THE LONG-TERM EVOLUTION PROJECT

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ABSTRACT. The Long-Term Evolution Project (LTEP), realized in collaboration by the IAS-Reparto di Planetologia (Rome, Italy) and the Astronomical Institute of SAV (Bratislava, Czechoslovakia), has been developed with the aim of giving a general insight into the dynamical evolution of short-period comets. The motion of all the known short-period comets has been investigated over a long time span (over 800 years) taking care, as far as possible, to eliminate the sources of possible discrepancies within the computations. An internally consistent data-set and an atlas of orbital evolutions are the first outputs of this project. The main characteristics of the LTEP are discussed, together with some general remarks on its importance for cometary studies, its limitations and the future developments.

1. INTRODUCTION

The orbital evolution of short-period comets is characterized by a wide range of possible regimes of motion that a single comet can pass through, or that can be observed comparing the histories of different objects. Peculiar dynamical events are rather frequent: strong gravitational interactions, mainly with Jupiter, leading sometime to a temporary energetic binding to the planet, and resonant motion about low-order resonances with Jupiter and Saturn. Our observational evidence of these processes is very limited and heterogeneous. The discovery probability of a comet depends to a great extent on its orbital elements (in particular on the perihelion distance and revolution period), and on its variable absolute brightness. The osculating orbits of individual comets are of very different accuracy, mainly depending on the number of apparitions linked up by the computation. Every strong

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perturbation outside this interval tends to degrade appreciably the reliability of the extrapolation beyond it. Moreover, the nongravitational effects, not only differing from one comet to another but also changing with time in each individual case, represent a severe constraint independent of the accuracy of the starting orbit.

These general remarks on the dynamical properties of short-period comets outline the difficulties that arise when approaching the study of the evolution of their orbits; although many authors have dealt with this problem, their results do not provide a complete and homogeneous coverage of the cometary population as a whole.

The wide choice between the special perturbation methods available to investigate cometary motion results in fact in a widespread variety of accuracy and stability characteristics of the integrations carried out by different authors. This, together with the numerical model adopted (e.g. number of perturbing bodies included in the computations, modelling of the nongravitational forces) can play an important role for the critical effect that strong gravitational interactions can have on the reliability of the results (see, e.g., Carusi et al., 1981). Furthermore, the number of objects taken into consideration and the time span covered by the integrations are, again, variable parameters from author to author.

The Long-Term Evolution Project (LTEP) has been carried out in order to provide a more general approach to the study of the dynamical evolution of short-period comets: all the known sample has been integrated for a long time span (over 800 years), taking care to eliminate, as far as possible, the main sources of discrepancies within the computations. The resulting data-base will then allow the development of different kinds of studies: focusing on the evolution of peculiar objects as well as investigating the behaviour of the population of periodic comets as a whole.

2. OUTLINE

The aims of generality of the LTEP point out some of its basic characteristics:

- The comet sample includes all the known short-period comets: their number, updated to 31 December 1983, is 126.

- The source of the starting data is, as far as possible, the same for all the comets: osculating elements are taken from the 4th edition of the "Catalogue of Cometary Orbits" (Marsden, 1982), unless new comets were discovered after its publication or more accurate elements became available; in this case data were extracted from the corresponding IAU or Minor Planet Circulars. The starting elements almost invariably correspond to the last apparition of the comet included in the orbit computation; predicted elements were always discarded.

- In order to satisfy the requirements of computing all the orbital evolutions using the same numerical integration technique, while ensuring a high rate of accuracy to them, many tests and comparisons have been carried out using different integrating methods. These will be described in detail in the next section. The best compromise between accuracy and computational speed has been reached by the single-step RADAU integrator (Everhart; 1974a, 1974b, 1985) solving barycentric equations of motion.
- The perturbations of all the planets on the comet motion have always been included. In order to speed up the computations and to keep, as much as possible, the same gravitational environment during the integration of different objects, only the motion of the comet has been computed; positions and velocities of the Sun and the planets have been read from the JPL DE-102 Long Ephemeris (Standish et al., 1982).
- Each comet has been followed for a time span of 821 years covering the arc from 1585 to 2406 AD, as representative of a reasonable fraction of the active lifetime of a short-period comet. It appeared to be long enough to recognize the basic dynamical behaviours of each object, without rendering completely unrealistic the results for the propagation of the errors due to the numerical approach (round-off errors, truncation errors, numerical instabilities after repeated planetary close encounters).
- Two evolutions for each comet have been computed: one backwards, from the starting date to JD 2300000.5 (1585 February 1.0), the other forwards, from the starting date to JD 2600000.5 (2406 June 17.0), adding up to a total of exactly 300,000 days.
- The forces acting on a comet were assumed to be of purely gravitational origin; nongravitational effects have always been neglected, mainly because the related parameters are not yet known for all the short-period comets and are variable with time. Their inclusion is planned for further developments of the project.
- A quality class has been defined in order to give an estimate of the reliability of every integration. It is determined by the accuracy of the starting elements and by the importance of the nongravitational forces acting on the comet.

