SPATIAL DISTRIBUTION OF QUASARS

Yaoquan Chu and LiZhi Fang
Center for Astrophysics
University of Science and Technology of China
Hefei, Anhui, The People's Republic of China

1. INTRODUCTION

The distribution of quasars has become one of the most interesting problems in observational cosmology. This is due mainly to the development of theory of the formation of large scale structure in the universe. In recent years, several scenarios of clusteringhave been proposed. In the adiabatic case, the clustering process is from larger scales to smaller ones, i.e., the first systems to form out would be on the scale of superclusters, then these systems fragment to form smaller scale systems such as galaxies. In the isothermal case, the clustering is from smaller scales to larger ones, namely, galaxies condense out at first and larger scale systems, such as clusters and superclusters, then form later via hierachical build-up processes. In the universe contain two components, the scenario of clustering might be different from both standard adiabatic and isothermal cases(1). According to this new scenario, there should be two kinds of small scale objects, one is formed due to fragment of larger scale systems, another is formed before large scale systems form.

Therefore, one of the crucial problems in cosmology is to distinguish which scenario is the right one. For doing this test we need information of the largescale structure at different cosmological time, i.e., different redshifts. However, up to now, most of the information of the structure of the universe is limited to a very tiny fraction of the observable universe, it comes from the study of distributions of galaxies, all of which with redshifts $Z < 1$. From the $Z < 1$ information, it would not be able to judge which one of scenarios agree with the actual situation better. Hence the distribution of quasars become important. It can tell us the distribution of matter (at least the luminous matter) in the time $Z \sim 1 - 3$, just earlier than that of $Z < 1$ galaxies.

Work on the quasars distribution has often been hampered by the lack of good complete samples. Recently, a large number of new identifications of quasars have been done from the surveys using

A. Hewitt et al. (eds.), Observational Cosmology, 627–638.

https://doi.org/10.1017/S0074180900159650 Published online by Cambridge University Press

objective prism or grating prism techniques. The total number of confirmed quasars is about 3000 and quasars' candidate about 10^4. Several surveys are already available in studying the spatial distributions of quasars.

The quasars surveys have been investigated in Sect. 2 from the view of point of clustering analysis; In Sect. 3 the statistical methods employed in the analysis are discussed. The statistical results and their implication in the clustering scenario are given in Sect. 4 - 6.

2. QUASARS SURVEYS

The difficulty of to do the statistical analyses of quasars clustering is the lack of available samples of quasars. Most of homogeneous surveys for quasars listed only brighter quasars, or covered only very small area. For example, the Schmidt-Green sample have only 46 confirmed quasars but spread 10714 square degree on the sky; Braccesi et. al. found 8 confirmed quasars in their 37.2 square degree UVX survey; Kron and Chiu's sample cover only 0.1 square degree. These samples can not be used to do statistical analyses.

In recent years, a large number of quasars surveys have been carried out. A complete catalogue of main slitless-spectrum surveys can be found in "The Asiago Catalogue of Quasars Edition 1985" (2). About 10000 quasars canditates have been identified using these methods. In complementary to this methods, the ultraviolent-excess (UVX) search methods is best suited for quasars with redshift $Z < 2$.Several wide field samples which are comparatively available for the tests are listed as follows:

a, Savage and Bolton's survey of two $5^\circ \times 5^\circ$ fields near the South Galactic Pole; (3)

b, CTIO optical quasars sample given by Osmer; (4)

c, South Galactic Pole sample of Shanks et. al.; (5)

d, $01^h\ 12^m$, -35° field sample given by Savage et. al.; (6)

e, Smith and He's survey of a 40 square degree field centred at $01^h\ 44^m$ and $-40^\circ\ 00'$; (7)

f, S.A. 94 field optical quasars sample given by Barbieri et. al.

3. STATISTICAL METHODS

The advances and widespread availability of computers have made the statistical analysis of large catalogue of data to be a tractable work. Now the distribution on one, two and three dimension space of quasars and quasars' candidates have been investigated using variously statistical methods, many of which have already been used in investigations of galaxy clustering. The most popular method is the correlation function (CF). Of considerable practical importance has been the fact that the correlation function have been widely used in study of galaxy and cluster of galaxies distribution. It makes fairly easy to compare the results between quasars and galaxies distribution. Another popular statistic is the nearest neighbor test (NNT). The basic idea of NNT is to compare the mean nearest neighbor distance between objects with that expected for a random distribution sample. The NNt is a useful complement to the CF approach, because the probability of not finding a objects in a given distance region depends on all order of correlation function. Although the NNT do not give the information about the type of clustering, but the test is powerful for detecting the presence of weak cluster.

