RADIAL VELOCITIES AND PROPER MOTIONS OF GLOBULAR CLUSTER STARS

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ABSTRACT
We describe techniques and results for obtaining radial velocities and proper motions for individual stars in globular clusters. One set of these results for the nearby northern clusters M92 and M13 are discussed in terms of new dynamical models and distance estimators.

1. Introduction

In order to make realistic models of globular clusters, one needs to know something about both the stellar velocities and the light profile. Most earlier studies of clusters have only used the photometry and in a few cases, central velocity dispersions (see, for example, Illingworth 1976) derived from the width of the lines in integrated spectra either by Fourier techniques or direct least-squares fitting in wavelength space. Gunn and Griffin (1979, hereinafter GG) first made dyamical models of a cluster (M3) using radial velocity data for stars throughout the cluster. The fact that those data badly disagreed with the most sophisticated model constructed till that time (Da Costa and Freeman 1976) and demanded both significant anisotropy in the outer parts of the cluster and a significantly steeper mass function (all within the framework of the King (1966)-Mitchie (1963) models, about which we will say more later) says quite certainly that without such data one cannot even approximately understand the dynamics of globular clusters. It is clear that if one had in addition proper motions, essentially all of the ambiguity of reconstructing the three-dimensional velocity distribution from the observations would disappear, and one would have in addition a very powerful handle on the distance. The proper motions available are not quite sufficiently accurate to allow this to be done straightforwardly, as we shall see, but some considerable leverage on the models and quite good distance estimates can still be obtained.

2. Radial velocities.

Radial velocities of stars at these brightness levels (B < 15, say) and to the requisite accuracy (about 1 km/sec) are now being obtained routinely by two techniques which yield comparable efficiencies. The

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original Griffin cross-correlation technique, applied at the coude of the 200-inch Hale telescope (Griffin and Gunn 1974) has been used in all our work. This technique involves the use of an infinite-contrast mask at the focal plane of a high-dispersion spectrograph, the mask having transparent slots which correspond in location to absorption lines in the stellar spectrum. The mask is scanned across the spectrum (or, in our and other cases, the spectrum across the mask) and the total flux through the mask recorded. The position of the mask which corresponds to a minimum in the flux represents the best agreement between the mask and spectrum and directly yields a radial velocity relative to those of a set of standard stars. In practice the scanning is done rapidly and periodically and the scans accumulated in a real or software-simulated multichannel analyzer; the position of the minimum is found in later analysis. Possible problems resulting from spectral-type mismatch between the template star and the object observed, the fact that the shift to match is constant in wavelength and the doppler shift proportional to the wavelength, zero-point and scale drifts, are all unimportant in practice in properly designed equipment, and are discussed extensively in Griffin and Gunn (1974). This technique is some thousands of times faster than photographic spectroscopy, and probably yields more accurate results, if for no other reason than that the zero point of the instrument can be monitored on essentially arbitrarily short time scales. In a modification incorporating an echelle as the dispersing element, Mayor (see Mayor, et al. 1983) in the Coravel machine has solved at least the problem of the shift mismatch and has gained the high luminosity efficiency of echelle spectrographs. This instrument has been used to obtain high-quality velocities in 47 Tuc, as reported by Da Costa and Freeman in this volume. The advantages of the technique are speed and stability; the photomultiplier is used only as a flux collector and even large magnetic perturbations to its electron optics have negligible effects on the results. The disadvantage is that time spent scanning away from the cross-correlation dip is time wasted; this can be minimized to some extent by clever control of the scanning, and this is done to advantage in Coravel. It need not be a major loss for the globular cluster problem, since the line widths in the stars alone demand instrumental profiles some twenty km/s wide and the velocity dispersions are smaller than this--thus not much "continuum" need be scanned once the cluster velocity is found. One looks fondly today at CCD quantum efficiencies, but the rapid time response required and the large photocathode areas required by the realities of optics make their use in this configuration impossible.

