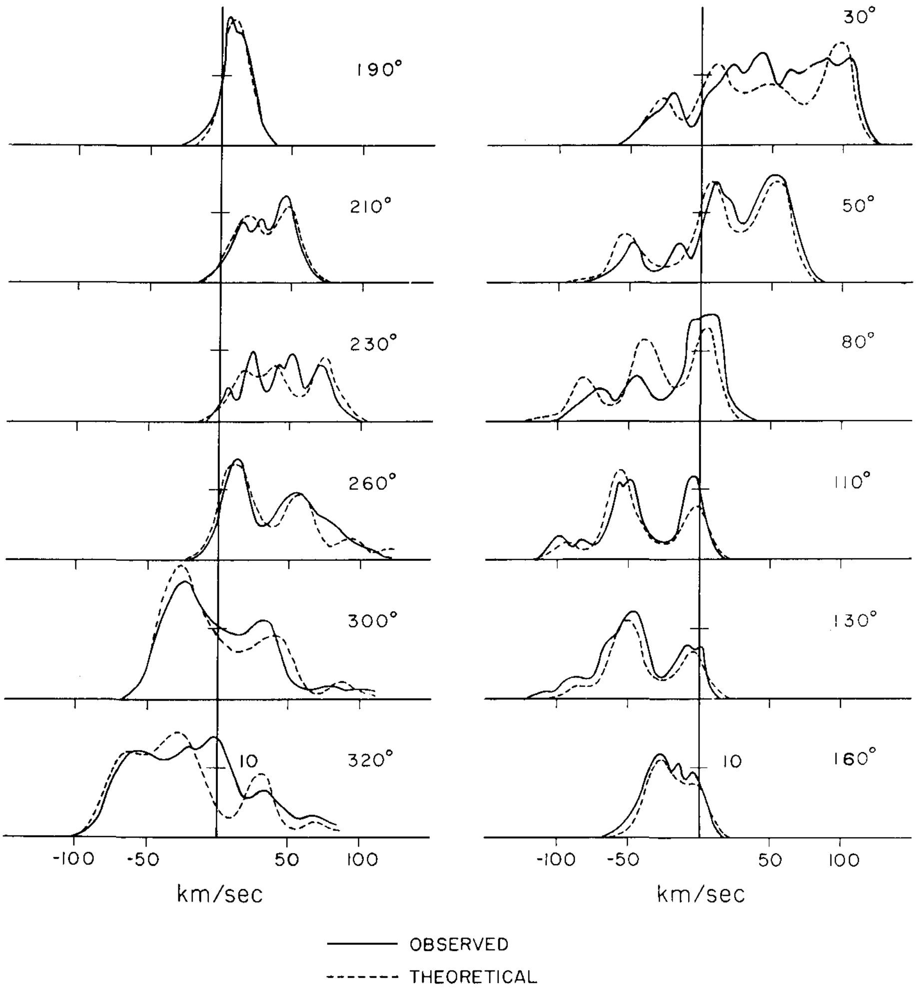


Fig. 11. Theoretical velocity-intensity profiles and their comparison with observations (after Yuan). Data for $|l^{\mathrm{II}}| < 30°$ are not analyzed since they are dominated by features in the central regions not related to a simple spiral structure.

* For example, in the direction of the galactic center, Yuan (1969a, p. 881) pointed out that the observed motion of the gas is compatible with a theoretically expected inwards motion of 7 km/sec, provided that there is an overturning motion (Henderson, 1967) with an amplitude of 5 km/sec.

Independently of Yuan's work, Burton (1970), Burton and Shane (1970) used the density wave theory to analyze their observational data by constructing theoretical contour maps in the longitude-velocity plane. These are restricted to certain northern directions, but more detailed than Yuan's analysis. The results obtained are more satisfactory than those obtained by using a circular model. It is found that an additional expansion field is needed within 5 kpc at least in the region around $l^{II}=30°$. This may well be associated with the oval shaped distortion of the mass distribution near the resonance ring, as suggested by the nonlinear analysis of Contopoulos.

B. IONIZED HYDROGEN

The continuum survey by Westerhout (1958) shows that the bulk of the galactic continuum radiation comes from a narrow range of latitude centered about the galactic plane and that most of the ionized hydrogen must be concentrated in a ring somewhat outside of the 4 kpc radius ($l=\pm 26°$). From this distance outwards, there is a decline of the density of ionized hydrogen to practically nothing at 13 kpc, while the amount of neutral hydrogen increases to a peak value. As mentioned above, this feature is consistent with that discovered by Roberts (1966, 1968) in a number of external galaxies.

The theoretical curve shown in the figure (Figure 12) is a rough estimate of the abundance ratio of ionized hydrogen to neutral hydrogen at various galactocentric distances. The estimate is based only on the strength of the gravitational field in the density wave pattern (See Figure 3 in Shu, 1970b). The calculations were made by Stuart Feldman (private communication), based on data analyzed by Mezger (1969). Other factors* that may influence the amount of ionized hydrogen produced are not taken into account. It is remarkable that there is still general agreement between theory and observations. Within the ring of resonance at a radial distance of about 4 kpc, there is little ionized hydrogen expected, and indeed very little is observed. The rather sudden decline of the abundance of ionized hydrogen in the transition region from 5 kpc inwards is also well described by the theoretical results of James Mark (private communication).