The whole LTEP required about 25 CPU hours, using an UNIVAC 1100/82 machine, and a total of about 70 hours of I/O were necessary to handle the input and output data and to generate auxiliary and back up files. The total amount of time covered by the integrations is about 100,000 years; the mean length of the time step was 43.6 days but, since an automatic step-size control was provided, it fell down to a fraction of an hour in case of extremely close approaches to Jupiter, or grew up to one year for comets moving temporarily well outside the planetary region. The longest integration was that of P/Encke (15,128 time steps), the shortest that of P/Wilk (2,074 time steps), with an average of 6,877 time steps per evolution (backwards and forwards).

All the information about every cometary evolution has been retained; the output files, stored on tape, contain the Julian Date and the six heliocentric ecliptic coordinates of the comet at each integration step. These can be considered as the basic data files; from them, auxiliary files are generated for specific purposes.

3. THE NUMERICAL METHOD

The advantages of suitable testing procedures for the selection of a numerical integration technique have been outlined by many authors (e.g. Krogh, 1973; Lapidus and Seinfeld, 1971). In our case the information needed was mainly concerned with computational speed requirements, in order to guarantee the feasibility of the whole project, and with the accuracy that the integrator could ensure over the wide range of orbital characteristics of the short-period comets sample. The procedure described by Everhart (1974a) was chosen because of its good accuracy/speed ratio. The test was the "closure error test", which consists in performing forward-backward integrations parametrized on the numerical approach in question (the core integrator, the force formulation, the accuracy requirements, etc.). For each test-integration, the relative closure error Δr, defined as the normalized difference between the given starting point and the computed one, and the number of function calls N_f, i.e. the number of force evaluations needed to carry out the whole integration, are computed. Characteristic curves can then be isolated on the corresponding (Δr, N_f) plane, which allow both to analyze the behaviour of individual methods and to make comparisons between them.

This procedure was applied to some 100 years test-integrations of cometary orbits; they were chosen among the most representative of the sample, in order to have an estimate of the efficiency of each numerical approach over different possible regimes of motion. Some of the results obtained are shown in fig. 1: the moderately perturbed, but quite dissimilar, orbits of comets P/Encke and P/Halley, and the more chaotic orbit of P/de Vico-Swift have been integrated several times, requesting an increasing accuracy to the two best integrators at our disposal: the multi-step DVDQ, developed by Krogh (1970), and the single-step RADAU, by Everhart (1974a, 1974b, 1985).

Connecting the points in the (Δr,N_f) plane, corresponding to the same orbit and obtained using the same integrator, a family of curves is