It would seem that it would be much more efficient to record all the spectrum at once and do the analysis later in the computer. Since there are many lines, one need obtain only very poor signal-to-noise in the spectrum to obtain very good velocities. The requirement is clearly for a detector with no noise of its own, high quantum efficiency, and excellent dimensional stability. The first requirement today means very high-gain intensification, whatever the ultimate detector. This approach was used by the Stromlo group with an intensified SEC vidicon to obtain a few velocities in NGC6397 (Da Costa et al. 1977). It has since been applied by the Latham and his collaborators at Harvard to a large number of problems, most notable in this context to a followup of Gunn and Griffin's velocity measurements in M3. The original measurements strongly suggested that the incidence of spectroscopic binaries in M3 was very small compared to the field, but the statistics were limited and the result has been called into question by Harris and McClure (1983). Latham and collaborators (1983,1984) have measured all the stars in the GG sample again, and have in fact found one binary; this confirms the greatly reduced numbers but is at clear variance with GG's suggestion that there may be none. Latham et al. have used an echelle spectrograph with a pulse-counting intensified Reticon detector of the Shectograph (Shectman and Hiltner 1974) type. The efficiency, as expected, is somewhat higher than that of the cross-correlation technique, but it is not clear whether the ultimate accuracy is as high. The latter question is irrelevant for globular cluster work in any case, since the errors are small compared to the cluster velocity dispersions.

The ideal machine for this problem has not been built yet but will doubtless be soon. An echelle spectrograph with a resultion of about 0.3 Angstroms with coatings for high throughput in the 4000-6000 A range with a low-noise violet-sensitive CCD as a detector would be at least an order of magnitude more sensitive than any instrument now being applied to the problem. It should not be difficult to build an instrument of this sort with 30 percent quantum efficiency on the sky, and with some effort even higher efficiencies might be achieved. At 15th magnitude, such a spectrograph would yield 20 electrons per resolution element per second; with a thousand stellar absorption lines of about 20 percent average depth at this resolution, one km/sec accuracy would be obtained in of the order of ten seconds, longer for very metal-weak objects just inversely as the line strengths. A CCD with very good low-level transfer efficiency, very good geometric properties, backside illumination with control of the surface potential profile for efficient acculation of electrons generated very close to the surface by violet photons, and very low noise (clearly 2-3 electrons would be desirable) is required. While such chips so not exist today, we can look forward confidently to their existence within a couple of years. Such a machine would allow easy extension of the techniques discussed here to the most distant globular clusters and to dwarf spheriodals.

3. Proper Motions

Proper motions of individual stars in globular clusters measured by the techniques available today require very long time baselines. The field has been completely dominated by Cudworth and his collaborators, who have made use of the Yerkes plate collection, which contains first-epoch plates for the bright northern clusters taken in the first few years of this century. Since one-dimensional velocity despersions for clusters are of the order of five kilometers per second and the nearest clusters are about ten kpc distant, the relative proper motions are of order 10 milliarcseconds per century. Cudworth and Monet (1979) have measured proper motions in M13 with an accuracy of 13 mas/century,

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Cudworth in M3 (1979a) and M5 (1979b) to 20 mas, and in M15 (1976a) and M92(1976b) to 30 mas; all quoted errors are one-component standard deviations.

None of these results are sufficiently accurate to use the inferred motion of individual stars from the proper motions in dynamical analyses, though in the case of M13 its small distance and the relatively great accuracy of its proper motions allows one to approach this goal. What one can do is to use the proper motions in a statistical analysis, if the errors are well enough understood. Since globular clusters are pressure-supported structures, the measurement errors simply add in quadrature to the random velocities, and any systematics such as anisotropy should show in large enough samples. Since the fields are relatively small, it has not been possible to tie to the the background well enough to measure any rotation in the plane of the sky (though it is in principle possible to detect it as an induced anisotropy parallel to the equatorial plane in the random proper motions) or indeed even well enough to measure the absolute proper motion of the cluster, which is expected to be huge on the scale of the relative internal velocities.

