Math. Proc. Camb. Phil. Soc. (1984), **95**, 511
Printed in Great Britain

Corrigenda
Volume 89 (1981), 285–292

'Lattice isomorphisms of Lie algebras'

BY D. A. TOWERS
University of Lancaster

(*Received* 20 *October* 1983)

Lemma 4·1, which appears on p. 289 of [**2**], is false. Nevertheless, Theorem 4·2, the only result dependent upon it, is true. The following supplies the necessary modification to the proof.

Replace Lemma 4·1 by

LEMMA 4·1. *Let S be a three-dimensional non-split simple Lie algebra, and let R be an irreducible S-module. Then, for any $s \in S$, R has an* ad *s-invariant subspace of dimension less than or equal to two.*

Proof. See [**1**], p. 23.

Now replace the second paragraph of the proof of Theorem 4·2 by the following.

Suppose that S^* is three-dimensional non-split simple. Let A^* be a minimal ideal of L^*, and put $U^* = S^* \dotplus A^*$. Then $U = S \dotplus A$ where A is a minimal abelian ideal of U, and S is a two-dimensional abelian subalgebra of U. Let $0 \neq s \in S$, $0 \neq a \in A$ and let $f(\theta)$ be the polynomial of smallest degree for which $af(\operatorname{ad} s) = 0$. It follows from the fact that S is abelian that $\{x \in A : xf(\operatorname{ad} s) = 0\}$ is an S-submodule of A, and hence that it coincides with A. Clearly then $f(\theta)$ is the minimum polynomial of $\operatorname{ad} s|_A$.

Suppose first that there is an $s_1 \in S$ for which the minimum polynomial for $\operatorname{ad} s_1$ has degree two, and let this polynomial be $f(\theta) = \theta^2 - \lambda_2\theta - \lambda_1$. Pick $s_2 \in S$ such that $S = ((s_1, s_2))$. Then $((s_1, s_2, a, as_1, as_2))$ is a subalgebra of L (for any $a \in A$), since

$$(as_1)s_1 = \lambda_1 a + \lambda_2 as_1,$$
$$(as_1)s_2 = (as_2)s_1 = \alpha_1 a + \alpha_2 as_1 + \alpha_3 as_2,$$

since $a(\operatorname{ad}(s_1+s_2))^2 \in ((a, a(s_1+s_2)))$, and

$$(as_2)s_2 = \beta_1 a + \beta_2 as_2.$$

Now, $((as_2)s_1)s_1 = \lambda_1 as_2 + \lambda_2(as_2)s_1$, so

$$(\alpha_2\lambda_1 + \alpha_3\alpha_1)a + (\alpha_1 + \alpha_2\lambda_2)as_1 + \alpha_3^2 as_2 = \lambda_2\alpha_1 a + \lambda_2\alpha_2 as_1 + (\lambda_1 + \lambda_2\alpha_3)as_2.$$

Since $f(\theta)$ is irreducible, $\alpha_3^2 \neq \lambda_1 + \lambda_2\alpha_3$, and so $as_2 = \gamma_1 a + \gamma_2 as_1$. Hence A has dimension at most two, and $U = S + S_1$ where $S_1 = \{s + sa : s \in s\}$, and $S \cap S_1 = \{0\}$. But this means that $U^* = S^* \cup S_1^*$ where $S^* \cap S_1^* = \{0\}$, so U^* is at least six-dimensional, and A^* is at least 3-dimensional, which is impossible.

Hence we may assume that, for every $s \in S$, $\operatorname{ad} s$ has an eigenvector. But this implies that $A + ((s))$ is semiabelian for each $s \in S$ and hence that A is one-dimensional. This means that $U^* = S^* \oplus A^*$, contradicting Lemma 3·3.

REFERENCES

[1] GEIN, A. G. Projections of a Lie algebra of characteristic zero. *Izvestija vysš. ucebn. Zaved. Mat.*, no. 4, **191** (1978), 26–31.
[2] TOWERS, D. A. Lattice isomorphisms of Lie algebras. *Math. Proc. Cambridge Philos. Soc.* **89** (1981), 285–292.