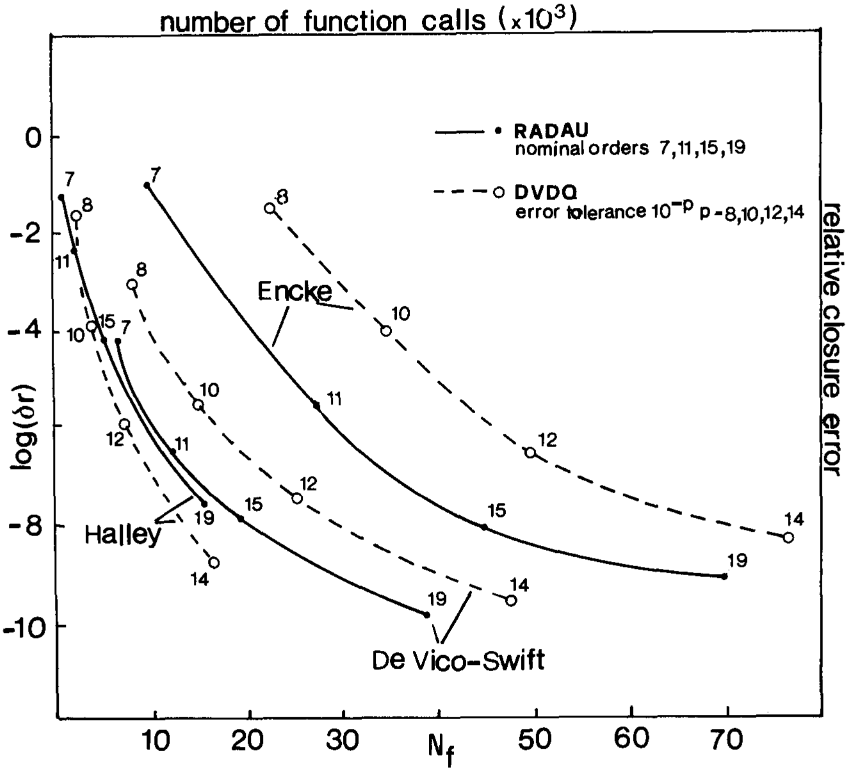

Figure 1. Some of the results obtained applying the closure error test to 100 years integrations of short-period comet orbits. For each of the test objects, curves have been fitted through the points obtained using the same integrator, but requesting an increasing accuracy in the computations. This was done changing the parameter that controls the local error tolerance of DVDQ, and varying the nominal order of the integrations with RADAU.

drawn. They define some basic characteristics of the methods tested through their slope (related to the order of the integrator, i.e. the gain in precision when halving the step-size), and their position within the plane (the more the line is displaced towards the lower-left corner of the plane, the faster and more accurate the integration has been). The distance between the lines corresponding to the same object allows a quantitative estimate of the difference in computational speed and accuracy of the two integrators; note the considerable improvements that can be achieved by a proper choice. The crossing of the lines related to comet P/Halley, reverting the results obtained for the other two objects, shows the different sensitivity of the integrators when treating different types of orbits.

Several test-integrations were performed, focused mainly on the numerical method and on the formulation of the equations of motion used. They led to the final choice of the double precision, 19th order version of RADAU, integrating barycentric equations of motion.

Notwithstanding these efforts in optimizing, as far as possible, the numerical approach to the problem, some general comments on the reliability of our integrations are needed. As to the testing procedure used: the "closure error" is just an estimate of the internal accuracy of an integrator (Everhart, 1974a; Zadunaisky, 1979); round-off and truncation errors will anyway affect the accuracy of the results, i.e. their convergence to an exact solution. Even more dramatic is the loss of significance that can arise from the uncertainty with which the osculating period of a short-period comet is known: 5 to 6 significant digits of the best determined orbits contrast with only 1 or 2 in the case of some of the one-apparition comets.

The propagation of both of these errors is not easily predictable as it depends upon the stability characteristics of the numerical method, and upon the orbit itself. The reliability of the results becomes critical especially when strong gravitational interactions occur (see e.g. Carusi et al., 1985), which is a rather frequent event during the dynamical evolution of short-period comets.

It is necessary to point out that our integrations should be regarded as a representative sample of evolutions of observable short-period comets passing, near their midpoints, close to the osculating orbits of all known objects of this type.

4. THE ATLAS OF COMETARY EVOLUTIONS

The basic output of the LTEP has been collected into a book: "Long-Term Evolution of Short-Period Comets" (Carusi et al., 1984), in which the dynamical history of each comet is presented in a compact form, with the aid of plots and tables. Each comet is described in a General Section, whose format is the same throughout the book (as an example, the one relative to comet P/Oterma is reported in fig. 2). A short text is followed by a table showing the evolution of the orbital elements at equispaced intervals of 50,000 days; the core of the section consists of four plots showing the behaviour of the Tisserand invariant of the comet with respect to Jupiter, of its inclination, perihelion and aphelion distances in time, thus allowing an overall view of the main events characterizing each evolution. In the case of comet P/Oterma, for example, the rather appreciable variation in absolute value of its Tisserand invariant arount 1770 implies the occurrence of a strong perturbation by

P/Oterma

43	1	

The orbital evolution of P/Oterma is quite unique in two respects. At the beginning its perihelion is far outside the orbit of Jupiter, and the Tisserand invariant relative to Jupiter is extraordinarily high. It is only a deep encounter with Saturn in the XVIIIth century which permits the comet to be captured by Jupiter into a temporary satellite orbit, to make three revolutions in an orbit of extraordinarily small aphelion distance and, after another symmetrical close encounter and stay in a satellite orbit, to be ejected into an orbit similar to the previous one. This evolution is assisted by permanently low eccentricity and inclination.