If one can convice oneself that the errors are well understood, it is possible, of course, to derive a tangential velocity dispersion up to the distance scale factor. With accurate radial velocities one can derive a radial velocity disperison, and the ratio of these, calculated (with not very much model sensitivity) from a dynamical model allows a distance to be determined. This technique was first applied by Cudworth to his M3 proper motions and the model of GG constructed using their radial velocities; the derived distance of 9.6 kpc compared well with accepted values between 9.2 and 10.5, but the errors were large enough not to be able to choose within the favored range. We will discuss a model of M13 below for which a quite good distance estimate can be obtained.

4. Models for M13 and M92

The two nearest bright northern clusters, M13 and M92, both show dynamically significant rotation in the radial velocity data which we have obtained for them, a fact which significantly complicates the construction of dynamical models--particularly as for both the effects of anisotropy are also important in the outer parts, as was the case for M3. One must use models with three integrals to describe such systems, and we discuss such models in this and the following sections.

The models used are three component modified King(1966a) models, with distribution functions of the form

$$f_j\ (E,J,J_z) = e^{-\beta J^2}\ e^{-A_j \Omega J_z}\ (e^{-A_j E}-1)\ ,$$

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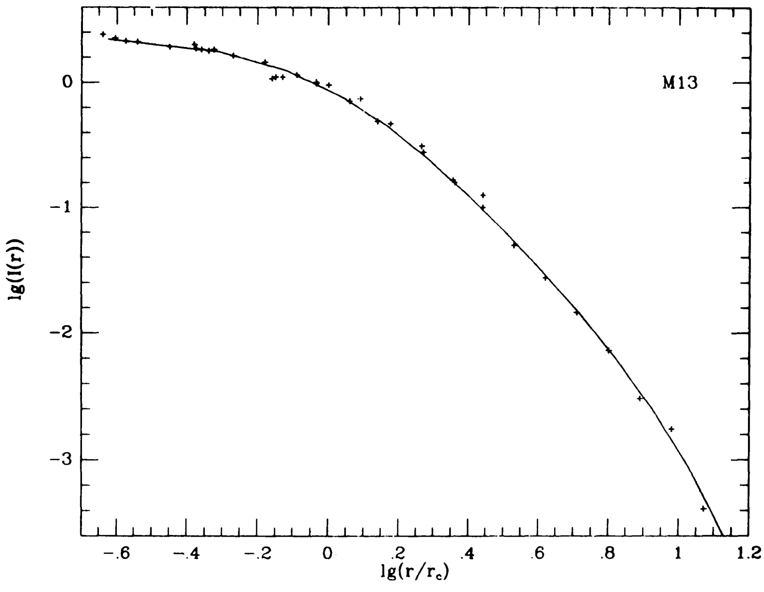

Figure 1a.
The M13 radial light profile. Crosses are the observational points, and the solid line represents the best fitting M13 model.