The data analyzed by Mezger (1969) are based on the work of Reifenstein *et al.* (1970), and of Wilson *et al.* (1970). In Mezger's original presentation of the data, the five giant H II regions near or inside the 4 kpc circle were evenly distributed inside the circle, for their distances are not well known. Actually, four of these may well be associated with the very center of the galaxy**. A fifth one (G347.6+0.2) is not far

* One important factor is the relative velocity between the gas and the density wave. This factor is especially important in the outer regions where the relative velocity is small and hence the shock is weak. In particular, there should be little star formation near co-rotation. For most part of the data shown in Figure 12, this factor is however not very important.

** In a private communication, dated May 11, 1970, Dr Mezger kindly informed me that these H II regions have physical characteristics different from those in the spiral arms. The author wishes also to thank Drs B. F. Burke and W. W. Shane for the discussion of the data and their interpretations.

from features associated with the '3 kpc arm'. Thus, the giant H II regions inside of the resonance ring are not relevant to the problem at hand.

A similar analysis could be made from the data of Hodge (1969) for external galaxies. Since the distribution of neutral hydrogen may not be as uniform in the region in question as in the Milky Way, definite conclusions cannot be drawn until more detailed H I observations become available.

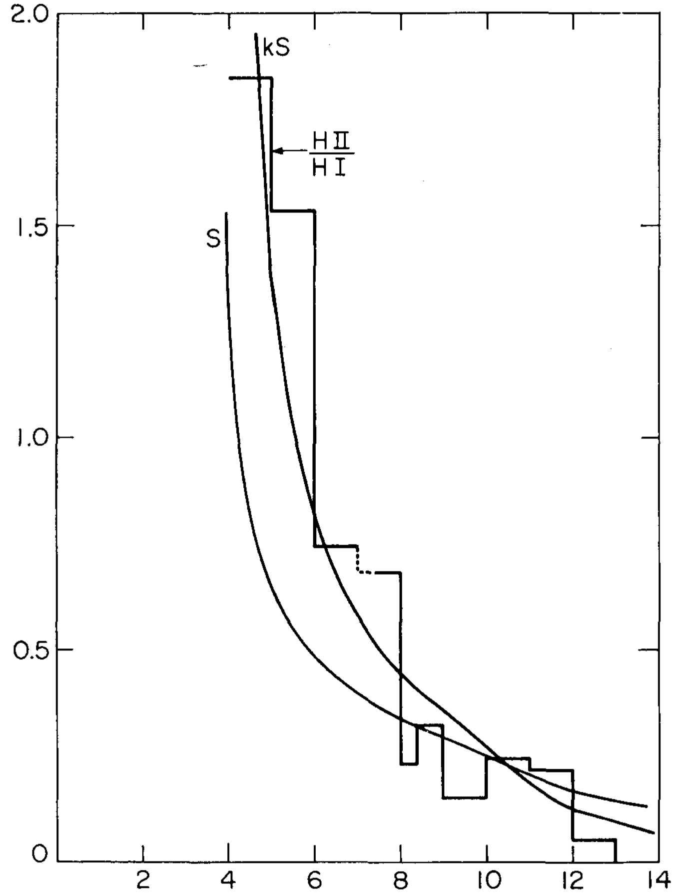


Fig. 12. Relative abundance distribution of ionized hydrogen to neutral hydrogen (arbitrary scale). Horizontal scale is galacto-centric distance in kpc. The theoretical curve (marked kS) is a rough estimate of the relative abundance, based only on the strength of the gravitational field in the spiral wave.

The kinematics of ionized and neutral hydrogen are essentially identical, based on the observations made and analyzed by Dieter (1967), by Kerr *et al.* (1968), and by Mezger *et al.* (1970). Their data suggest that the very young stars are co-moving with the gas and that they are essentially distributed along the spiral arms defined by neutral hydrogen, in agreement with the gravitational theory of spiral structure.

C. STARS

The very young stars* are associated with their H II regions (Figure 13). For somewhat older stars in the solar vicinity, Strömgren (1967) has discussed their migration on the basis of their current kinematical data and their age, in order to determine the places of their origin. The stars are expected to be formed inside of spiral arms. The program had not been entirely successful until the density wave concept was brought into the

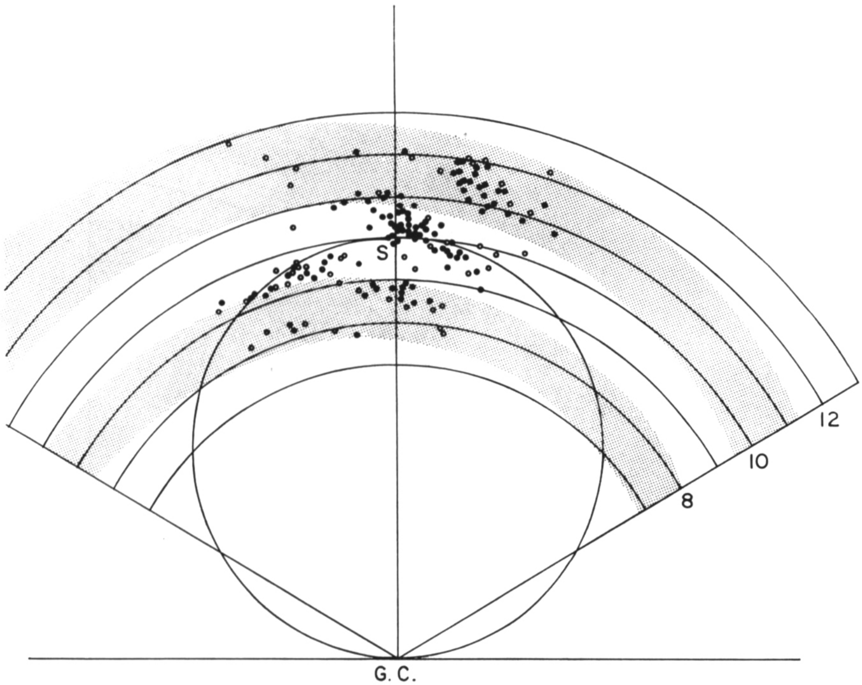


Fig. 13. Distribution of young stellar populations. The Orion arm might be a material arm. The Carina arm is probably part of a secondary feature.

