NUTATION IN SPACE AND DIURNAL NUTATION IN THE CASE OF AN ELASTIC EARTH

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In order to improve the representation of nutation, the effect of elasticity of the Earth on the nutation in space and diurnal nutation of the terrestrial rotation axis is considered and its amplitude is evaluated for the principal terms.The choice between several methods taking this effect into account is discussed. A comparison with the effect induced on nutation by the existence of a liquid core in the Earth's interior shows that the consideration of elasticity alone cannot give any amelioration in the representation of nutation.

INTRODUCTION

For geophysical studies of the rotation of the Earth, it is necessary to take into account all effects acting on the motion of the instantaneous rotation axis which can be mathematically represented. The elasticity of the Earth is one of these effects.

The effect of elasticity on the free motion of the instantaneous rotation axis within the Earth is well known : it converts its Eulerian period of 305 days into the Chandlerian period of 430 days. The effect of elasticity on the tidal variation of gravity and on the tidal deviation of the vertical at each point of the Earth is also well known and is usually considered in the reduction of the observations.

But the modification of the coefficients of nutation due to elasticity is not well known and it is not clear if it is necessary or not to take it into account in the reduction of the observations. Poincaré (1910) and Jeffreys & Vicente (1957) have evaluated this modification as negligible. In contrast Fedorov (1963, 1977) and Mc Clure (1973) considered that it must be taken into account; however the modification proposed by these authors is not always correctly understood or not always admitted for the diurnal nutation (Mc Carthy 1976).

The question of the axes to which the coefficients of the nutation in space must be referred (Jeffreys 1959,Atkinson 1973) is also of great importance in the case of a non rigid Earth.

D. D. McCarthy and J. D. Pilkington (eds.), Time and the Earth's Rotation, 169–174.
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https://doi.org/10.1017/S0074180900036196 Published online by Cambridge University Press

In this paper, we have computed the modifications of the coefficients of nutation, in the case of an elastic Earth, referred to three axes which can be considered : the Gz axis fixed within the Earth, the instantaneous axis of rotation and the axis of angular momentum. The results show how elasticity modifies the representation of nutation and whether or not it is necessary to take this modification into account.

1. SOLUTION FOR NUTATION IN SPACE AND DIURNAL NUTATION IN THE CASE OF AN ELASTIC EARTH

In the reduction of the observations, we have to express the position of a terrestrial reference frame (Gxyz) (G being the Earth's center of mass) with respect to a non-rotating reference frame. It is thus necessary to express the motion in space of the Gz axis. We do not consider here the Chandlerian free motion nor the forced annual motion of the rotation axis within the Earth, which are not predictable, so we limit the motion in space of the Gz axis to its lunisolar precession and nutation.

Considering the lunisolar torque and the variation of the Earth's inertia tensor due to rotational and tidal deformations, the complex solution for the coordinates of the pole of rotation in (Gxyz) can be written :

$$m = -\, iE_{☾} \sum_j R(j)\, A'_{21j}\, e^{-i(\omega_j t + \beta_j)} \qquad (1)$$

with : $E_{☾} = \dfrac{3Gm_{☾}(C-A)}{c_{☾}^3 \Omega^2}$ (2), $R(j) = 1 - \dfrac{k_2 n_j}{k_s \Omega}$ (3), $A'_{21j} = A_{21j} / (1 - \dfrac{A}{C}\dfrac{n_j}{\Omega})$ (4)

G being the gravitational constant,
Ω the Earth's angular velocity, $n_j = \Omega - \omega_j$,
(A,A,C) the principal moments of inertia of the Earth in the non deformable case,
$m_{☾}$ and $c_{☾}$ respectively the lunar mass and the Earth-Moon mean distance,
k_2 and k_S respectively the Love number of degree 2 and the secular Love number,
A_{21j} the jth Doodson's real coefficient of degree 2 and order 1 for the total lunisolar term of frequency ω_j in the tidal potential,
$\omega_j t + \beta_j$ a linear combination of the mean sidereal time Φ, the mean lunar and solar longitudes ⊙ and ☾, the mean longitudes of lunar and solar perigees p and π and the longitude ☊ of the lunar ascending node.