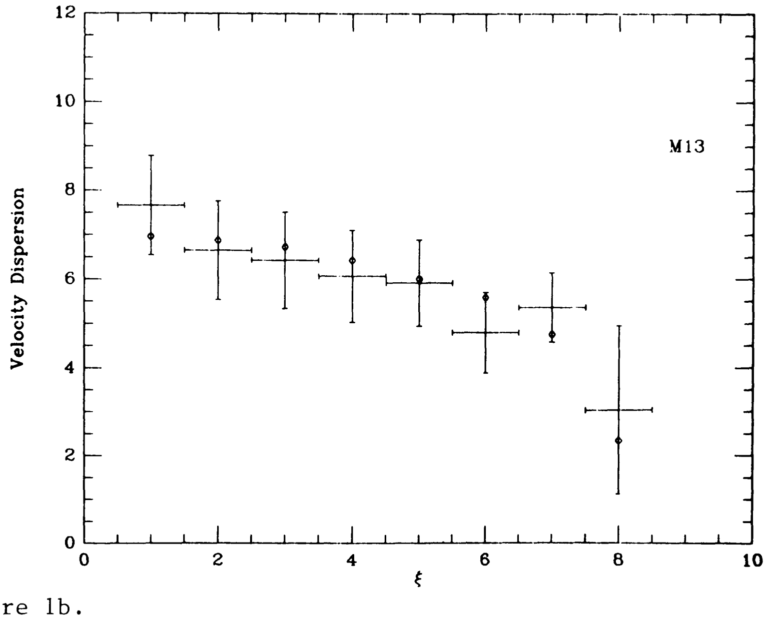

Figure 1b.
Velocity dispersion profile for the M13 model, with the stars binned in radius with 20 stars per bin. The bins are centered at 0.35, 0.77, 1.16, 1.74, 2.42, 3.08, 4.62 and 11.82 core radii.

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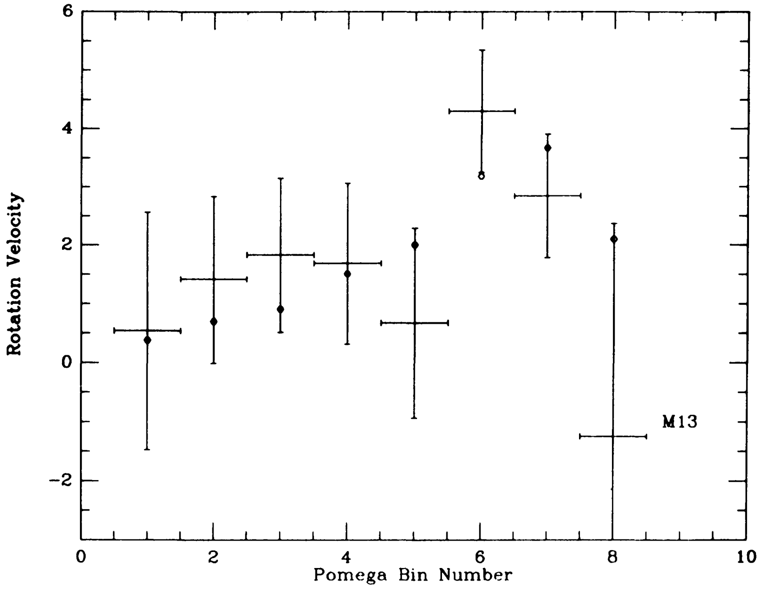

Figure 1c.
Rotation curve for the M13 model, with the stars binned in polar radius with 20 stars per bin. The bins are centered at 0.115, 0.327, 0.517, 0.803, 1.26, 2.13, 3.77 and 9.16 core radii.

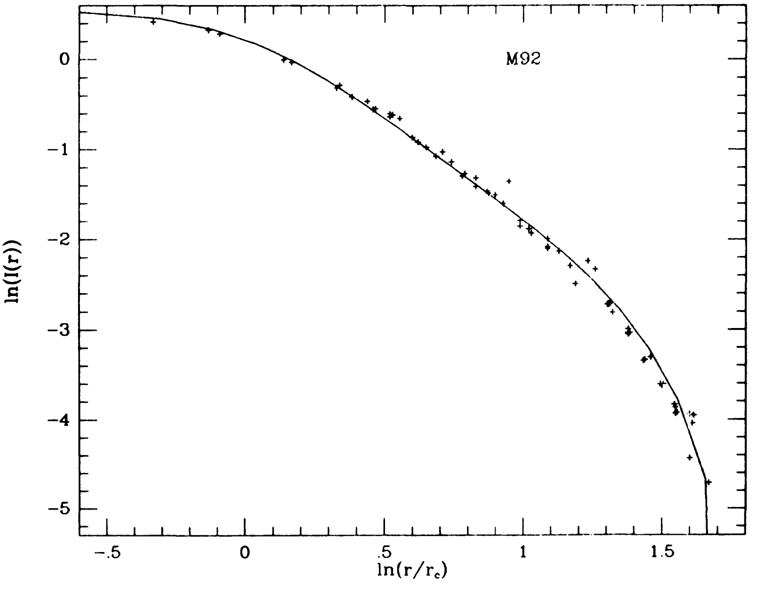

Figure 2a.
The M92 radial light profile. Crosses are the observational points, and the solid line represents best-fitting M92 model.