picture. A comparison of the analyses for B 8–B 9 stars with and without the gravitational field (Figure 14; see Lin *et al.*, 1969; Yuan, 1969b), shows clearly the existence of a spiral field with an amplitude of about 5% of the mean field. A larger field of 7% or a smaller field of 4% would yield considerably worse agreement (Cf. Section D).

Another well known phenomenon is the 'vertex deviation' of the velocity distribution of local stars, particularly the A stars. Asymmetries in the field and in the origin of these stars have been suggested as the cause for this vertex deviation. With the help of the density wave theory, we can actually make detailed calculations for stars of one

* In a study of the space distribution and kinematics of supergiant stars, Humphreys (1969) found that "in the direction $l = 285°–300°$, systematic motions of 10 km/sec were found between the two sides of the arm as expected from Lin's density wave theory of spiral structure".

https://doi.org/10.1017/S1539299600000071 Published online by Cambridge University Press

particular age group. In Figure 15, we show the comparison of the theoretical results (Yuan, 1970b) with the observational data (Eggen, 1965) for stars in the age group of 300–500 million years. It is seen that the density wave theory gives the desired explanation of the observed phenomena: the A stars show a more pronounced vertex deviation than both the B stars and the F stars (Eggen, 1969). (Another approach to this problem has been adopted by Mayor (1970), but it would take us too far afield to discuss all the issues involved.)

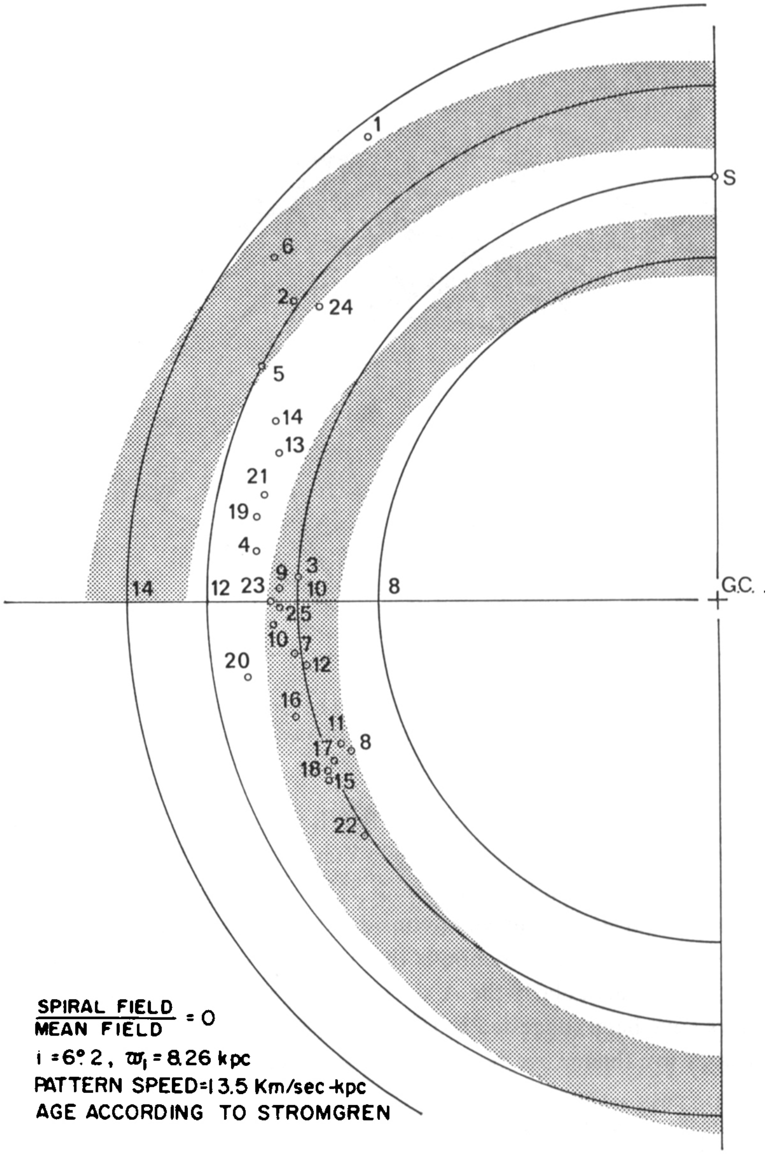


Fig. 14a.

https://doi.org/10.1017/S1539299600000071 Published online by Cambridge University Press

D. SUMMARY OF PHYSICAL PARAMETERS

At this point, it is important to emphasize that we have used the same spiral pattern with the same physical parameters in our analysis of each of the various phenomena. These parameters are shown in Table II. In particular, we note that the dispersion of