Beside these, more statistical methods have been employed. For example, the power spectrum (PS) is fourier or spherical harmonic trasform of the correlation function; The binning analysis (BA) is also known as the cluster cell methods; The percolation test,which have beensuggested by Zeldovich and his colleagues, seems to be sensitive to the clustering pattern.

4, CLUSTERING OF QUASARS

All the statistical analyses of quasars clustering are listed in Table 1. It can be seen from Table 1 that the results from different researcher are not completely the same with each other. Osmer (4) and Webster (9) showed no clustering in the quasars distribution from the CTIO sample, while Arp had claimed that there are some quasars groups and other inhomogeneity structure in this sample. For the quasars of two $5^\circ \times 5^\circ$ survey done by Savage and Bolton, Chu and Zhu (10) found weak clustering on about 100 Mpc size in one $5^\circ \times 5^\circ$ sample and no clustering in the other one. But Deng et. al. gave same traces of possible void structure.

The differences among the results is due mainly to the sample, and the statistical methods. Namely, the number of quasars in all survey are not large enough to do statistical analyses with high confidence; The criteria of clustering in different statistical methods may not be equivalent; The last but not least point is selection effects.

In spite of the above-mentioned differences, a conclusion seems to already be acceptable: The clustering of quasars is rather weak,

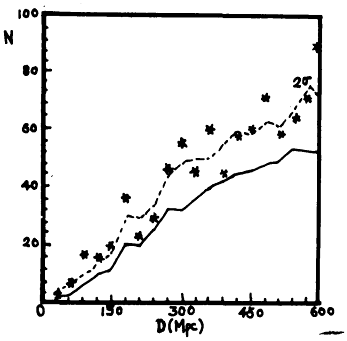

Fig.1 CF for QSOs in (02^h,-50)

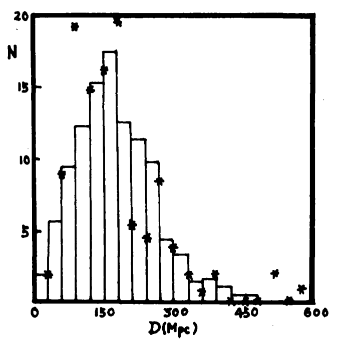

Fig.2, NNT for QSOs in (02^h,-50)

https://doi.org/10.1017/S0074180900159650 Published online by Cambridge University Press

Table 1: Statistical Analysis for Quasars Clustering

Survey Name	Survey Tech.	Area	No. of Quasars	Analysis Author	Statistical Methods*	Results
CTIO 21^h-04^h -40°	Prism+ Grism	15x(5°x5°)	174	Osmer(1981)	NNT CF BA	No
			108	Weberst (1982)	PS	No
SB 02^h--50°	Prism+ UVX	5° x 5°	116	Chu & Zhu (1983)	NNT PS CF	Weak
			43 $1.9<Z<2.5$	Fang, Zhou et. al. (1986)	CF PE	No
SB 22^h--18°	Prism+ UVX	5° x 5°	124	Chu & Zhu (1983)	NNT PS CF	No
			29 $1.9<Z<2.5$	Fang, Zhou et. al. (1986)	CF PE	No
SGP	UVX	11.5deg^2	293	Shanks et. al. (1983)	CF	Yes
01^h12^m -35°	Prism+ UVX	$5^\circ.75$x$5^\circ.3$	325	Savage et. al. (1984)	CF	Weak
			106 $1.6<Z<2.5$	Fang, Zhou et.al. (1986)	CF PE	No
S.A. 94	Prism+ UVX	2^h43^m - 3^h04^m $-2^\circ 18'$ - $2^\circ 51'$	190 350	Barbieri & Chu (1986)	CF	Weak
NGC 450	UVX	25deg^2	140 94 60	Gosset, Surdet & Swings (1985)	MBA NNT CF PS EKS	Yes

* NNT: Nearest Neighbours Test
CF: Correlation Function
BA: Binning Analysis
PS: Power Spectrum
PE: Percolation
EKS: Extended Kolmogorov Smirnov
MBA: MUltiple Binning Analysis

at least, it is weaker than that of galaxies. For instance, Fig. 1 shows the correlation function for the quasars in the field (02^h, -50°) of Savage-Bolton survey. In Fig. 1 the observed data are systematically deviated from the mean expected correlation function for a random distribution. This is an evidence for a clustering in this field. However, the deviation between the observed and the random data is only 2σ level. This means the clustering is rather weak. The nearest neighbor test also shows the same result(Fig.2).