42 revolutions. 3 apparitions. Quality class 5.

Epoch	T	a	e	ω	Ω	i
1958.0606	1958.06105	3.95999	0.14449	354.872	155.109	3.992
1585.0201	1588.05116	8.54140	0.25940	194.385	82.782	4.208
1721.1225	1732.09061	8.02791	0.21650	189.229	80.212	4.163
1858.1117	1861.01223	7.30148	0.21030	235.409	38.038	3.095
1995.1010	2002.12217	7.27569	0.24400	55.745	330.875	1.933
2132.0901	2135.12183	6.98534	0.16419	120.951	282.430	1.652
2269.0725	2278.05037	7.62596	0.20376	180.688	259.562	1.868
2406.0617	2403.07087	7.56274	0.20343	182.673	252.106	1.912

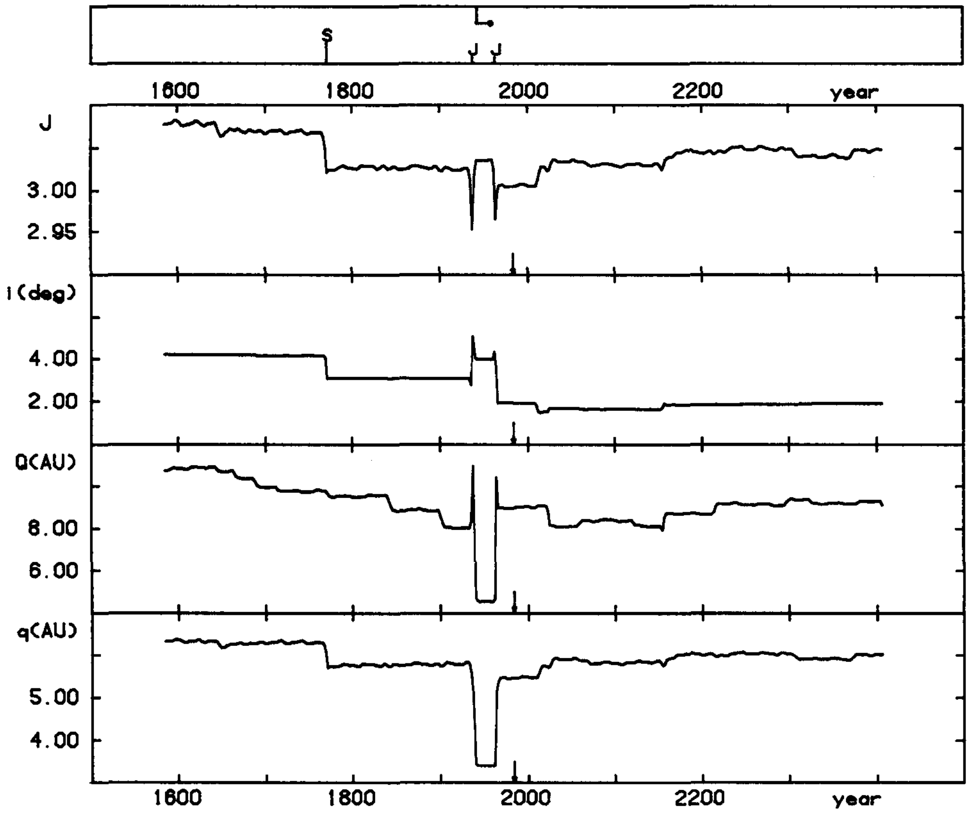

Figure 2. An example of the General Section: the orbital evolution of P/Oterma. For more explanations see text.

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a planet other than Jupiter (in fact it was shown to be Saturn). Besides, the strong reduction of the perihelion distance of the comet, due to a close encounter with Jupiter in 1937, is clearly responsible of its discovery, indicated in the narrow window over the main plots. Here additional general information is given: the time of the first observation is marked by a vertical bar; beginning at the lower end of it a horizontal line covers the time span between the first and last observations. A dot indicates the starting date of the integration, corresponding to the elements listed in the first line of the table. At the bottom of the window, letters J, S, and U indicate the passage of the comet within 0.5 AU from Jupiter, Saturn, and Uranus, respectively. No encounters with Neptune or Pluto have been found, and the ones with the terrestrial planets are not listed.