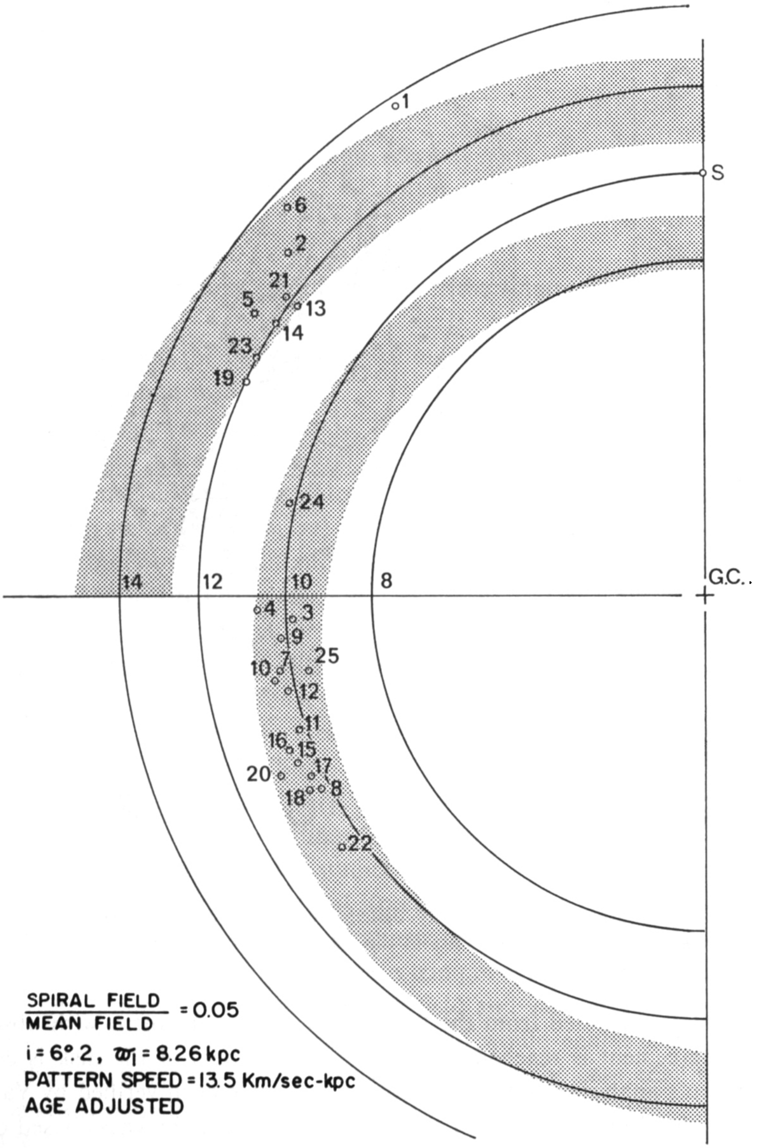


Fig. 14b.

Figs. 14a, b. Places of formation of stars relative to the spiral arms (after Lin, Yuan and Shu), as determined by migration studies (a) without a spiral gravitational field; and (b) with a spiral gravitational field.

https://doi.org/10.1017/S1539299600000071 Published online by Cambridge University Press

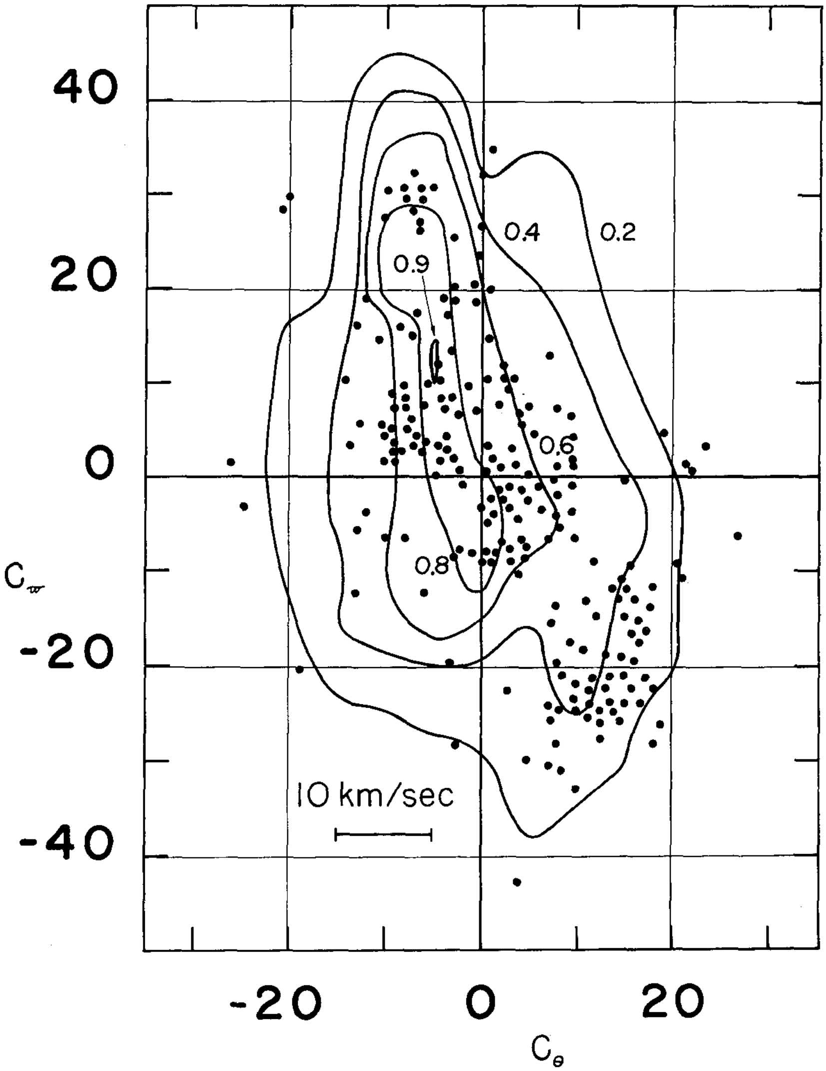


Fig. 15. 'Vertex deviation' in the velocity distribution of A stars compared with theoretically expected velocity distribution for stars in the age group of 300–500 × 10^6 yr (after Yuan).

stellar velocities predicted from the theory is in good agreement with observational data. This agreement is reached only after the thickness of the disk is considered, following Shu's analysis (1968). Shu's theory has since been verified by Vandervoort (1970a) who developed a theory of galactic disks of finite thickness based on the use of adiabatic invariants.