Weak clustering in the quasars distribution is very significant. It implies that at the time Z~2 the visible objects, at least quasars, cluster weakly at the scale of superclusters. Therefore, the superclusters of galaxies should be formed after the formation of quasars, or the formation of galaxies and quasars are due to different mechanism, the former has strong clustering, the later weak. Both above-mentioned pictures are inconsistent with the adiabatic scenarios.

5, REDSHIFT DEPENDENCE OF QUASARS CLUSTERING

It is very interesting to see from Table 1 that all results of samples containing UVX quasars (or quasars' candidates) always show more strong clustering than others. We know that the redshift of quasars found by objective prism and grism are higher than that of quasars found by UVX methods. Therefore the stronger clustering in UVX samples may imply that the strength of quasars clustering depends on the redshift of quasar: more larger redshift more weaker clustering.

For showing the redshift dependence, we should do the clustering analyses for quasars in different redshift ranges. This analysis have be done using Savage-Bolton sample, which consists of two classes of quasars identified by both objective prism technique and the UVB two colour method, the redshifts in this sample spreads on a more broad region than that of other surveies. It is convenient to do the comparision between quasars with larger and smaller redshilft.

The results of the nearest neighbor test are given in Table 2, in which N is the number of quasars, $\langle D_{\langle} \rangle$ denote the sample's

Table 2: Nearest Neighbor Test for Savage-Bolton Sample

redshift	N	quasar data $\langle D_{\langle} \rangle$ Mpc	Monte Carlo data $D^*_{\langle}$ Mpc	$\hat{\sigma}$	δ	1-P(δ)
		(02^h, -50°) field				
Z < 2	62	141.7	159.0	79.6	-1.72	96%
Z > 2	48	201.0	205.9	83.2	-0.40	66%
		(22^h, -18°) field				
Z < 2	57	146.7	165.8	77.9	-1.84	97%
Z > 2	26	207.1	193.0	75.9	> 0	-

https://doi.org/10.1017/S0074180900159650 Published online by Cambridge University Press

mean of nearest neighbor separation, $D^*_<$ and $\hat{\sigma}$ are the mean and standard deviation for Monte Carlo samples respectively. As a measure of statistical significance, we define the following function

$$\delta = N^{1/2} \frac{\langle D_< \rangle - D^*_<}{\hat{\sigma}} \quad (1)$$

The distribution of δ is asymptotically normal with mean 0 and variance being 1. Therefore 1 - P(δ) in Table 2 is the probability of clustering to be found in the sample. The main results from Table 2 are that an apparent clustering at 95% significant level for the quasars of Z < 2 in both fields, and there is no evidence of clustering for Z > 2. These results can be more clearly seen in Figs. 3 and 4, in which the distributions of the nearest neighbor distance for each field are ploted. The observed Z< 2 distribution

