

CRITICAL REMARKS ON THE POSSIBILITY OF DETERMINING
VARIATIONS OF THE GEOCENTER POSITION USING GEOSTATIONARY
SATELLITE OBSERVATIONS

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ABSTRACT*

The possibility of determining the variations of the Earth's center (expressed in terms of the geopotential harmonic coefficients \bar{C}_{11} , \bar{S}_{11} , or of some functions of them), using geostationary satellites, was discussed by Burša and Šíma (1978) after discerning the difference between observed and theoretical positions of libration points. Although their effect on geostationary orbits cannot be expected to be of high magnitude, the question of determination of these terms arose. It was suggested that the "resonant phenomenon" observed here might be analogous to the resonant effects in close satellite orbits, because the satellite is at the 1/1-resonance (1 nodal revolution per 1 sidereal day).

The rate of change of a satellite's semimajor axis a (Lagrange planetary equation) at the 1/1-resonance can be written as follows (Klokočník and Kostelecký, 1979):

$$\left(\frac{da}{ds}\right)_{1/1} = 2n_s a_e \sum_{\gamma=1}^{\infty} \left[C_{\gamma} \sin(\gamma\phi_{1/1}) - S_{\gamma} \cos(\gamma\phi_{1/1}) \right] + O(e) \quad , \quad (1)$$

where $\phi_{1/1} = \omega + \Omega + M - S$ is the resonant angle, $E(a, e, I, \omega, \Omega, M)$ are the usual orbital elements, n_s is the satellite's mean (Keplerian) motion, a_e is the semimajor axis of the best-fitting Earth's ellipsoid, and C_{γ} , S_{γ} are the lumped geopotential coefficients. The coefficients are defined as follows:

$$\begin{Bmatrix} C \\ S \end{Bmatrix}_{\gamma}^E = \sum_{i=0}^{\infty} (-1)^i \tilde{F}_{\gamma+2i,\gamma,i} \left(\frac{a_e}{a}\right)^{\gamma+2i-1} \begin{Bmatrix} \bar{C} \\ \bar{S} \end{Bmatrix}_{\gamma+2i,\gamma}$$

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with the harmonic geopotential coefficients (Stokes constants) $\bar{C}_{\gamma+2i,\gamma}$, $\bar{S}_{\gamma+2i,\gamma}$ of order γ and degree $\gamma + 2i$, $\gamma = 1, 2, \dots$; $i = 0, 1, 2, \dots$; F are certain functions of the inclination functions.

Formal setting $\gamma = 1$, $i = 0$ into Eq. 1 shows the dependence of \bar{a} on \bar{C}_{11} , \bar{S}_{11} ; these terms may be interpreted as a shift (Δ) of true geocenter in some reference frame in which geostationary satellites are observed. If we substitute sufficiently large values of \bar{C}_{11} , \bar{S}_{11} and integrate Eq. 1, then their influence in the total da/ds begins to compete with the equatorial flattening \bar{C}_{22} , \bar{S}_{22} - effect. It is then natural to ask whether such a non-geocentricity of the reference frame may show itself a dominant resonant effect in a geostationary orbit? The answer is negative.

The reason for the negative answer is just the above mentioned formal substitution, which is illegitimate. Our Lagrange planetary equation (LPE) for the geostationary orbits was derived from general LPE, where all tesseral harmonics are kept. Both these equations are valid only in an actual inertial reference frame. The satellite's motion (and all observations) are considered in a shifted non-inertial frame, there \bar{C}_{11} , $\bar{S}_{11} \neq 0$. Therefore, the original LPE must be transformed into another form compatible with that quasigeocentric system.

According to (Klokočník and Kostecký, 1980, Eq. 14):

$$\frac{d\bar{a}}{ds} = \frac{2}{n_s \bar{a}} \frac{\partial R}{\partial M} + \Delta (\dot{S} - n_s) \sin(\bar{\phi}_{1/1} - \lambda_{11}) + 0 \quad , \quad (2)$$

where Δ , λ_{11} stay instead of \bar{C}_{11} , \bar{S}_{11} (since they are more convenient in the derivations); the values with the bars are related to the quasigeocentric reference frame and R is the disturbing potential. It is important that the terms \bar{C}_{11} , \bar{S}_{11} (or Δ , λ_{11}) are not included in this potential.

When integrating (2) by means of successive approximations, and supposing the resonant case (i.e. $\bar{\phi}_{1/1} \rightarrow 0$, $\dot{S} - n_s \rightarrow 0$), we arrive at the situation, where the limit of the second term on the right side of Eq. 2 approaches 1. This results in the conclusions that there is no resonant effect on the geostationary orbit owing to \bar{C}_{11} , \bar{S}_{11} and that the resulting small effect would be of the same amplitude on any satellite's orbit. Our "non-inertiality" is just a geometrical feature, which could be measured only if the "measuring accuracy" (orbit determination) would be comparable with the magnitude of the effect (i.e., with the amplitude of Δ). It should be noticed that Burša and Šima (1979a,b) obtained the same result by another approach.

Orbital variations due to \bar{C}_{11} , \bar{S}_{11} can be found from very precisely determined orbits of close satellites, e.g., as a part of the experiment with two counter-orbiting drag-free satellites suggested by Van Patten et al. 1976 and Breakwell et al. 1978 for testing Lense-Thirring effect

for improving the individual harmonic coefficients of low order m and degree n (originally suggested for $n, m > 1$). (Relative geocentric coordinates of observing stations also need to be known with the accuracy comparable with Δ).

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