E. LOCAL STRUCTURE

All the successes cited above should not obscure the fact that we can only suggest a

TABLE II

Certain physical parameters in the Milky Way (mostly near the Sun)

1. rms radial velocity predicted for stars in the solar neighborhood	$\leqslant 37$ km sec^{-1}
2. Spiral pattern (primary component), pattern speed	13.5 km sec^{-1} kpc^{-1}
3. In the solar vicinity	
(a) arm spacing (between Perseus and Sagittarius arms)	3.5 kpc
(b) amplitude of spiral gravitational field	5% of mean field
(c) amplitude of variation of projected mass density	10% of mean
(i) in stars	5%
(ii) in gas	5%
(d) rms turbulent velocity of the gas (adopted)	7 km sec^{-1}
(e) magnetic field (adopted for dynamical consistency)	5 μG

tentative picture for the various spiral features in the solar vicinity. We suggest the following picture*.

(1) The Sagittarius and the Perseus arms belong to the primary spiral pattern. (So does the Norma-Scutum arm).

(2) The Carina-Cygnus arm (Bok, 1932, 1969; Bok *et al.*, 1970; Kerr 1969; Dickel *et al.* 1970) belongs to a secondary spiral pattern. (See detailed explanation below).

(3) The Orion arm is essentially a material arm at high inclination, or, as suggested by Frank Shu (private communication), it might be part of the 'long-wave' pattern.

The most important observational support for the above point of view is the absence of giant H II regions in the Orion and the Carina arms. There are some giant H II regions in the Carina direction, but they are so far out that they may very well belong to another spiral arm. Yuan (1969a) has also presented other evidence to show that the Orion arm differs from an ordinary major spiral arm.

It must be stressed again that, in these outer parts of the Milky Way, there could easily be coexistence of many features, as suggested by the photographs of many external galaxies. It is therefore not surprising that the solar vicinity is so difficult to analyze, since we can observe many minor features clearly. Alternative interpretations should be examined by more detailed studies.

The opportunities offered by such local structures should not be forgotten. We can examine the detailed structure of each spiral arm by using diverse methods – optical, radio, infrared, etc. – and by studies of various components – young stars, older stars, neutral hydrogen, ionized hydrogen, magnetic field and various molecules. The fact that the various arms have different characteristics would enable us to learn more about the physical processes in the galaxy. A prominent case that has received considerable attention is the mystery in the Perseus arm (Münch, 1965; Miller, 1967; Rickard, 1967). Here, two components of gaseous motion have been detected, one agreeing quite well with the motion of the stellar association, the other appearing to

* Cf. Prof. Bok's lecture. See also Kerr and Kerr (1970) for additional evidence for a gap between Sagittarius and Carina arms.

expand away from it. It is not yet known for sure whether this is a special phenomenon or a consequence of the density waves*.

5. General Remarks: Future Outlook

I have presented a broad survey of the comparison of observational features with the deductions from the density wave theory, – both for the Milky Way and for some external galaxies. I hope that this will convince everyone that the density wave concept introduced by Lindblad, including the material concentration of both gas and stars, is the essential basis for the spiral structure of disk-shaped galaxies. At this point, it seems appropriate to summarize some of the remaining points to be clarified and some of the natural implications to be considered in order to provide a guide for future research.

A. UNDERSTANDING THE DYNAMICAL MECHANISM

The theory of density waves in stellar systems is very similar to the theory of certain plasma waves – but there are also essential differences. (See Lin *et al.*, 1969, Appendix C). Thus, the understanding of electromagnetic plasma waves and the density waves go hand in hand. Indeed, many of the problems we face are those associated with inhomogeneous plasmas, for which the theory is still not fully developed at this time. Yet it is precisely here that we might find the key to the containment problem of thermo-nuclear reactors. Thus, the understanding of natural phenomena could lead to useful applications. But this is not our main concern here. We must attempt to understand better some of the observed phenomena in nature.

We have not yet fully understood the origin and the development of spiral patterns in the galaxies. I have reported briefly on the work done up to now on (1) numerical experiments, (2) the existence of an unstable mode over the whole galactic disk, and (3) the initiation of the density waves by Jeans instability. Further work appears to be essential and challenging. Various possible mechanisms for exciting the waves should be studied.

In connection with the last possibility, the reflection of the density wave from the inner Lindblad resonance ring presents a very interesting problem. So far, there is only James Mark's linear analysis of such waves. Total absorption by Landau damping was found. Going beyond the linear theory, Contopoulos (1970) has made calculations of the nonlinear response of the stellar system to a given spiral field near resonance, and Roberts (1969), of the gaseous component; but self-sustained waves have not yet been considered in either case. Nonlinear self-sustained waves in gaseous disks have been recently treated by Berry and Vandervoort (1970), extending an earlier work in the linear approximation by Fujimoto (1968).