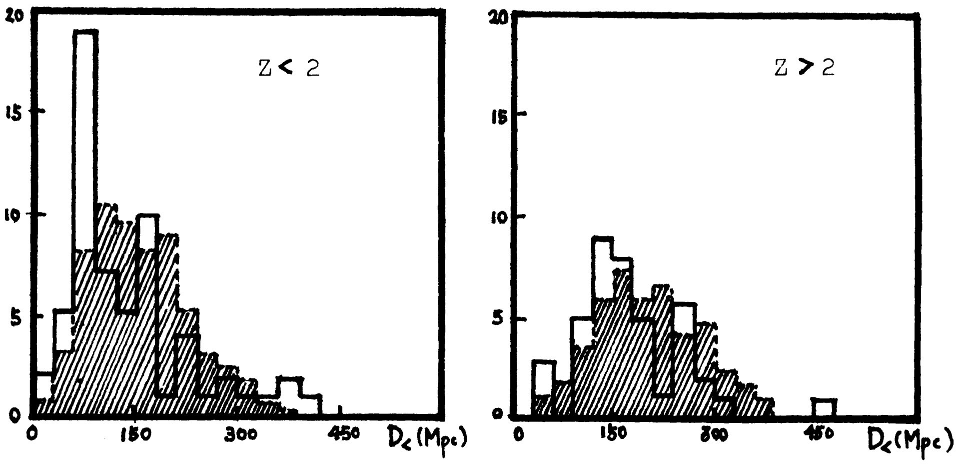

Fig.3 The nearest neighbor distribution for quasars with Z< 2 and Z >2 for (02^h, -50°) of Savage-Bolton sample.

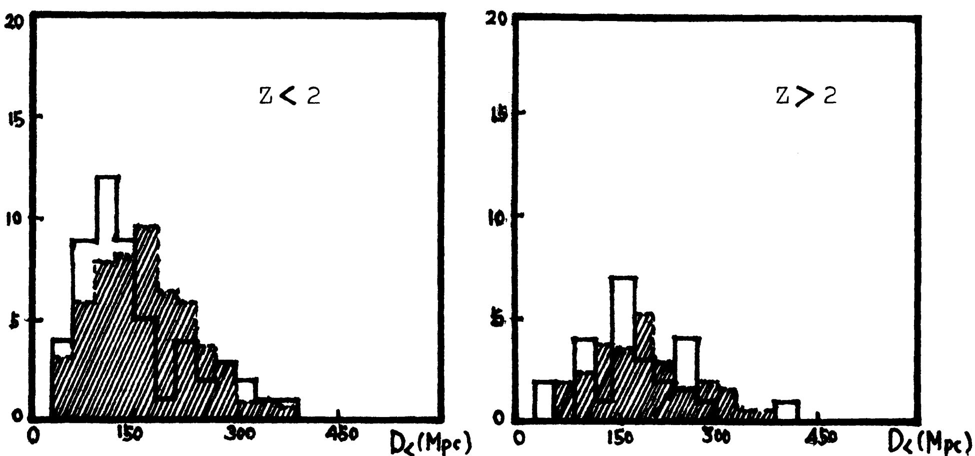

Fig. 4 The nearest neighbor distribution for quasars with Z< 2 and Z >2 for (22^h, -18°) of Savage-Bolton sample.

https://doi.org/10.1017/S0074180900159650 Published online by Cambridge University Press

(solid line) deviate obviously from the Monte Carlo samples (dashed line) on scale of 50 - 100 Mpc. It means that the distribution of $Z < 2$ quasars does have $\sim$100 Mpc clustering. The $Z > 2$ distribution does not show the difference from that of random sample, namely, there is not distinguishable inhomogeneity.

The clustering of quasars in the sample of Shanks et. al. (5) (see Table 1) has been analysed by 2-dimensional correlation function method. They found that the UVX stars are clustered, while 'probable' quasars identified by objective prism show no evidence of clustering. Therefore, the difference of clustering between the UVX stars and the objective prism quasars can also be seen as an evidence for the difference between $Z < 2$ and $Z > 2$ quasars.

Quasars listed in Smith-He catalog (7) are identified by the objective prism method. By using power spectrum analysis, it is found no evidence for clustering of quasars in this catalogy. CTIO sample is also given by the objective prism method. Most of redshifts of quasars in this sample is of $Z > 2$. Osmer already claimed no evidence for large clustering can be found from CTIO sample.(4,9)

In one word, a common result in these studies on large structure of quasars from different samples is that the inhomogeneities on the scales of about 100 Mpc exist in the distributions of $Z < 2$ quasars, but not of $Z > 2$ quasars.

