15. SUPERNOVAE AND THE INTERSTELLAR MEDIUM

Introductory Report

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1. Introduction

The nature of the supernova event is still poorly understood. A variety of models has been proposed and the interpretation of the observations remains ambiguous. About all that is certain is that in the supernova event much of the matter is ejected at speeds of typically 10 000 km sec^{-1} and that about 10^{49} erg of visible light is emitted. The amount of matter ejected and the bolometric correction to be applied to the optical radiation are very uncertain. The discovery of a pulsar in two supernova remnants (Crab Nebula and Vela X) suggests that frequently a neutron star or other condensed object results following the outburst.

In most models it is assumed that some instability near the end of the evolution of a star causes the interior to collapse; and that, following this, the envelope is driven out, possibly by the pressure from neutrinos generated in the core. Colgate and White (1966) have made a detailed study of the effect of this pressure pulse on the envelope. A strong, essentially adiabatic shock moves out and gains speed because of the steep density gradient. The outermost parts of the star come off at relativistic speeds, so that the whole event may be important as an acceleration mechanism for cosmic rays. Pacini (1967) and several authors following him have drawn attention to the large rotational energies expected in neutron stars and the possibility that electromagnetic waves generated by rotating magnetic neutron stars could energize supernova remnants even after the initial explosion.

2. Simple Hydrodynamical Expansion

To isolate some of the more important aspects of the expansion of the supernova remnant we shall assume that in the supernova explosion at some initial moment an amount of energy ε_0 is released in a small volume and imparted to a mass of gas M_0 which acquires an initial expansion velocity V_0. We treat the system as spherically symmetric and consider its interaction with the surrounding interstellar medium (hydrogen density per cm^3 n_{is}, mass of hydrogen atom m). We suppose for the moment that the interaction can be described in macroscopic hydrodynamical terms and that magnetic field and relativistic particle effects can be neglected. Four phases of the expansion can be distinguished.

Phase I – As long as $(4\pi/3)n_{is}mR^3 \ll M_0$, the expansion of the remnant will be essentially free and much of the developments will depend on the initial conditions.

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Phase II – As the remnant expands, the mass of the interstellar matter that has been swept up will gradually increase and exceed M_0. As first pointed out by Shklovskii (1962), we then enter the phase where the behavior of the expanding gas can be described by the simple similarity solution discussed by Sedov (1959). In this solution an amount of energy ε_0 (but no mass) is injected into a uniform medium at time $t=0$. A shock propagates into the undisturbed medium behind which the gas is compressed. As the amount of matter swept up becomes very large compared to M_0, the assumption of no initial mass ejection becomes less serious. In the similarity solution the ratio of thermal to kinetic energy is constant in time and we may write for the total energy

$$\varepsilon = \varepsilon_0 = \eta \tfrac{2}{3}\pi n_{is} m R^3 V^2 , \tag{1}$$

with R and V the radius and velocity of the shock. For a gas with $\gamma=\frac{5}{3}$ we have $\eta=1.37$ from Sedov's solutions. Of course the solution is only valid if $(4\pi/3)n_{is}mR^3 \gg M_0$. Integrating Equation (1) with $R=0$ at $t=0$ we have for $\gamma=\frac{5}{3}$

$$R = \left(\frac{75}{8\pi}\right)^{1/5} \left(\frac{\varepsilon_0}{n_{is}m\eta}\right)^{1/5} t^{2/5} = \tfrac{5}{2} V t . \tag{2}$$

The shock separating the hot gas from the undisturbed medium is strong. Consequently the density behind the shock front n^* is equal to $4n_{is}$, while for the temperature T^* we have

$$T^* = \frac{3}{32}\frac{m}{k} V^2 . \tag{3}$$

The hot gas will radiate and by the time that the integrated radiation becomes comparable with ε_0 the adiabatic solution can no longer be valid. At high temperatures ($T^* \gg 10^6$ K) free-free radiation of hydrogen dominates for material of interstellar composition. The emission per unit mass is proportional to $nT^{1/2}$. With $T \propto V^2 \propto t^{-6/5}$ and the total mass $M \propto R^3 \propto t^{6/5}$, the total radiative loss rate is proportional to $t^{3/5}$ and the time integrated loss to $t^{8/5}$ (cf. Shklovskii, 1968). Hence the loss is small at early times (see also Heiles, 1964) and it is easily verified that for a wide range of energies the nonadiabatic effects are negligible until temperatures near or below 10^6 K are reached. But at those temperatures radiation by heavy elements becomes much more important than that by hydrogen. Pottasch (1965) has computed the total radiative loss from a plasma of standard composition which is ionized and excited collisionally. Over the interval $10^5 < T < 4 \times 10^6$ the loss rate per unit volume can be represented reasonably well (to within a factor of 2) by

$$\begin{aligned} j &= Q n_e^2 T^{-1} \\ Q &= 8 \times 10^{-17}\ \text{cgs} . \end{aligned} \tag{4}$$