There should perhaps be more work done to connect the density wave concept with our knowledge of the nature of the individual orbits in a spiral gravitation field. It is

* In a paper presented Aug. 25, 1970, W. W. Roberts showed that the latter is the case.

possible that one can construct self-sustained solutions from the orbital studies in a spiral gravitational field without linearizing the Boltzmann equation. Such an approach would also clarify the relationship of the modern theories with the original Lindblad concepts (Lindblad and Langebartel, 1953).

B. MILKY WAY

Since Prof. Bok has discussed the desiderata for future observational studies, I shall be rather brief on this subject.

(1) The study of large-scale distribution and motion of neutral hydrogen deserves continued effort. Even with the incorporation of the density wave concept (Yuan, 1969, 1970a; Burton, 1970; Burton and Shane, 1970), the analysis still needs improvement. At the same time, the complication of the local features needs to be resolved. In a sense, this complication is a blessing in disguise, since we are perhaps offered a variety of different features for close scrutiny. The proximity of such features allows us to examine them by diverse methods of observation. The clarification of their nature, with the help of the density-wave concept, would lead to deeper understanding. I would particularly like to call attention to the extensive work done on the Carina features and the mystery in the Perseus arm. In this connection, I wish again to stress *coexistence*. Besides the grand design, with both its short-wave and long-wave patterns, there might also be material arms, spiral patterns of density waves at higher pattern speeds, patterns with more than two arms, or even isolated spiral arms of density waves.

(2) I have spoken very little about the central region of the Milky Way, partly because we believe that there are no density waves of the type we discussed (as indicated by the deficiency of H II regions), but largely because there are many observed features lacking proper interpretation. Another class of problems lacking a full dynamical explanation is that of the high latitude features (Oort, 1970). Both classes of problems are fruitful areas for future research.

(3) Finally, I should mention the subject of the interstellar medium, including gas, dust particles, magnetic field, and cosmic ray particles. Clearly the nature of the nearby spiral arms, and especially the nature of the Orion arm, greatly influence our concept of the interstellar medium; for observational data are often derived from such regions. One should remember the special nature of the Orion arm. It is also important to keep in mind that the interstellar medium is extremely inhomogeneous on the scale of the spacing between spiral arms, because of the existence of a spiral gravitational field. There has as yet been very little discussion in the literature on the implications of such inhomogeneities.

C. EXTERNAL GALAXIES

We should perhaps now pay more attention to external galaxies for further work on general spiral features, for their structure can be more readily observed. The spiral patterns of only three such galaxies have been analyzed in terms of the density wave theory (Shu *et al.*, 1970). This work should obviously be extended.

A closer examination of the distribution of the various components of the galaxies would be very instructive. (Cf. Morgan, 1970). We mentioned the problem of the position of the dust lane relative to the stellar arm. Zwicky (1957) has stressed the different distributions of the 'blue' and the 'red' components in M 51. Sharpless has done extensive work in isolating the two components in many external galaxies. A great deal of data have been made available by Courtès and his collaborators (Carranza *et al.*, 1968; Carranza *et al.*, 1969; Crillon and Monnet, 1969). I look forward with great expectation to the time when the distribution of neutral hydrogen in many external galaxies can be resolved into spiral arms. A comparison of the spiral structure of neutral hydrogen, ionized hydrogen, blue supergiants, non-blue stellar population and dust lanes would be very instructive. So far, such a comparison is rather incomplete both in the Milky Way and in external galaxies.

With high resolution instruments, it would be very desirable to examine the conditions near the ring structure in external galaxies. This may be expected to throw light on the mechanism just discussed in Section A.

I shall close my remarks with some brief comments on barred spiral galaxies. A theory attributing spiral structure to material arms has been studied by Prendergast (1962, also unpublished notes) and by Freeman (1970). This may represent one class of bar-shaped spirals. I share the feeling expressed by many astronomers that there is a gradual transition from bar-shaped galaxies with a very open structure (e.g., NGC 1300) to disk-shaped galaxies with a short bar structure near the center or no bar structure at all. The spiral structure may then also have a gradual transition from essentially material arms to a pattern of density waves. I look forward to the complete understanding of such problems through further theoretical and observational investigations.

Bibliography

GENERAL REFERENCES

1. *Galactic Structure*
   Stars and Stellar Systems; vol. V
   (ed. by A. Blaauw and M. Schmidt), University of Chicago Press, Chicago, 1965.
2. *Galactic Structure* (Ithaca)
   Lectures in Applied Mathematics, vol. IX
   (ed. by Jürgen Ehlers), American Mathematical Society, Providence, R.I., 1967.
3. *Proceedings of IAU Symposium No. 38*, held at Basel, Switzerland, August 29–September 4, 1969
   (ed. by W. Becker and G. Contopoulos), Reidel, The Netherlands, 1970.

SPECIFIC REFERENCES TO INDIVIDUAL ARTICLES

References to articles in the above volumes will be made with the following abbreviations:
1. *Galactic Structure;*
2. *Galactic Structure (Ithaca);*
3. *Basel Symposium.*

Other references are cited in the conventional manner.

Arp, H.: 1969, *Astron. Astrophys.* **3**, 418.
Barbanis, B.: 1968a, *Astrophys. J.* **153**, 71.
Barbanis, B.: 1968b, *Astron. J.* **73**, 784.

