LETTER TO THE EDITOR

Dear Editor,

On processor sharing and random service

Ramaswami's interesting paper [2] on the sojourn-time distribution in the $GI/M/1$ system with processor-sharing discipline leads to the following conclusion.

For a $GI/M/1$ system with processor-sharing service discipline the distribution $G_n(t)$ of the sojourn time of a customer who meets upon his arrival n customers present is identical with the distribution $w_n(t)$ of the waiting time of a customer who meets upon this arrival $n+1$ customers present in a $GI/M/1$ system with random service discipline, $n \geqq 0$.

The motivation of this conclusion proceeds as follows. From Ramaswami's Theorem 1 it is easily seen that his function $H(z,s)/(1-z)$ satisfies the same differential equation and conditions as Cohen's function $w(p,\rho)$, cf. [1], form. (3.43) SSQ, p. 444, and so these functions are identical. From

$$\frac{1}{1-z}H(z,s) = \sum_{n=0}^{\infty} z^n \int_{0-}^{\infty} e^{-st}dG_n(t), \qquad |z|<1,\ \mathrm{Re}\, s \geqq 0,$$

$$w(p,\rho) = \sum_{i=0}^{\infty} p^i \int_{0}^{\infty} e^{-\rho t}dw_i(t), \qquad |p|<1,\ \mathrm{Re}\, \rho \geqq 0,$$

the statement follows.

Mathematical Institute
University of Utrecht

Yours sincerely,
J. W. COHEN

References

[1] COHEN, J. W. (1982) *The Single Server Queue*, revised edition. North-Holland, Amsterdam.

[2] RAMASWAMI, V. (1984) The sojourn time in the $GI/M/1$ queue with processor sharing. *J. Appl. Prob.* **21**, 437–442.