(1) is the expression of the diurnal nutation of the rotation axis within the Earth.

Let θ, Ψ, Φ be the Euler's angles between the (Gxyz) system and the non-rotating system (GXYZ) defined by the mean ecliptic and equinox of the epoch t_o : θ is the obliquity of the ecliptic, Ψ the longitude of the equinox and Φ the angle of rotation of the Earth. Using expression (1) for m and Euler's kinematical relations, we obtain, by an

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integration assuming sin θ = constant, the following expression for the lunisolar precession and nutation of the Gz axis:

$$\Delta \theta_{z} + i\Delta\Psi_{z}\sin\theta = E_{\mathbb{C}}\left[iA_{210}\Omega t + \sum_{j\neq o} R(j)\, A'_{21j}\frac{\Omega}{n_j}\, e^{-i(-n_j t+\beta_j)}\right] \quad (5)$$

The motion of the Gz axis with respect to the (GXYZ) system can be obtained by using the expressions for the lunisolar precession and nutation of either the rotation axis or the angular momentum axis, respectively given by (6) and (8); the additional motions of these axes with respect to the Gz axis are given by (9) and (10) in the (GXYZ) system, and by (1) and (12) in the (Gxyz) system:

$$\Delta \theta_{r} + i\,\Delta\Psi_{r}\sin\theta = E_{\mathbb{C}}\left[iA_{210}\Omega t + \sum_{j\neq o} R(j)\, A''_{21j}\frac{\Omega}{n_j}\, e^{-i(-n_j t+\beta_j)}\right] \quad (6)$$

with : $A''_{21j} = A_{21j}(1-\frac{n_j}{\Omega})/\ (1-\frac{A}{C}\frac{n_j}{\Omega})$ (7)

$$\Delta \theta_{H} + i\Delta\Psi_{H}\sin\theta = E_{\mathbb{C}}\left[iA_{210}\Omega t + \sum_{j\neq o} A_{21j}\frac{\Omega}{n_j}\, e^{-i(-n_j t+\beta_j)}\right] \quad (8)$$

$$\delta\,\theta_{rz} + i\delta\Psi_{rz}\sin\theta = E_{\mathbb{C}}\sum_j R(j)A'_{21j}\ e^{-i(-n_j t+\beta_j)} \quad (9)$$

$$\delta\,\theta_{Hz} + i\delta\Psi_{Hz}\sin\theta = E_{\mathbb{C}}(A/C)\sum_j R\,A'_{21j}\, e^{-i(-n_j t+\beta_j)} \quad (10)$$

with : $R = 1 - \frac{k_2}{k_s}$ (11)

$$(H/C\Omega) = -\ iE_{\mathbb{C}}R\sum_j A'_{21j}\quad e^{-i(\omega_j t+\beta_j)} \quad (12)$$

2. COMPARISON WITH THE CASE OF A RIGID EARTH AND WITH THE CASE OF AN EARTH MODEL INCLUDING A LIQUID CORE

In the case of a rigid Earth, $k_2 = 0$, thus R(j) = 1 and R = 1 .

In the case of an elastic Earth, as compared to the case of a rigid Earth, the amplitude of the circular nutation, j , in space of the Gz axis or of the rotation axis is multiplied by the factor R(j) . The corresponding modified coefficients of nutation in longitude and obliquity are computed in Table 1 for the principal terms. R(j) is also the factor of modification , due to elasticity, of the amplitude of the term of frequency ω_j in the expression (1). This factor being between 0.94 and 1.06 for the considered diurnal waves, we see that the diurnal nutation of the rotation axis within the Earth is practically not affected by elasticity. This is confirmed by the values computed in Table 1 .

In contrast, the motion in space of the angular momentum axis

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Table 1 - Amplitudes (in 10^{-4}") of the coefficients ($\Delta\Psi \sin\theta, \Delta\theta$) of the principal nutations in space and of the corresponding terms of diurnal nutation within the Earth .