Barbanis, B.: 1970, *Basel Symposium*, p. 343.
Barbanis, B. and Woltjer, L.: 1967, *Astrophys. J.* **150**, 461.
Berry, C. L. and Vandervoort, P. O.: 1970, *Basel Symposium*, p. 336.
Bok, B. J.: 1932, Ph.D. Thesis, Harvard University.
Bok, B. J.: 1959, *Observatory* **74**, 61.
Bok, B. J.: 1970, *Basel Symposium*, p. 457.
Bok, B. J., Hine, A. A., and Miller, E. W.: 1970, *Basel Symposium*, p. 246.
Burton, W. B.: 1966, *B.A.M.* **18**, 247.
Burton, W. B.: 1970, *Astron. Astrophys., Suppl.* **2**, 261, 291, 339.
Burton, W. B. and Shane, W. W.: 1970, *Basel Symposium*, p. 397.
Carranza, G., Courtès, G., Georgelin, Y., Monnet, G., and Pourcelot, A.: 1968, *Ann. Astrophys.* **31**, 63.
Carranza, G., Crillon, R., et Monnet, G.: 1969, *Astron. Astrophys.* **1**, 479.
Contopoulos, G.: 1970a, *Basel Symposium*, p. 303.
Contopoulos, G.: 1970b, Report of IAU Commission 33.
Contopoulos, G.: 1970c, *Astrophys. J.* **160**, 113.
Crillon, R. et Monnet, G.: 1969, *Astron. Astrophys.* **2**, 1.
Dickel, H. R. *et al.*: 1970, Basel Symposium, p. 213.
Dieter, N. H.: 1967, *Astrophys. J.* **150**, 435.
Eggen, O. J.: 1965, *Galactic Structure*, p. 117.
Eggen, O. J.: 1969, *Astrophys. J.* **155**, 701.
Feldman, Stuart: 1970, private communication.
Freeman, K. C.: 1966, *Monthly Notices Roy. Astron. Soc.* **133**, 47; **134**, 1, 15.
Freeman, K. C.: 1970, *Basel Symposium*, p. 351.
Fujimoto, M.: 1968, *Astrophys. J.* **152**, 391.
Fujimoto, M.: 1969, *Publ. Astron. Soc. Japan* **21**, 288.
Fujimoto, M. and Miyamoto, M.: 1969, *Publ. Astron. Soc. Japan* **21**, 194.
Fujimoto, M. and Miyamoto, M.: 1970, *Basel Symposium*, p. 440.
Goldreich, P. and Lynden-Bell, D.: 1965, *Monthly Notices Roy. Astron. Soc.* **130**, 125.
Henderson, A. P.: 1967, Unpublished Ph.D. Thesis, University of Maryland.
Hodge, P. W.: 1969, *Astrophys. J.* **155**, 417.
Hohl, F.: 1970, *Basel Symposium*, p. 368.
Hohl, F. and Hockney, R. W.: 1969, *J. Comput. Phys.* **4**, 306.
Humphreys, Roberta M.: 1969, Ph.D. Thesis, University of Michigan.
Hunter, C.: 1963, *Monthly Notices Roy. Astron. Soc.* **126**, 299.
Hunter, C.: 1965, *Monthly Notices Roy. Astron. Soc.* **129**, 321.
Hunter, C.: 1969a, *Astrophys. J.* **157**, 183.
Hunter, C.: 1969b, *Studies Appl. Math.* **48**, 55.
Hunter, C.: 1970, *Basel Symposium*, p. 326.
Hunter, C. and Toomre, A.: 1969, *Astrophys. J.* **155**, 747.
Julian, W. H. and Toomre, A.: 1966, *Astrophys. J.* **146**, 810.
Kalnajs, A. J.: 1965, Ph.D. Thesis, Harvard University.
Kalnajs, A. J.: 1970, *Basel Symposium*, p. 318.
Kerr, F. J.: 1964, *The Galaxy and the Magellanic Clouds, IAU Symposium* No. 20, p. 81.
Kerr, F. J.: 1969, *Ann. Rev. Astron. Astrophys.* **7**, 39.
Kerr, F. J.: 1970, *Basel Symposium*, p. 95.
Kerr, F. J., Burke, B. F., Reifenstein, E. C., Wilson, T. L., and Mezger, P.: 1968, *Nature* **220**, 1210.
Kerr, F. J. and Kerr, Maureen: 1970, *Astrophys. Letters*, in press.
Lin, C. C.: 1966, *SIAM J. Appl. Math.* **14**, 876.
Lin, C. C.: 1967a, *Ann. Rev. Astron. Astrophys.* **5**, 453.
Lin, C. C.: 1967b, *Galactic Structure (Ithaca)*, p. 66.
Lin, C. C.: 1968, in *Galaxies and the Universe (Columbia)*, p. 33.
Lin, C. C.: 1970, *Basel Symposium*, p. 377.
Lin, C. C. and Shu, F. H.: 1964, *Astrophys. J.* **140**, 646.
Lin, C. C. and Shu, F. H.: 1966, *Proc. Nat. Acad. Sci.* **55**, 229.
Lin, C. C. and Shu, F. H.: 1967, *IAU Symposium* No. 31 (Noordwijk), p. 313.