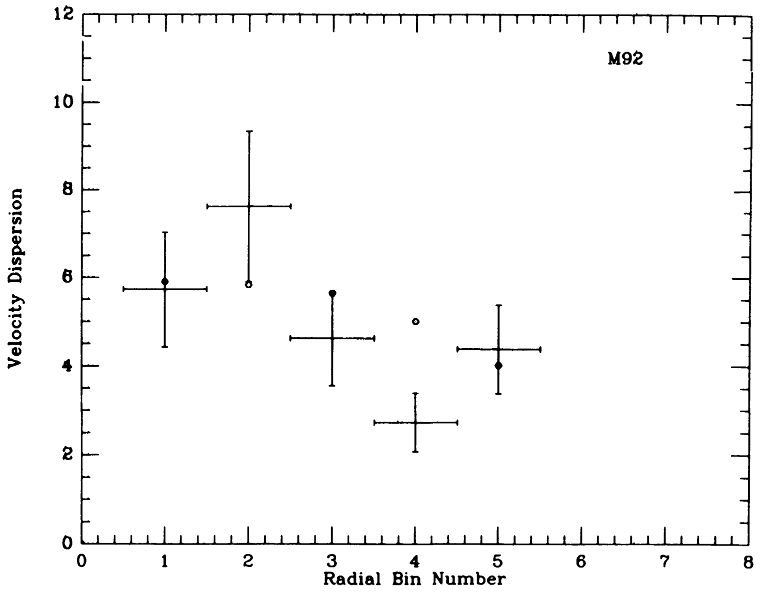

Figure 2b.
Velocity dispersion profile for the M92 model, with the stars binned in radius with 10 stars per bin. The bins are centered at 0.62, 1.97, 3.63, 7.47 and 13.9 core radii.

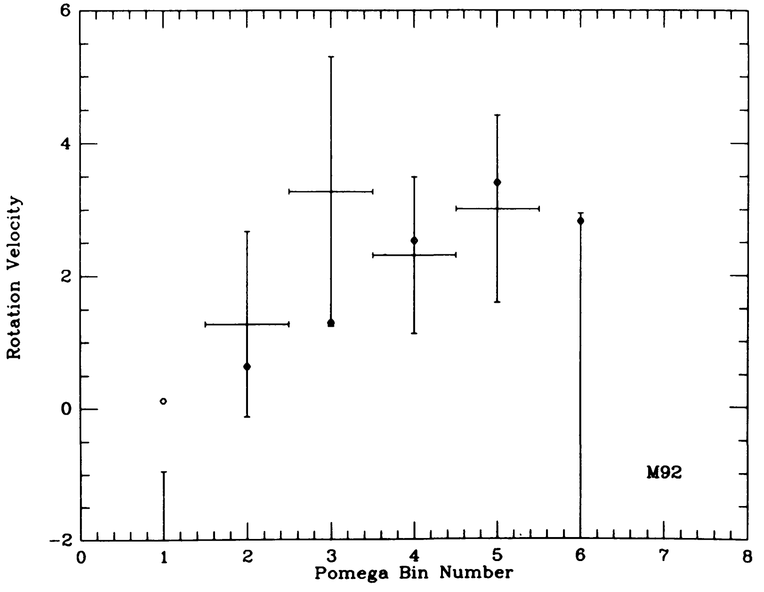

Figure 2c.
Rotation curve for the M92 model, with the stars binned in polar radius with 10 stars per bin. The bins are centered at 0.356, 0.928, 1.79, 3.40 and 7.67 core radii.

