

# Time series recursions and stochastic approximation

Victor Solo

This thesis is concerned with the asymptotic theory, both convergence and Central Limit Theorem, for recursive estimators of time invariant parameters in Time Series models. This theory derives from a similar theory for classical Stochastic Approximation schemes so that these too, are given some consideration.

The thesis is divided into two parts. In Part I after an introductory chapter, a general approach is offered for the construction of recursions from a Prediction Error or Gaussian Likelihood viewpoint. This approach also allows other methods (for example, Model Reference) to be seen in context. Next, an heuristic argument is used to intuit the asymptotic properties of a general class of recursions (including the Prediction Error Recursions) and Time Series Stochastic Approximation schemes. The third chapter of Part I contains simulations illustrating some of the above ideas.

In Part II attention is turned to a rigorous analysis of various recursive and Stochastic Approximation schemes. In Chapter 4 a general regression problem is considered and strong convergence proved under various conditions. Chapter 5 is concerned with deriving limit laws for classical Stochastic Approximation in a general setting: in Chapter 6 this is extended to allow dependent noise. Chapter 7 contains a Central Limit Theorem and Invariance Principle for a well known Prediction Error Recursion which is seen to be asymptotically efficient. Finally, in Chapter 8, a proof is given of the convergence, without monitoring, of a

---

Received 16 November 1978. Thesis submitted to the Australian National University, September 1978. Degree approved, March 1979. Supervisors: Professor E.J. Hannan, Dr P.C. Young.

slightly modified form of the  $RML_1$  Recursion.