The above mentioned conclusion on the evolution of quasar clustering imply that the dark matter in the universe consists, at least, of two kinds of noninteracting particles, one is dominant component with larger velocitydispersion, such as massive neutrinos; one is non-dominant component which is more weakly interacting, more massive particle with smaller velocity dispersion (11). Since more weakly interacting particles decoupled at higher temperature, its number density should be lower than neutrinos. Thus it is possible that the universe is still dominated by massive neutrinos, while the more massive particles are only to be a non-dominant component. The Jeans mass and length (or the free streaming length) of the more massive particle component are much smaller than that of neutrinos, for instence, the Jeans mass of m=1 kev noninteracting particles can be equal to about 10^{12} $M_\odot$. In such a two component dark matter universe the small scale $\sim 10^{12}$ $M_\odot$ perturbations can avoid to be erased by neutrino free streaming, and it can be kept in the non-dominant component until the recombination. Thus the small scale perturbations can also collapse before the formation of large scale structure. Therefore, the picture for small scale structure formation in two component dark matter model is quite different from the universe containing only one kind dark matter. In the latter the small scale structure formed only after the large scale structure collapses, namely, the clustering process is from larger scales to smaller ones. The first objects to be condensed out would be that of mass about 10^{15} $M_\odot$, then smaller scale objects such as galaxies form due to fragmentation. Thus, all the visible objects with small scales should have redshifts less than 2. However, in the two component dark matter model, it is possible to form small scale structure before the collapse of the large scale structure. After the recombination, small scale clustering can take place in both the non-dominant dark matter and the baryons. These clustering processes are independent of large scale clustering.

According to this clustering picture, there should be two kinds of visible objects with the scale less than superclusters. One is to be formed by fragmentation of the large scale structure; one is not to link strongly with the formation of large scale structure. The distribution of the first kind of objects should have marked large scale inhomogeneity, while the second kind should have not such structure. Thus, this scenario leads to the following conclusion: the small scale objects with $Z > 2$ should distribute more uniformly, especially on the scale of superclusters, than $Z < 2$ objects. This is just the results of statistical analyses to be discussed above.

6, OTHER EVIDENCE FOR THE EVOLUTION OF QUASAR CLUSTERING

Binggeli pointed out (12) that the major axis of a cluster tends to point to the nearest neighbor cluster. If the distance between a cluster and its nearest neighbor is less than 35 Mpc, the angle between the major axis of the cluster and the direction to the nearest neighbor is always smaller than 45°. No such correlation exists if the nearest neighbor distance is large than 35 Mpc. Therefore, the orientation correlation is also an evidence of large scale inhomogeneity in the distribution of clusters.

A similar study for radio double sources has been done recently (13). Using a complete sample of radio double sources by Condon et. al.(14), one found some statistical evidences of the correlation between the orientation of the double sources and the direction to their nearest neighbor radio sources, the correlation scale is also on about tens Mpc, but the correlation is not so obvious as that of clusters of galaxies.(Fig. 5). Not all of radio sources are quasars. Since these sources are selected by the criterion of the separation between two components to be less than 1.5 arcmin, many of them should probably be quasar.

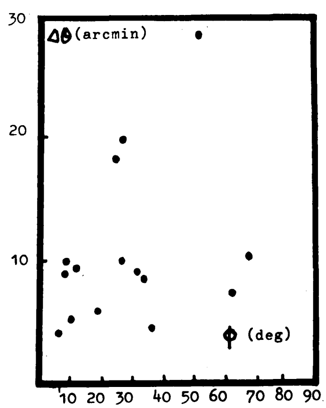

Fig. 5, Orientation distribution for radio double sources

Recently, Zhu (15) did the analyses on the alignment of quasars. She also took 5°x5° survey of Savage-Bolton as sample. The result shows that the phenomenon of alignment in quasars is not obvious, i.e., no identifiable difference from sample given by Monte-Carlo method.

The Ly-α absorption lines in quasar spectrum might indicate the intergalactic clouds. The redshift distribution of Ly-α absorption

https://doi.org/10.1017/S0074180900159650 Published online by Cambridge University Press

lines should then show the clustering of intergalactic clouds. Sargent et. al. studied (16) six quasars with very rich absorption lines, the redshifts of which cover from 1.7 to 3.3. They did not find any evidence of the inhomgeneity in the absorption line distribution. By using different method, which is especially available to analyse the inhomogeneity with scale of a few hundreds Mpc, it is found that (17) the absorption lines distribution inhomogeneous, but the amplitude of the inhomogeneity is very small.

An evidence for the difference between $Z < 2$ and $Z > 2$ quasars has also been found from the research on the evolution of optically selected quasar. Veron (18) have built the luminosity function of quasars at various redshifts, using the number-magnitude relation for quasars and the redshift distributions at various apparent magnitudes. He found that the evolution law for small redshift is quite different from that for large one; the evolution is very fast for $1 < Z < 2.3$, while at some $Z > 2.3$ the evolution has to stop and even to reverse. This shows again that $Z \simeq 2$ is a crucial time of the large scale structure formation.