Lin, C. C., Yuan, C., and Shu, F. H.: 1969, *Astrophys. J.* **155**, 721.
Lindblad, B.: 1963, *Stockholm Obs. Ann.* **22**, No. 5.
Lindblad, B. and Langebartel, R.: 1953, *Stockholm Obs. Ann.* **17**, No. 6.
Lindblad, P. O.: 1960, *Stockholm Obs. Ann.* **21**, No. 4.
Lindblad, P. O.: 1962, in *Interstellar Matter in Galaxies*, Benjamin, New York, p. 222.
Lynden-Bell, D.: 1967, *Galactic Structure (Ithaca)*, p. 150.
Lynden-Bell, D.: 1970, *Basel Symposium*, p. 331.
Lynden-Bell, D. and Ostriker, J. D.: 1967, *Monthly Notices Roy. Astron. Soc.* **136**, 273.
Lynds, B. T.: 1967, *Sky Telescope* **34**, 18.
Lynds, B. T.: 1970, *Basel Symposium*, p. 26.
Mark, James W-K.: 1970, private communication.
Marochnik, L. S. and Suchkov, A. A.: 1969, *Astrophys. Space Sci.* **4**, 317.
Mathewson, D. S.: 1968, *Astrophys. J.* **153**, L47.
Mathewson, D. S.: 1969, *Proc. Astron. Soc. Australia* **1**, 209.
Mathewson, D. S. and Nicholls, D. C.: 1968, *Astrophys. J.* **154**, L11.
Mayor, M.: 1970, *Astron. Astrophys.* **6**, 60–66.
Mezger, P. G.: 1970, *Basel Symposium*, p. 106.
Mezger, P. G., Wilson, T. L., Gardner, F. F., and Milne, D. K.: 1969, *Astron. Astrophys.* **4**, 96.
Miller, J. S.: 1968, *Astrophys. J.* **151**, 473.
Miller, R. H. and Prendergast, K. H.: 1968, *Astrophys. J.* **151**, 699.
Miller, R. H., Prendergast, K. H., and Quirk, W. J.: 1970, *Astrophys. J.* **161**, 903.
Miyamoto, M.: 1969, *Publ. Astron. Soc. Japan* **21**, 319.
Morgan, W. W.: 1970, *Basel Symposium*, p. 9.
Morse, P. M. and Feshbach, H.: 1953, *Methods of Theoretical Physics*, p. 1092.
Munch, G.: 1965, *Galactic Structure*, p. 203.
Oort, J. H.: 1967, *IAU Symposium* No. 31, p. 279.
Oort, J. H.: 1970, *Basel Symposium*, pp. 1, 142.
Prendergast, K. H.: 1962, in *Interstellar Matter in Galaxies*, Benjamin, New York, p. 217.
Reifenstein, E. C., Wilson, T. L., Burke, B. F., Mezger, P. G., and Altenhoff, W.: 1970, *Astron. Astrophys.* **4**, 357.
Rickard, J.: 1968, *Astrophys. J.* **152**, 1019.
Roberts, M. S.: 1966, *Astrophys. J.* **144**, 639.
Roberts, M. S.: 1968, in *Interstellar Ionized Hydrogen*, Benjamin, New York, p. 617.
Roberts, W. W.: 1969, *Astrophys. J.* **158**, 123.
Roberts, W. W.: 1970, *Basel Symposium*, p. 415.
Roberts, W. W. and Yuan, C.: 1970, *Astrophys. J.* **161**, 877.
Sandage, A.: 1961, *The Hubble Atlas of Galaxies*, Carnegie Institution, Washington, D.C.
Shane, W. W. and Bieger-Smith, G. P.: 1966, *Bull. Astron. Inst. Neth.* **18**, 263.
Shu, F. H.: 1968, Ph.D. Thesis, Harvard University.
Shu, F. H.: 1969, *Astrophys. J.* **158**, 505.
Shu, F. H.: 1970a, *Astrophys. J.* **160**, 89.
Shu, F. H.: 1970b, *Astrophys. J.* **160**, 99.
Shu, F. H.: 1970c, *Basel Symposium*, p. 323.
Shu, Frank H., Stachnik, Robert V., and Yost, Jonathan C.: 1970, *Astron. J.* (Abstract only), in press.
Strömgren, Bengt: 1967, *IAU Symposium* No. 31, p. 323.
Toomre, A.: 1964, *Astrophys. J.* **139**, 1217.
Toomre, A.: 1969, *Astrophys. J.* **158**, 899.
Toomre, A.: 1970, *Basel Symposium*, p. 334.
Vandervoort, P. O.: 1970a, *Astrophys. J.* **161**, 187.
Vandervoort, P. O.: 1970b, *Basel Symposium*, p. 337.
Vaucouleurs, G. de: 1970, *Basel Symposium*, p. 18.
Weaver, H.: 1970, *Basel Symposium*, p. 126.
Westerhout, G.: 1958, *Galactic Structure*, p. 196.
Wilson, T. L.: 1969, Ph.D. Thesis, M.I.T.
Wilson, T. L.: 1970, *Basel Symposium*, p. 140.
Wilson, T. L., Mezger, P. G., Gardner, F. F., and Milne, D. K.: 1970, to be published.

Woltjer, L.: 1970, *Basel Symposium*, p. 439.
Yabushita, S.: 1968, *Monthly Notices Roy. Astron. Soc.* **140**, 109.
Yabushita, S.: 1969a, *Monthly Notices Roy. Astron. Soc.* **142**, 201.
Yabushita, S.: 1969b, *Monthly Notices Roy. Astron. Soc.* **143**, 231.
Yuan, C.: 1969a, *Astrophys. J.* **158**, 871.
Yuan, C.: 1969b, *Astrophys. J.* **158**, 889.
Yuan, C.: 1970a, *Basel Symposium*, p. 391.
Yuan, C.: 1970b, *Astron. J.* (Abstract only), in press.
Zwicky, F.: 1957, *Morphological Astronomy*, Springer-Verlag, Berlin, p. 198.