mathematical statement is then identified with its proveability in this informal sense. This account leads to an explanation of the logical operators in terms of the informal notion of 'proof', in contrast to the classical explanation in terms of 'truth'. Not surprisingly, some classically accepted methods of reasoning are no longer acceptable. What must be surprising, at first sight, is the remarkable coherence of the intuitionistic logic that results when the unacceptable methods are dropped.

Brouwer felt it necessary to use the conceptually rather exotic notion of a free choice sequence in developing his intuitionistic analysis. While this notion continues to fascinate logicians and may have interesting sheaf theoretic models Bishop showed quite convincingly that Brouwer's free choice sequences are unnecessary for analysis. The apparent difficulties are overcome by suitable redefinitions of the basic notions. For example, if the standard definitions are used, the proof that every continuous function $f: [0, 1] \to \mathbb{R}$ is uniformly continuous is non-constructive. The tactic is to redefine continuity so that the result does follow constructively. This turns out to be possible without losing the continuity of the standard continuous functions. Of course, indiscriminate use of this sort of tactic of redefinition would make the whole subject worthless. Bishop's achievement was to demonstrate that an intelligent limited use of this tactic suffices to allow the constructivisation of a great deal of classical analysis.

All the mathematical results obtained in the book under review are valid from the classical standpoint. but some classical results do not have constructive proofs, and there is an obvious question to be faced. Why should any mathematician restrict his methods to those allowed in this book? The devoted constructivist would say that constructive methods are correct, non-constructive methods are meaningless and one ought not to use meaningless methods. Bridges' view is that constructive proofs give more information—e.g. a constructive existence proof gives a procedure for constructing the object proved to exist. But the availability of such extra information has not yet had any significant impact on classical mathematics. I would answer the question first by giving the conventional answer that the restriction to constructive methods leads to a discipline having its own intrinsic interest, with a distinctive range of problems. The present book amply demonstrates this. But it fails to give any indication of how the subject may come to interact fruitfully with other disciplines in the future.

To my mind we will not have to wait long before there will be a significant interaction between constructive mathematics and theoretical computer science. Also, the fact that the internal logic of a topos is intuitionistic means that many of the proofs of constructive mathematics can be carried over to the development of mathematics inside many topoi. So topos theory is another area which one can expect to interact significantly with constructive mathematics.

In conclusion, for those mathematicians who wish to gain a working knowledge of constructive analysis following the Bishop school this is a useful book that has no rival (except for Bishop's out of print book). But those who seek logical analysis or philosophical explanation of the ideas of constructive mathematics will have to look elsewhere.

P. ACZEL

Collected Papers of G. H. Hardy, edited by a committee appointed by the London Mathematical Society, Vol. VII (Clarendon Press, Oxford, 1979), 897 pp., £30·00.

It is an honour to be asked to review the seventh and last volume of the Collected Papers of the late Professor G. H. Hardy, ably edited by Professor Rankin and Dr. Busbridge. But it is an impossible task to give a brief appreciation of a volume of 897 pages.

The first half consists of research papers, some written in collaboration with Bochner, Littlewood or Titchmarsh; this part is for the specialist. Any mathematician will find much to enjoy in the rest of the book—elementary notes, addresses, reviews, obituary notices, and problems from the *Educational Times*. Who would not be amused at the complex curve

$$(x+iy)^2 = \lambda(x-iy)$$

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which is a parabola, a rectangular hyperbola and an equiangular spiral? Did undergraduates in 1907 really believe that there are points at infinity which lie on a line and that, if you could get there, you would find $1=0$? His interest in games is shown by notes on batting averages at cricket (Jack Hobbs was one of his idols) and about match and medal play at golf. There is also a hint of another of his interests, a reference to Agatha Christie's thriller "The Murder of Roger Ackroyd".

The research papers are devoted to Fourier Transforms, Integral Transforms and Integral Equations. It was not until 1918 that the words "integrable in the sense of Lebesgue" appear. Before then, Hardy, like Hobson and other English writers, used the word "summable", which had to be discarded because it was needed in the theory of summability of divergent series and integrals. Then follow 80 pages of miscellaneous papers on such varied topics as set theory, differential equations and genetics.

Amongst the addresses and invited lectures are two about Ramanujan and an excellent "Introduction to the Theory of Numbers" from the *Bulletin of the American Mathematical Society*. There is also his famous attack on the Mathematical Tripos, his Presidential Address to the Mathematical Association in 1926. "I do not want to reform the Tripos, but to destroy it." I should like "to give first classes to almost every candidate who applied; to crowd the syllabus with advanced subjects, until it was humanly impossible to show reasonable knowledge of them under the conditions of the examination." Did he really believe this? At any rate, six of the twelve obituary notices are about men who were trained under this system. A student of the history of mathematics in this century would find the obituary notices and book reviews of great interest.

Hardy's development as a mathematician is well illustrated by his contributions to *The Educational Times and Journal of the College of Preceptors*, a monthly journal devoted to educational matters. Each issue contained mathematical problems posed by contributors and solutions by the proposers and others. He set his first problem in 1898 whilst still an undergraduate and continued to contribute until the journal died at the end of the 1914–18 war. This first problem was "Give in a symmetrical form the general equation of a circle through two fixed points". His second asked for the coordinates of the vertex of the parabola inscribed in the triangle of reference, given that the focus is $(\alpha', \beta', \gamma')$ in trilinear coordinates. Most of his problems were concerned with evaluating complicated definite integrals. His first interest in the summability of divergent series appeared in 1899; his problem was to sum the divergent series

$$\sum_0^\infty \frac{(-1)^n(n+1)}{4n+1}.$$

His last contribution in 1917 was geometrical; he found the extraordinary formula

$$ds^2=(dn^2\alpha-dn^2\beta)(d\alpha^2-d\beta^2)$$

for the line element on a sphere of unit radius.

Although he was an analyst, he became Savilian Professor of Geometry at Oxford in 1919. He took his duties seriously and gave very interesting lectures to undergraduates. His views on "What is Geometry?" are clearly expounded in his 1925 Presidential Address to the Mathematical Association. It may suprise people to know that he gave a course of lectures on Relativity, and told us with glee that the Riemann–Christoffel Tensor had appeared as a clue in an American thriller.

There is no more I can say. I greatly enjoyed reading this volume because Hardy gave me my introduction to real mathematics.

E. T. COPSON

McBRIDE, A. C. *Fractional Calculus and Integral Transforms of Generalized Functions* (Research Notes in Mathematics 31, Pitman, 1979), 179 pp., £7·50.

This book is concerned with the study of certain spaces of generalized functions and their application to the theory of integral transforms defined on the positive real axis. Dr. McBride has