

Congruences on orthodox semigroups

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The *kernel* of a congruence on a regular semigroup S is the set of congruence classes which contain idempotents of S . The *regular kernel* of a congruence ρ on an orthodox semigroup (a regular semigroup whose set of idempotents forms a subsemigroup) is defined to be the set of maximal regular subsemigroups of the elements of the kernel of ρ . It is proved that a congruence on an orthodox semigroup is uniquely determined by its regular kernel. The regular kernel of a congruence on an orthodox semigroup is characterized as a "regular kernel normal system" (by analogy with Preston's kernel normal systems of inverse semigroups) and an explicit method of constructing the congruence associated with a prescribed regular kernel normal system is provided. These results are simplified in the case of idempotent-separating congruences and inverse semigroup congruences on orthodox semigroups.

Two congruences on an orthodox semigroup S are defined to be *idempotent-equivalent* if they induce the same partition of the set E_S of idempotents of S . Such congruences are examined from the point of view of their regular kernels. Those partitions of E_S which are induced by congruences on S are characterized and the maximal and minimal congruences inducing a given partition of E_S are determined. A construction of the regular kernels of the meet and join of two idempotent-equivalent congruences ρ and σ on S (in terms of the regular kernels of ρ and σ) is obtained.

The final chapter is concerned with one-sided congruences on inverse semigroups. These are examined from the point of view of their kernels and a theory of one-sided kernel normal systems of inverse semigroups and

Received 10 November 1969. Thesis submitted to Monash University, July 1969. Degree approved, September 1969. Supervisor: Professor G.B. Preston.

idempotent-equivalent one-sided congruences on inverse semigroups is developed.