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A note on generalized Hall planes

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We prove that if π is a generalized Hall plane of odd order with associated Baer subplane π_0 then π is a Hall plane if and only if there is a collineation σ of π such that $\pi_0\sigma \cap \pi_0$ is an affine point.

Generalized Hall planes were introduced by Kirkpatrick in [4]. The author [2] pointed out that such planes of odd order are derivable from translation planes of semi-translation class $1 - 3a$. Ostrom further observed that derivable translation planes of class $1 - 3a$ are semifield planes.

Kirkpatrick [5] has characterized the Hall planes of odd order as those generalized Hall planes with associated Baer subplane π_0 such that every affine line of π_0 is the axis of an involutory homology fixing π_0.

This note shows that an easy characterization is also available by assuming there is a collineation group which does not fix π_0.

We shall assume the reader is familiar with "derivation" as developed by Ostrom in [6].

THEOREM. *An affine translation plane* π *of odd square order is a Hall plane if and only if there is a Baer subplane* π_0 *of* π *and a group of collineations* G *such that*

(i) G *contains a subgroup* H *which is transitive on* $l_\infty - l_\infty \cap \pi_0$

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and fixes π_0 *pointwise, and*

(ii) there is an element σ *of* G *such that* $\pi_0\sigma \cap \pi_0$ *is an affine point.*

Proof. Assume π satisfies *(i)* and *(ii)*. By [4], [2], and Ostrom's observation, π is derivable from a semifield plane $\bar{\pi}$ where coordinates are chosen so that π_0 in π is $\{(x, y) \mid x = 0\}$ in $\bar{\pi}$ and $\bar{\pi}$ is a dual translation plane with respect to (∞).

Let $\sigma \in G$ such that $\pi_0\sigma \cap \pi_0$ is an affine point. By (2.6) [3], the *full* collineation group of π is the group "inherited" from $\bar{\pi}$. That is, σ is also a collineation of $\bar{\pi}$.

The points of π and $\bar{\pi}$ are identical so $(x = 0)\sigma \cap (x = 0)$ is an affine point of $\bar{\pi}$. Hence σ must move (∞). It is well known that in this situation $\bar{\pi}$ must be desarguesian so that π is a Hall plane by [1].

Conversely, if π is a Hall plane then $\bar{\pi}$ is desarguesian and the $((\infty), x = 0)$-, $((0), x = 0)$-, $((0), y = 0)$-central collineations of $\bar{\pi}$ which induce collinations of π satisfy *(i)* and *(ii)*.

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