From what we said above about the unimportance of the free-free emission in the early phases, it is clear that negligible error is incurred in our calculations if we make use of Equation (4) also at higher temperatures. We shall estimate the radiative loss rate assuming that all matter is concentrated in a shell with volume $(\pi/3)R^3$, electron

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density $n_e = 4n_{is}$ and temperature T^*. This results in an overestimate, because behind the shock the density actually decreases while the temperature goes up. The radiative loss rate of the remnant now becomes

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = -\frac{\pi}{3} R^3 Q\, 16 n_{is}^2 \frac{32}{3} \frac{k}{m} \frac{1}{V^2}$$

or making use of Equation (2) and introducing a suitable constant $\mathfrak{A}$ (equal to 9.3×10^{-2})

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = -\frac{3200}{9}\left(\frac{75}{8\pi}\right)^{1/5} \frac{\pi k Q}{m^{6/5}\eta^{1/5}} n_{is}^{9/5} \varepsilon_0^{1/5} t^{12/5} = -\frac{17}{10}\mathfrak{A} n_{is}^{9/5} \varepsilon_0^{1/5} t^{12/5}, \tag{5}$$

corresponding to a time integrated loss of

$$\int_0^t \frac{\mathrm{d}\varepsilon}{\mathrm{d}t}\,\mathrm{d}t = -\tfrac{1}{2}\mathfrak{A} n_{is}^{9/5} \varepsilon_0^{1/5} t^{17/5}. \tag{6}$$

The validity of the adiabatic solution requires

$$\int_0^t \left|\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}\right| \mathrm{d}t \ll \varepsilon_0 . \tag{7}$$

Denoting the variables at the time when the radiative loss is equal to $\frac{1}{2}\varepsilon_0$ by t_{rad}, R_{rad}, and V_{rad} we have

$$\begin{aligned} t_{\mathrm{rad}} &= \mathfrak{A}^{-5/17} \varepsilon_0^{4/17} n_{is}^{-9/17} \\ R_{\mathrm{rad}} &= (15/4\pi)^{2/5} (\eta m)^{-1/5} \mathfrak{A}^{-2/17} \varepsilon_0^{5/17} n_{is}^{-7/17} \\ V_{\mathrm{rad}} &= (2/5)(15/4\pi)^{2/5} (\eta m)^{-1/5} \mathfrak{A}^{3/17} \varepsilon_0^{1/17} n_{is}^{2/17} . \end{aligned} \tag{8}$$

Poveda and Woltjer (1968) derived analogous equations. They used a different criterion to estimate the validity of the adiabatic solution, namely that per unit volume of the shocked gas the thermal energy exceeds R/V times the radiative loss. This criterion leads to the same results for t_{rad}, if the constant Q (and therefore also $\mathfrak{A}$) is multiplied by a factor of 3, a value of V_{rad} that differs by about 25 per cent, and a value of R_{rad} that differs even less.

Inspecting Equation (8) we note that V_{rad} in particular is very insensitive to the physical parameters involved.

Phase III – When the radiative losses become dominant, the structure of the object changes. The matter that passes through the shock front cools rapidly and the density becomes high. In this phase the remnant can be described as a rather thin shell ploughing through the interstellar medium. The thermal energy in the object now is small and we may consider the shell to move with constant linear momentum. Consequently, as first discussed by Oort (1951), the equation of motion simply becomes

$$R^3 V = C_1 \tag{9}$$

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or upon integration

$$R = (4C_1 t + C_2)^{1/4} \tag{10}$$

with C_1 and C_2 constants.

Phase IV – After the shell has slowed down for some time the expansion velocity becomes comparable to the thermal or random motions in the surrounding interstellar gas. The object gradually loses its identity and becomes part of the interstellar medium.