7, QUASAR CLUSTERING AND DARK MATTER CLUSTERING

The result of quasar clustering can be used in finding information of the clustering of dark matter in the universe. In a dark matter dominant universe, the total density inhomogeneity should be dominated by the density distribution of dark matter. Therefore the information of dark matter distribution is necessary in studying the large scale structure in the universe. Obviously, it is difficult to determine the length scale and the amplitude of the inhomogeneity of dark matter. Nevertheless, some information about such inhomogeneity has been found from the gravitational effects of the dark matter. For instance, the stochastic perturbation in the gravitational field due to the density inhomogeneity will lead to luminosity fluctuations for distant sources. Therefore, the amplitude of the inhomogeneity can be found from the observed differences of luminosities between sources which originally had the same luminosity.

As mentioned in Sects. 4,5, the spatial distributions of quasars from many catalogues and surveys show that the distributions of quasars are quite uniform or of very weak clustering,especially for the quasars with large redshifts. Since the lensing effect is one of the reasons for giving the observed differences or inhomogeneities of luminosities of quasars, hence an upper limit to the amplitude of inhomogeneity of dark matter can be derived, assuming that all the luminosity inhomogeneity of quasars comes from the

... 5 are only on the number density inho-
... as a measure of the lumi-
...ectrum analy-

https://doi.org/10.1017/S0074180900159650 Published online by Cambridge University Press

$$I^{l}_{uv} = \frac{2}{\sum_{j=1}^{n} m_j} \left| \sum_{j=1}^{n} m_j \exp[i(ux_j + vy_j)] \right|^2 \quad (2)$$

where m_j is the magnitude of j-th quasar. In order to increase the power of thetest for luminosity inhomogeneity, we introduce, as in the case of number inhomogeneity, the statistics Q:

$$Q = \Sigma_I / \Sigma_v \quad (3)$$

with

$$\Sigma_I = \sum_{(u,v)} I^{l}_{uv} \quad (4)$$

$$\Sigma_v = \frac{1}{n}\left(\sum_{j=1}^{n} m_j\right) \sum_{(u,v)} 2, \quad (5)$$

where the sums in Σ_I and Σ_v are summed over all the (u,v) with $\lambda_{uv} > \lambda_c$. If any inhomogeneity exists, Q will deviate from unity and Q-1 can be used as a measure of the inhomogeneity amplitude.

Table 3: The M-PSA Statistics Q for quasars of Savage-Bolton Sample.

$(02^h, -50^\circ)$ field			$(22^h, -18^\circ)$ field		
$1/\lambda_c$	all quasars	Z > 2 quasars	$1/\lambda_c$	all quasars	Z > 2 quasars
1	0.50	0.95	1	2.66	1.12
2	1.63	0.85	2	0.73	1.00
3	0.85	0.68	3	1.17	1.05
4	1.39	1.15	4	0.70	1.18
5	1.00	0.96	5	0.80	1.32
6	0.90	1.24	6	0.79	0.80
7	1.00	1.04	7	1.15	1.22
8	1.15	1.30	8	0.82	0.76
9	1.08	0.99	9	0.94	1.07
10	1.04	0.97	10	0.95	1.09

The results of M-PSA for quasars in the Savage-Bolton survey are shown in Table 3. We note immediately from the Table that the mean of (Q-1) is about 0.1. The luminosity distribution of quasars with Z > 2 are more uniform than that of all quasars, especially for longer length scale $1/\lambda$ =1,2,3.

It appears that the mean luminosities of quasars located at different directions are the same to within about 10%. From this we find that the upper limit to the total density inhomogeneity:

$$\delta < \lambda_{10}^{-1/2} h_0^{-1/2} \Omega^{-1} \quad (6)$$

seems to be less than the inhomogeneity amplitudes of galactic distribution by a factor of three to five. This conclusion should be considered as a very rough one. Nevertheless, the link between the distribution of quasars with Z > 2 and the large scale inhomogeneity of dark matter merits our attention.