Argument	Axis of angular momentum			Rotation axis				Gz axis		
	Nutation in space	Diurnal Nutation		Nutation in space		Diurnal Nutation		Nutation in space		
		Rigid Earth	Elastic Earth	Rigid Earth	Elastic Earth	Rigid Earth	Elastic Earth	Rigid Earth	Elastic Earth	Jeffreys-Vicente's Earth-model
Ω	-68722 92277	- 2 12	- 1 8	-68722 92277	-68726 92280	- 2 12	- 2 12	-68708 92267	-68713 92270	-68376 92020
2Ω	827 -902	0 0	0 0	827 -902	827 -902	0 0	0 0	827 -902	827 -902	
$\odot - \pi$	501 0	1 1	0 0	501 0	501 0	1 1	1 1	501 -1	501 -1	
$2\ \odot$	-5054 5509	-29 1	-20 1	-5054 5509	-5045 5501	-29 1	-29 1	-5084 5537	-5075 5528	-5249 5743
$3\ \odot - \pi$	-197 215	-2 0	-1 0	-197 215	-196 215	-2 0	-2 0	-199 217	-198 216	
g	269 0	5 5	4 3	269 0	269 3	5 5	5 5	269 -10	269 -7	
2 ☾	-812 885	-67 2	-47 2	-812 885	-793 867	-67 2	-65 3	-881 949	-861 930	-903 978
2 ☾ - Ω	-136 183	-13 2	-9 1	-136 183	-132 180	-13 2	-12 2	-150 194	-146 191	
2 ☾ + g	-104 113	-13 0	-9 0	-104 113	-100 110	-13 0	-13 0	-118 126	-114 122	

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remains unchanged in the case of an elastic Earth as compared to the case of a rigid Earth, but its diurnal nutation is multiplied by the factor R (R = 0.70 if k_2 = 0.30) . The corresponding values for the principal terms are given in Table 1 .

The modified coefficients of nutation of the Gz axis, computed by using the factors $\alpha(j)$ of Jeffreys&Vicente(1957) for the corresponding circular nutations in space of the rotation axis, in the case of an elastic Earth's model including a liquid core, are also given in Table 1 for the three principal terms. We can see that the modifications are much larger than in the simpler case of an elastic Earth.

The computations of Table 1 have been made by using the coefficients of nutation in space of the rotation axis given by Kinoshita (1977) for a rigid Earth and the values k_2 = 0.30, k_S = 0.96, (A/C) = 0.996.

CONCLUSION

Table 1 shows that :

- In the case of an elastic Earth, the rotation axis and the angular momentum axis are separated by 0".0020.

- The modifications of the coefficients of nutation due to elasticity are lower than 0".0001 except for the terms of arguments ☊ , 2☾ -☊, 2☾+ g for which it reachs 0".0004, for the term of argument 2 ☉ for which it reachs 0".0009 and for the term of argument 2☾ for which it reachs 0".0020 .

- If these modifications were the most important, the best method to take them into account must be to use the modified coefficients of nutation given in Table 1 for an elastic Earth and referred to the Gz axis , in order to avoid diurnal terms referred to the (Gxyz) system.

- Using the present IAU coefficients (Woolard 1953) referred (with a precision of 0".0001) to the axis of angular momentum and unchanged by elasticity, the simplest method to take elasticity into account has been given by Fedorov (1963, 1977) : it consists of multiplying the diurnal nutation of the axis of angular momentum for a rigid Earth by the factor R .

- The influence of elasticity on the coefficients of nutation is negligible with respect to the one due to the presence of a liquid core in the Earth's interior.

We can then conclude that the only good method to improve the representation of nutation must be to use the modified coefficients of nutation in space obtained from observations and from the most complete Earth's model.

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The proposition of Atkinson (1973) consists to use the coefficients of nutation in space referred to the Gz axis. It seems that this method remains the simplest and the most satisfying one in the case of an elastic Earth and in the case of a more complete Earth's model.

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