3. Effects of Relativistic Particles

If in Phase I or II relativistic particles are present and coupled effectively to the gas, the situation will not change too much. During the expansion the relativistic particles will quickly lose their energy, transferring it to the gas and the final situation will not be very different from that described before, provided that the initial cosmic-ray energy is added to ε_0. In Phase III the situation is different. According to Equation (9) the kinetic energy of the remnant (without cosmic rays) is proportional to V or to R^{-3}. The cosmic-ray energy on the other hand is proportional to R^{-1}. Hence as the object expands, the cosmic-ray energy tends to increase in relative importance and its dynamical effect on the shell cannot be neglected.

Following Kahn and Woltjer (1967) we consider a simple model of a shell of swept-up interstellar matter (mass per unit solid angle $\frac{1}{3}n_{is}mR^3$) in which at one time the radius was R_0 and the total cosmic ray energy E_0. Assuming all cosmic rays to be confined we have at later times $E = E_0 R_0 / R$. The force exerted by the cosmic rays per unit solid angle of the shell is $\mathbf{P}R^2$ where the pressure $\mathbf{P}$ is equal to one-third of the energy density $(3E/4\pi R^3)$. The equation of motion becomes

$$\tfrac{1}{3}\varrho_0 R^3 \frac{\mathrm{d}^2 R}{\mathrm{d}t^2} + \varrho_0 R^2 \left(\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2 = \frac{E_0 R_0}{4\pi R^2}, \tag{11}$$

where the first term represents the mass of the shell multiplied by the acceleration, the second the rate of change of momentum of the matter swept up by the shell, and the third the cosmic-ray force; all terms refer to a unit solid angle. Upon integration we have

$$\left(\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2 = \frac{3}{4\pi}\frac{E_0 R_0}{n_{is} m R^4} + \frac{C}{R^6} \tag{12}$$

with C a constant of integration. If $E_0 = 0$, Equation (12) leads to Equation (9). Multiplying with the mass M we can write Equation (12) as

$$MV^2 = E\left[1 + \frac{R_0^2}{R^2}\left(\frac{M_0 V_0^2}{E_0} - 1\right)\right] \tag{13}$$

with M_0 now the mass of the shell at the time that the radius was R_0. Hence in sufficiently late phases, the kinetic energy tends to be one half of the cosmic ray energy.

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4. Other Effects

Several important factors are not yet included in our analysis. Interstellar magnetic fields will be compressed along with the interstellar gas. Current evidence indicates a systematic magnetic field in the solar neighborhood of about 3μG; possibly the random component of the field is somewhat stronger. In a medium with a density of $0.1\ \mathrm{cm}^{-3}$ the corresponding Alfvén velocity is about $20\ \mathrm{km\ sec}^{-1}$. The field is likely to have two effects: (i) When the expansion velocity drops much below $100\ \mathrm{km\ sec}^{-1}$, the compression at the shock will be decreased. (ii) More importantly, already at higher expansion velocities the field will affect the compression further behind the shock, for the main pressure will ultimately be provided by the compressed fields. (In the absence of the field this pressure will be provided by the matter compressed after cooling.) As a consequence rather strong fields will occur behind the shock and this is particularly important in the analysis of the radio emission in supernova remnants. The dynamics of a supernova shell propagating through a medium with a uniform magnetic field have been further discussed by Bernstein and Kulsrud (1965).

Inhomogeneities in the interstellar gas will affect the propagation of the shock and the appearance of the supernova shell. At the same time, Rayleigh-Taylor type instabilities may well lead to a rather inhomogeneous shell. Especially in the early phases, when the velocity of expansion is high, it would be likely that dense concentrations of matter would not be accelerated too much by the passing shell and could be left behind. In fact, the stationary filaments in Cas A may show this effect.

5. Supernova Energetics and the Interstellar Medium

From Equations (8) and (9) we obtain for a remnant in Phase III

$$R^3 V = R_{\mathrm{rad}}^3 V_{\mathrm{rad}} = (\tfrac{2}{5})\,(15/4\pi)^{8/5}\,(\eta m)^{-4/5}\mathfrak{A}^{-3/17}\varepsilon_0^{16/17} n_{is}^{-19/17} . \qquad (14)$$