Or, in dimensionless form,

$$\tilde{f}\,(\vec{\xi},\vec{u}) = \alpha_j C_j \exp(-\tfrac{1}{2}\mu_j u^2 \xi^2 \xi_t^{-2})\ \exp(\mu_j \tilde{\Omega}\tilde{\xi}+u\phi)\ [\exp(\tfrac{1}{2}\mu_j u^2+\mu_j W)-1]$$

for each of the three mass classes, labeled with j. The notation is the same as that of GG; the new quantities Ω and $\tilde{\Omega}$ are the central angular velocity and its dimensionless counterpart (the "units" of the latter are scale velocities per core radius). It is assumed that the three mass classes are in thermal equilibrium in the central regions, so that the central velocity dispersions are inversely proportional to the square roots of the masses. The models are rotating, with a linear rotation curve in the core, and are anisotropic in the outer parts. These models, and the techniques used to construct them, are described in Lupton and Gunn (1984), following the notation and precepts outlined in GG. In particular, the use of the total angular momentum as an approximate integral is shown to introduce errors of less than about 1% in the density distributions.

5. The Observations

a). Light Distribution. In globular clusters, almost all of the light comes from the stars on the giant branch, with a smaller contribution from the stars on the horizontal branch and near the turnoff. This means that essentially all the light comes from a relatively small number of stars, and that the light distribution is seriously affected by small number statistics. The problem is especially serious in the core, where for example in M3 the integrated light is uncertain to about 6% (GG). Seriously affected as well are the measurements of the small ellipticities of the relatively slowly-rotating systems discussed here, though the flattening is obvious on the POSS prints. We have used the ellipticity data of Kadla et al (1976) and our own photographic photometry from out-of focus 1.2-meter Schmidt plates to determine the ellipticities.

The radial photometry is in better shape, consisting of photometry and star counts by King (1966b) and King et al. (1968), and of our own data. The brightness may be integrated to give an estimate of the total magnitude of the clusters being studied.

b). Velocities. The radial velocities used in this paper were measured with the Griffin-Gunn (1974) cross-correlation coude' spectrometer on the Hale telescope. Typical accuracies for a single measurement are about 1.2 km/s, to which must be added (as was the case for M3 as well) a "jitter" of about the same size to bring the cumulative chi-squared of the measurements down to an acceptable value. This extra error is believed to be due to motions in the atmospheres of the high luminosity giants studied, but its origin is immaterial to the dynamical analysis. Only stars brighter than about V = 14.0 can be studied easily, so these radial velocities are restricted to a relatively small number of stars in each cluster. 142 velocities are available for M13 while for M92, where velocities are very difficult to

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measure owing to the extreme weakness of the metal lines, we have only 49. These velocities are presented and discussed in detail in Lupton, Griffin, and Gunn (1984) (hereinafter LGG).

The proper motion data are those of Cudworth and Monet for M13 and Cudworth for M92 discussed above. Since the errors in these motions are large compared to the expected velocities we did not use them in the model-making procedure. We did, however, use our models in conjunction with the proper motions to estimate the distance to the cluster, and to compare a postiori the velocity ellipsoids observed with those predicted by the models.

6. Fitting Procedures

We have adopted fitting procedures which are analogous to those presented in GG, though the added complexity of the rotation and subsequent lack of circular symmetry both in the brightness and in the kinematics complicates things somewhat. As was done there, we have restricted ourselves to models which fit the azimuthally averaged brightness profiles essentially perfectly. We have used here a maximum-likelihood technique with an approximate distribution for the likelihood along with realistic estimates of the errors in the brightness profiles to judge the significance of deviations from the observed profile; we had no such tool for the M3 work, and it is possible that those models were overconstrained by the observed brightness distribution. The ellipticities played no role in the fitting, but were examined afterward.

The scale of the fit to the brightness gives the angular core radius and the total flux, which with an assumed distance gives the physical core radius r and the total luminosity. The shape parameters are essentially those described in GG, the inner core shape being determined by the total mass in massive remnants (taken here, as there, to be 1.2 MO white dwarfs), the slope in the "body" of the cluster, determined mostly by the depth of the dimensionless central potential, and the cutoff radius, determined here by a combination of the central potential, the anisotropy, and the rotation.

