CORRESPONDENCE.

SOLUTION OF A PROBLEM.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In my paper entitled "How dos an Increast Mortality afect Policy-Values?" (*J.I.A.* xxi, 98) I proposed for solution a problem which may be stated thus:

If
$$\frac{1+a_x}{1+a_{x+1}} > \frac{1+a'_x}{1+a'_{x+1}} > 1,$$

then will
$$\frac{a_x}{v(1+a_x)} > \left(\frac{1}{1+a'_x} - \frac{1}{1+a_x}\right)\frac{1}{v-v'};$$

a'_x being the value of an annuity calculated by the same mortality table as a_x, but at a higher rate of interest. I have recently received from Mr. J. C. Hopkinson a solution of the problem, which you may perhaps think worthy of being brought under the notice of your readers.

We have
$$\frac{1+a_x}{1+a'_x} - 1 = \frac{a_x - a'_x}{1+a'_x}$$
$$= \left(\frac{a_x}{a'_x} - \frac{1+a_x}{1+a'_x}\right)a'_x$$
$$< \left(\frac{a_x}{a'_x} - \frac{1+a_{x+1}}{1+a'_{x+1}}\right)a'_x$$
$$< \left\{\frac{a_x}{a'_x} - \frac{v'}{v}\cdot\frac{vp_x(1+a_{x+1})}{v'p_x(1+a'_{x+1})}\right\}a'_x$$
$$< \left(\frac{a_x}{a'_x} - \frac{v'}{v}\cdot\frac{a_x}{a'_x}\right)a'_x$$
$$< a_x \cdot \frac{v-v'}{v}.$$

Then dividing both sides of this inequality by $(v-v')(1+a_x)$, the desired result at once follows. It will be observed that we have made no use of the condition that $\frac{1+a_x}{1+a_{x+1}}$ and $\frac{1+a'_x}{1+a'_{x+1}}$ are both >1; and it seems that this condition is unnecessary, and that it would be

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sufficient to have $\frac{1+a_x}{1+a_{x+1}} > \frac{1+a'_x}{1+a'_{x+1}}$. It is worth noticing that this inequality leads to

$$\frac{vp_{x-1}(1+a_x)}{vp_x(1+a_{x+1})} > \frac{v'p_{x-1}(1+a'_x)}{v'p_x(1+a'_{x+1})},$$

whence

$$\frac{a_{x-1}}{a_x} > \frac{a'_{x-1}}{a'_x}.$$

I am, Sir,
Your obedient servant,

T. B. SPRAGUE.

26 *St. Andrew Square,*
Edinburgh,
9 *May* 1885.