For an object like the Cygnus Loop we can measure R and V and if n_{is} can be estimated ε_0 is obtained from Equation (14). If cosmic rays drive the shell in Phase III, as discussed in Section 3, then Equation (9) has to be replaced by Equation (13). If $R \gg R_0$ this leads to $R^2 V = R_{\mathrm{rad}}^2 V_{\mathrm{rad}}$ (since $M/E \propto R^4$) and subsequently to a value of ε_0 which is about a factor R_{rad}/R times the value derived from Equation (14). This factor is less than 1; it is of the order of $\frac{1}{2}$ to $\frac{1}{3}$. If the incoming gas swept up by the Cygnus Loop is ionized, then every proton that passes through the shock ultimately recombines and yields one Balmer photon. It can be shown quantitatively that the importance of direct collisional excitation of Balmer lines is quite minor in this case. If the incoming gas is neutral the collisional excitation might be somewhat more important, because in the process of collisional ionization some excitation would also take place. The photoelectric measurements of Parker (1967) show that the Balmer decrement in the Cygnus Loop is of the recombination type. Consequently we can find the rate at which photons are swept up by observing the rate at which Balmer photons are emitted. The available data for the Cygnus Loop suggests $n_{is} = 0.2$ or 0.3

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cm^{-3}. With $R=20$ pc and $V=100\ \text{km sec}^{-1}$, we obtain from Equation (14) $\varepsilon_0 = 4\times10^{49}$ erg. For IC 443 the value of ε_0 may be even smaller. Most of the other large supernova remnants are likely to be also in Phase III, but precise observational data on velocities and intensities are not available.

With regard to the younger supernova remnants, the Crab Nebula is undoubtedly still in Phase I. Its present kinetic energy is about 2×10^{49} erg. Cas A may be at the transition from Phase I to Phase II, while following Minkowski we assign Tycho's supernova remnant to Phase II. The radio isophotes indicate emission from a rather smooth shell; optically very little is seen. Minkowski (1964) has estimated $\varepsilon_0 = 3\times10^{51}$ erg from Equation (1) obtaining V from the radius and the known age of the object and taking $n_{is} = 1\ \text{cm}^{-3}$. The estimate is very uncertain because of the uncertainty in n_{is} and that in R and V. The latter two are proportional to the distance D which has been estimated from 21-cm absorption data; note that $\varepsilon_0 \propto D^5$. Tycho's supernova was of type I and therefore the chance that it exploded in the tenuous intercloud medium is much greater than that the event took place in a cloud. The regular shape of the object also would be unexpected if an interstellar cloud were involved. Consequently an interstellar density of 0.1 cm^{-3} may be more appropriate.

If we consider that a supernova remnant effectively transfers its kinetic energy to the interstellar medium when it has been slowed down to 10 km sec^{-1}, an object like the Cygnus Loop, which now has an energy of 2×10^{49} erg left, would give an effective input of 2×10^{48} erg; the remainder is radiated away. With one Cygnus Loop-type supernova per 60 years, the input rate becomes $1\times10^{39}\ \text{erg sec}^{-1}$ in the Galaxy. The total dissipation rate in the interstellar medium was estimated by Kahn and Woltjer (1967) to be $8\times10^{39}\ \text{erg sec}^{-1}$. Of course an additional energy input comes from H II regions and perhaps from rarer, more energetic supernovae like those whose huge expanding rings are seen in the Magellanic Clouds.

With regard to cosmic rays, it is easily shown that the total energy input into the Galaxy amounts to

$$\mathrm{d}E/\mathrm{d}t = ucM/q \tag{15}$$

irrespective of the confinement volume of the Galaxy (Woltjer, 1968)*. With the energy density $u = 1\times10^{-12}\ \text{erg cm}^{-3}$, c the velocity of light, the total mass of interstellar gas in the confinement volume $M = 4\times10^{9}\ M_\odot$, and the amount of matter traversed by a typical cosmic-ray particle $q = 4\ \text{g cm}^{-2}$ we have $\mathrm{d}E/\mathrm{d}t = 6\times10^{40}\ \text{erg sec}^{-1}$ or 10^{50} erg per supernova. Supernovae may inject cosmic rays in two phases. During the first explosion phase the outer envelope may come off at relativistic speeds (Colgate and White, 1966) and, in the cases where a rotating magnetic neutron star is left, effective acceleration is possible over a more prolonged period (Gold, 1969; Gunn and Ostriker, 1969). If the shell energy is as small as in the case of the Cygnus Loop, the contribution of the first process is unlikely to be large. In the rotating neutron stars, enough energy could be present; but in the one case about which we have information – the Crab

* See also the remark by Syrovat-skii in the Discussion following this Report. (Ed.)

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Nebula – it is unlikely that a large cosmic-ray energy is present. We know that the shell has a very low kinetic energy and those cosmic rays that are present will accelerate the shell effectively, since coupling between cosmic rays and shell motion is to be expected. Perhaps the main producers of cosmic rays are, again, rarer, more powerful supernovae; cosmic-ray acceleration in supernovae is still far more speculative than is frequently assumed.

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