Conclusion

In this book we have developed relativistic quantum field theory at finite temperature and density. We have studied extensively the theories of three of the four fundamental forces of nature: QED, QCD, and the Glashow–Weinberg–Salam theory of the weak interactions. In its nonrelativistic quantum mechanical guise, QED is responsible for the structure of atomic and molecular systems. Here we have focused on the properties of relativistic plasmas as realized in astrophysical environments. We have studied the screening of static electric charges, the propagation of collective excitations with the quantum numbers of the photon and the electron, shear and bulk viscosities, and thermal and electrical conductivities. We have also used the cold equation of state of dense electrons to calculate the masses and radii of white dwarf stars.

Spontaneous symmetry breaking is an important concept in both the strong and the electroweak interactions. When such symmetries are broken, the result is Goldstone bosons that reflect the underlying symmetry. In simple models illustrating this phenomenon, the spontaneously broken symmetry is restored at high enough temperatures, often via a second-order phase transition. An extension of these models to include gauge bosons reveals the Higgs mechanism, whereby one of the would-be Goldstone bosons combines with a gauge boson to produce a massive vector boson with three spin states. In simple enough models, this symmetry is restored at high temperatures.

QCD is the theory of quarks and gluons. We have studied it using perturbation theory and have found the limitations of the latter. The minimum extension is to sum the set of ring diagrams. This gives a contribution of order g^3 to the pressure at high temperature. Contributions of order g^4, $g^4 \ln g^2$, g^5, and $g^6 \ln g^2$ have all been computed at high temperature, with rather slow convergence. The ring diagrams spawned a more elaborate technique that goes under the title of hard thermal loops.

They are important for calculating various linear-response properties of quark–gluon plasma, such as the emission of electromagnetic radiation in the form of photons and lepton pairs. At asymptotically high temperatures asymptotic freedom forces $g^2(T)$ to go to zero, albeit only logarithmically. Since individual quarks and gluons are never observed at zero and low temperatures, due to confinement, only color-neutral objects, or hadrons, can exist there. Numerical calculations with lattice gauge theory show conclusively that for the physical three-color theory without quarks, there is a first-order phase transition separating the two phases. For two flavors of massless quarks it should be a second-order transition, and for three massless flavors it should be first order. The answer for two up and down quarks, which are light, and one slightly heavier strange quark is still not known with certainty. Cold dense quark matter has been shown to be color superconducting. Various ways of pairing quarks can occur, including two-flavor superconducting and color-flavor-locked superconducting.

At subcritical baryon densities, the most economical way to describe the system is in terms of nucleon and hyperon degrees of freedom. The simplest model that displays the main features of nuclear matter is the Walecka model, which is readily solved in the mean field approximation. Sophistications can include more interactions and more fields, and solving to a higher number of loops. Complications with the former occur at high densities when the baryons are densely packed and multiparticle interactions become important. Complications with the latter are due to the large, order of 10, coupling constants. In any case, the philosophy is to construct the most sophisticated Lagrangian possible, that reflects the symmetries of QCD and low-energy scattering properties, and then to calculate the partition function to the best of one's abilities. The goal is to extrapolate to high densities, such as those in a neutron star. In fact, dozens of such stars have been observed with masses measured to be twice that of a star composed of neutrons alone, thereby showing the crucial importance of including interactions and/or other degrees of freedom.

Hot hadronic matter occurs at subcritical energy densities and with small or zero baryon density. The symmetries of QCD, particularly chiral symmetry, again restrict the form of effective Lagrangians used to describe the properties of this matter. The equation of state at small temperatures is quite well determined. As the temperature rises, more and more of the hundreds of hadrons observed in particle physics experiments are created, and the interactions among them are complicated and generally unknown. Still, it is important to understand this type of matter for it is the ultimate fate of quark–gluon plasma created in high-energy heavy ion collisions, as explored at accelerators at Brookhaven National Laboratory and at CERN. Signatures of the formation of quark–gluon plasma include the thermal emission of photons and lepton pairs, J/ψ production, strangeness production, and the relative abundances of numerous species of mesons and baryons.

The early universe provides an ideal setting to study matter at extraordinarily high temperatures. If QCD, for example, does undergo a first-order phase transition with its physical parameters then one may study the nucleation of the low-density hadronic phase from the high-density quark–gluon phase and the subsequent evolution of the bubbles and drops. The resulting inhomogeneities in energy density, baryon density, and isospin density may even influence nucleosynthesis at later times. At an even earlier epoch it was suspected that the spontaneously broken symmetry of the combined electroweak interactions might have been restored. A mean field approximation yields a second-order phase transition, but this becomes a very weak first-order transition when a resummation of the ring diagrams is done. This might have bided well for baryogensis occurring at this time via nonperturbative field configurations or sphalerons. However, it turns out that the order and even existence of a transition depends on the value of the quartic coupling in the Higgs sector, or rather on the Higgs mass. Lattice calculations in the three-dimensional sector show that present limits on the as yet undiscovered Higgs boson preclude a phase transition.

The reader should now be in a position to read the current literature on finite-temperature field theory and to make original contributions. There are a large number and variety of topics that require investigation. Neutron stars are being discovered all the time. Refined calculations of dense nuclear matter are still needed. Comparing their computed mass, radius, glitch characteristics, and cooling rates with observation should be invaluable for learning about the matter inside the densest objects in the universe. Since this is likely to be the only environment where superconducting quark matter may exist, it is necessary to understand it thoroughly. It has been suggested that quark matter at modest densities is actually in a color-superconducting crystalline state; this need to be worked out. The matter formed in high-energy nuclear collisions at RHIC seems to be behaving as a near perfect fluid. What is the nature of quark–gluon matter just above the critical, or crossover, temperature? What are the correlations between quarks and gluons there and how strong are they? Lattice calculations may be the best approach for studying the strongly coupled region in this vicinity. Much has been accomplished, but more work needs to be done even though the first lattice calculations at finite temperature were made twenty-five years ago. Analytical results are always appealing and welcome; the order g^6 and g^7 contributions to QCD should be available in the near future. How far can one go? A topic that has not been covered in this text is the absorption of high-energy jets at RHIC. This may well provide important information on the nature of the matter the jets traverse.

The full equation of state of electroweak theory has not been computed to the same level as it has for QCD. The importance of this theory for the early universe, and the possibility that it affects baryogenesis, strongly suggests that more work ought to be done. The same is true of grand unified theories (GUTs), which attempt to unify the strong, weak, and electromagnetic forces. Supersymmetry and supersymmetric extensions of the standard model have been studied to some extent in the literature but not at the level that QCD has. Hawking radiation has been discussed briefly in this book. It is unique in the sense that so far it is the only concrete connection we have between quantum theory and gravity. How was it manifested in the early universe, and where might it possibly be manifested today? More generally, how can one use thermal field theory in a possible theory-of-everything, namely, string theory? What about dark matter and dark energy?

We hope that, in some way, this book stimulates people to make further progress. There is much to be done. There is work for all!