The radial velocity data was likewise fit to the model by a maximum likelihood technique, here a generalization of the one described in GG. Here we have three parameters to fit, the central velocity dispersion (the velocity scale in the models), the mean cluster velocity, and the position angle of the rotation axis on the sky. As in the spherical models, the run of dispersion with radius is a critical diagnostic for mass in low-mass stars and for anisotropy. The slope of the mean rotation curve in units of central velocity dispersion per core radius determines the dimensionless central angular velocity for any assumed inclination angle, but the inclination angle itself is very poorly determined, even if one uses the ellipticity data, as has already been noted by Wilson (1975) . That this should be so is an immediate consequence of the fact that the ellipticity is quadratic in the angular velocity, the observed velocity is proportional to the sine of the inclination angle, and the observed ellipticity to the square of that quantity. The isotropic component of the centrifugal potential affects the cluster density distribution, so one has a weak handle, but this near-degeneracy makes for a good deal of uncertainty in the final models.

The list of parameters to be fit is the one for the spherical three-component ones enumerated in GG: Three dimensional ones, the total light, the core radius, and the central velocity dispersion; three dimensionless parameters for the mass function, here taken to be the mass fraction in 1.2 MO white dwarfs, the mass fraction in lower main sequence stars, and the mean mass of those objects (these two parameters manifest themselves as the slope and lower mass cutoff for continuous mass functions), and two dimensionless model parameters, the depth of the central potential in scale velocity units and the anisotropy radius in core radii. To this list are added three parameters associated with the rotation: the dimensionless central angular velocity, and the two angles, the inclination and the position angle of the apparent axis. A daunting list, but GG outlined how the first set are quite unambiguously determined by the behavior of the data; the position angle is well determined, but as noted above, the combination Ω sin i is all that is determined well by the observed rotation curve.

The final test of the models is the agreement or lack thereof between the dynamical mass, $Av_o\ r_c/G$, where the dimensionless number A is calculated with the model, and the population mass, $L^*(M/L)$, where M/L is the mass-to-light ratio for the assumed mass function. It is important to note that this comparison is affected by errors in the distance, since the dynamical mass is proportional to the first power of the distance and the population mass to the square; hence claims of the presence or absence of "missing mass" in globular clusters based on these techniques should be viewed with requisite caution.

One last physical check on the models is to determine whether the anisotropy radius is in fact outside the radius where the local deflection relaxation time is of order 10 billion years; if it is not, it is difficult to understand how the anisotropy has survived till the present.

7. Results

a). M13. With an inclination angle of 60 degrees, near the most probable value, we easily obtained good fits to both the radial velocity and brightness data. The fits are shown in figures 1 and 2. The derived mass function parameters correspond to a main sequence slope of about 2.2, compared to a value of 2.0 for the best M3 model in GG and a value less than 1 for the solar neighborhood. This large value is driven largely by forcing agreement between the dynamical and population masses; adding remnants at the massive end or very low-mass stars at the other end to achieve the same end is precluded by the core shape in the first instance and the observed rapid fall of velocity dispersion with radius in the second.

The parameters are summarized in the Table 1; a surprising result is the very small value of the anisotropy radius, only five core radii.

We also built M13 models with an inclination angle of 30 degrees, which, of course, almost doubles the central angular velocity. These models, though not near the classical ratio of 0.14 for ordered kinetic to gravitational energy for the onset of instability, represent the most rapidly rotating models we can easily construct. The centrifugal potential pulls in the wings of the cluster, and to fit the observed light profile the anisotropy radius must be very small, only 2.2 core radii, at which radius the deflection relaxation time is of order one billion years. We thus regard this model as unlikely to represent the cluster, though the fit is very nearly as good as the best 60 degree model.