This upper limit may already be used to test models on the formation of large scale structure in the universe. For instance, the so-called 'biasing' model of the cold matter scenario predicted that light is not an unbiased trace of mass and that there are many (most) clumps of cold matter and baryons which are not shining. This hypothesis implies that galaxies are formed only in high density regions which are the peaks in the primordial fluctuation spectrum. If the peaks lie above some threshold τ (in units of rms density fluctuation), the amplitude of overall density inhomogeneity is lower, relative to galaxies, by a factor of τ roughly. In particular, τ=2.5 is considered by requiring the fit of the 2-point correlation function of galaxies on the scale of $5h_0^{-1}$ Mpc. Therefore, the biasing model agrees, marginally, with the upper limit. However, if the amplitude of luminosity inhomogeneity of $Z > 2$ quasars is confirmed, by further systematic search, to be smaller than 0.1, the biased dark matter scenario would be in difficulty.

References

(1),L.Z. Fang, Y. Chu & X. Zhu, Ap.S.S., 115(1985),99.
(2),C. Barbieri, Y. Chu, et. al., "The Asiago Catalogue of QSQs edition 1985", in press.
(3),A. Savage & J.G. Bolton, MNRAS, 188(1979), 599.
(4),P.S. Osmer, Ap.J., 247(1981), 761.
(5),T. Shanks, R. Fong, et. al., MNRAS, 203(1983), 181.
(6),A. Savage, et. al., MNRAS, ___(1984),
(7),X.T. He, Y. Chu, L.Z. Fang & M.G. Smith, Acta Astronomia Sinica, 27(1986), No.2,144.
(8),C. Barbieri & Y. Chu, preprint.
(9),A. Webster, MNRAS, 199(1982),683.
(10),Y. Chu & X. Zhu, Ap.J., 267(1983),4.
(11),L.Z. Fang, S.X. Li & S.P. Xiang, A.Ap., 140(1985),77.
(12),B. Binggeli, A.Ap., 107(1982), 338.
(13),L.Z. Fang, Y. Chu & X. Zhu, A.Ap., 143(1985), 241.
(14),J.J. Condon, M.A. Condon & C. Hazard, A.J., 87(1982),739.
(15),X. Zhu, Acta Astrophysica Sinica, 6(1986),no.1, 1.
(16),W.L.W. Sargent, P.J. Young, Ap.J.Suppl., 42(1980),41.
(17),Y. Chu, L.Z. Fang & Y.Z. Liu, Astrophys.Lett.,24(1984),95.
(18),P. Veron, in "Quasars and Gravitational Lensens", 24th Liege Astrophysical Colloquium, 1983.

DISCUSSION

BAHCALL: With the large number of QSOs available in your sample, can you separate the redshift subsamples into smaller bins (e.g., z = 0 to 1, 1 to 2, and 2 to 3) to see if the change of the clustering strength is gradual or rather sharp at z ~ 2?

CHU: I agree that this is a good idea, but so far the sample is not big enough for us to divide it into smaller bins.

https://doi.org/10.1017/S0074180900159650 Published online by Cambridge University Press

BOYLE: On the fields in which you find clustering, what fraction of the QSOs have spectroscopic confirmation and/or redshifts?

CHU: One of the fields which we studied is a field worked on by Savage and Bolton (1979). Most of the quasars in this sample have been observed spectroscopically, either using slit spectra or the objective prism method.

CLOWES: We have recently completed an automated search for quasars in SA 94 using the same objective-prism plate as Prof. Barbieri did for his visual search. Prof. Barbieri appears to have missed quasars, particularly in the central regions of the plate. I suspect that any 2-D clustering analysis of his data is likely to be invalid.

CHU: We think it will be interesting to reanalysis the quasar distribution using your catalogue in SA 94. By comparing both results we can get some idea of the selection effects.

ARP: For the clustering of quasars at redshifts less than $z < 2$, what are the redshift ranges involved in the individual clusters?

CHU: At the present stage of our statistical analysis no individual cluster can be identified.

DEKEL: Any clustering evolution should be compared with the growth rate in a theoretical model. Can you comment on the significance of the detected evolution in this respect?

CHU: No, but our motivation for doing the clustering analyses of quasars with redshifts $z > 2$ and $z < 2$ separately was to compare with the model that predicts stronger clustering for objects at $z < 2$ than for objects with $z > 2$.