The scale velocity of M13 is quite high, 7.8 km/sec, and the distance fairly small, the favored values clustering about 6.5 kpc. The characteristic proper motions are thus pretty large on the scale of such motions, being about 25 millarcseconds/century. With the phenomenally low errors achieved for the proper motions in this cluster by Cudworth and Monet, only about 12 mas/century, very good comparisons can be made for the tangential motions and an excellent distance can be obtained. The anisotropy is obvious in the data, and appears to set in at about the place predicted by the models, but the amplitude of the anisotropy appears too large in the proper motion data, as if the errors had been slightly underestimated resulting in too large a ratio of the corrected radial to tangential motions. These points are discussed in detail in LGG, as are the schemes used for deriving the distance from various "best fit" criteria to the observed proper motions. We obtain a distance of 6.5 (-0.4,+0.6) kpc, in excellent agreement with the accepted value.

b). M92. We have radial velocities for only 49 stars in this cluster, and so the fitting is much less certain. The kinematics of the outer parts is particularly poorly constrained, so the radius of the onset of anisotropy is very uncertain; in any case, very nearly isotropic models seem to fit well. The rotation is not as important dynamically as is the case in M13, and the rotation curve is poorly determined, so we have build models only for an inclination of 60 degrees.

The best-fitting model has an anisotropy radius of about twenty core radii, less extreme than M3 and much less so than M13, and in agreement with Cudworth's proper motion data, which indicates that the motions are isotropic over the area covered by his data, almost all of which come from within this radius. The mass function of the best model again has a slope of 2.2, and the comments made about that value for M13 we could repeat here, though the larger uncertainties must be kept in mind. The modulus of M92 is commonly thought to be slightly greater than that of M13, and our derived scale velocity smaller; these, combined with the vastly larger proper motion errors for this cluster make the dynamically derived distance estimate very uncertain indeed. We obtain a value of 6.1 (+1.0,-0.9) kpc, much smaller than the accepted 8.3 kpc value. Cudworth has indicated to us that the errors of the proper motions, which dominate this estimate, are likely to have been underestimated, and that a reanalysis using more modern techniques is underway.

The parameters again are summarized in the table.

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Table 1. M13 and M92 models: properties of best fit models

cl	r_c(pc)	R(pc)	v_o(km/s)	M/L_V	M_5	Td_7	W_o	ζ_T	$\tilde{\Omega}$
M13	2.17	22.9	7.858	2.39	5.84	17.	7.5	5.0	0.34
M92	0.74	61.0	6.077	2.39	3.92	1.6	11.5	20.	0.14

r_c is the core radius in pc, R the mean limiting radius, v_o the velocity scale (very nearly the central one-dimensional velocity dispersion), M/L_V the visual mass-to-light ratio, M_5 the total mass in units of 10^5 solar masses, Td_7 the reference central deflection relaxation time for the mean mass star, W_o the dimensionless central potential, X_t the dimensionless anisotropy radius, and OM the dimensionless angular velocity.

8. Discussion

We have discussed techniques for obtaining radial velocities and proper motions for globular cluster stars and the application of these data to the construction of dynamical models of the generalized King type, in particular the application of approximate three-integral models to photometric and kinematic observations of the rotating globular clusters M13 and M92. Excellent fits are obtained, and relatively unambiguous determination of the large number of parameters which characterize the models is achieved with the notable exception of the inclination of the rotation axis to the line of sight. The inclination is strictly unobtainable in the limit of small rotation but does not affect the other parameters (except, of course, the angular velocity) and is in practice unobtainable for more rapid rotation but confuses the fitting of other parameters.

The reliability of the models themselves is questionable on two fundamental points. First, the relatively arbitrary form of the distribution function, and, second, the assumption of thermal equilibrium among the mass classes. The points are related, because thermal equilibrium in a single mass class dictates a distribution function not qualitatively different from the one we are using, though one may quibble with the exact form of the energy cutoff and such. Many of our conclusions about anisotropy and the lower end of the mass function depend quantitatively but not qualitatively on thermal equilibrium and the concommitant mass segregation. The models, though complex, are probably the simplest ones which include the physics one knows to be necessary to model real clusters; what is in question is not whether there is enough, but whether it is correct. Only real evolutionary models (and real data on mass functions and hopefully someday kinematic data for stars of radically different mass) will finally tell. These models in any case represent probably the farthest one wishes to proceed with the formalism and philosophy originated by King(1966a); the next step will of necessity be very much more complicated.

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