When peculiar events, such as close encounters or resonant motion, are found, a Special Section is added. It can have two different formats, as shown in figs. 3 and 4 for P/Shajn-Schaldach and P/Kopff.

If a very deep and/or slow complex encounter with a planet takes place, resulting in drastic changes of the orbital elements of the comet, its motion within a sphere of 2 AU radius around the planet is traced (fig. 3). Tables containing both the heliocentric and planetocentric elements are added, together with plots following the variations of the distance from the planet, of the planetocentric binding energy and the trajectory of the comet during the encounter.

The second type of special section is presented when a comet exhibits libration cycles around some resonance with Jupiter (fig. 4): plots showing the jovicentric trajectory are then included, together with a polar diagram where the radius vector is the osculating mean motion of the comet. Further information, such as the number of apparitions or the quality class of the orbit can be found in the text.

The atlas is produced in separate sheets, in order to allow both an internal rearrangement of the ordering of the comets (e.g. by alphabet, period, perihelion distance, number of apparitions), and an easy updating of its content, as the LTEP continues to produce when new periodic comets are discovered or the orbital elements of known comets are improved.

5. CONCLUSIONS

The aims and the general characteristics of the Long-Term Evolution Project - an investigation on the orbital evolution of all the known short-period comets for a long time span - have been reviewed. Although some constraints on the reliability of the integrations are due to the uncertainty with which the osculating elements of the comets are known,

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P/Shajn-Schaldach

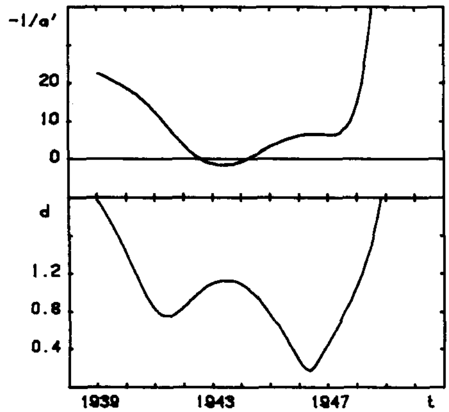
Encounter with Jupiter

1939 - 1948

61	3

Heliocentric elements

t	f	q	Q	a	e	i	ω	Ω
1939.065	292.07	4.2962	5.1840	4.7401	0.0936	10.81	179.83	208.89
1948.839	260.47	2.2346	5.2901	3.7624	0.4061	6.14	215.08	167.47



Jovicentric elements

t	d	a'
1939.065	1.9533	-0.045
1941.434	0.7468	-0.116
1946.335	0.1800	-0.150
1948.839	1.9815	-0.013

t	e'	i'
1939.065	38.52	27.11
1941.434	7.54	103.21
1946.335	2.20	59.20
1948.839	45.77	84.35

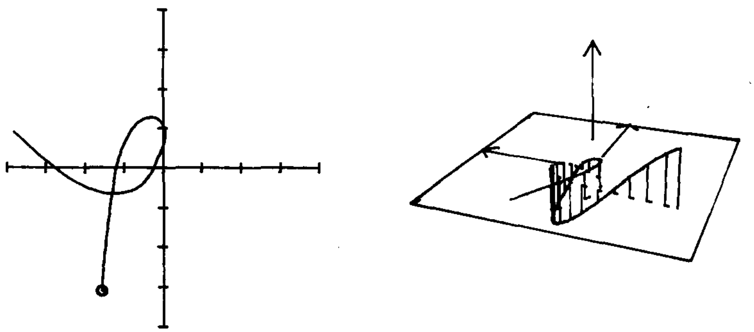


Figure 3. An example of the Special Section - Planetary Encounters: the 1939-1948 close encounter of P/Shajn-Schaldach with Jupiter. The period of a temporary satellite capture is represented by the negative part of the -1/a plot; the distance d and the semimajor axes a, a' are in AU. The two plots at the bottom of the section depict the orbital pattern of the comet: the one on the left shows the x-y projection of the comet trajectory in the frame centred on Jupiter and rotating with its radius vector. This plot extends from -2 to +2 AU in both directions, and a small circles indicates the beginning of the path. A three dimensional perspective view of the same path is presented on the right.

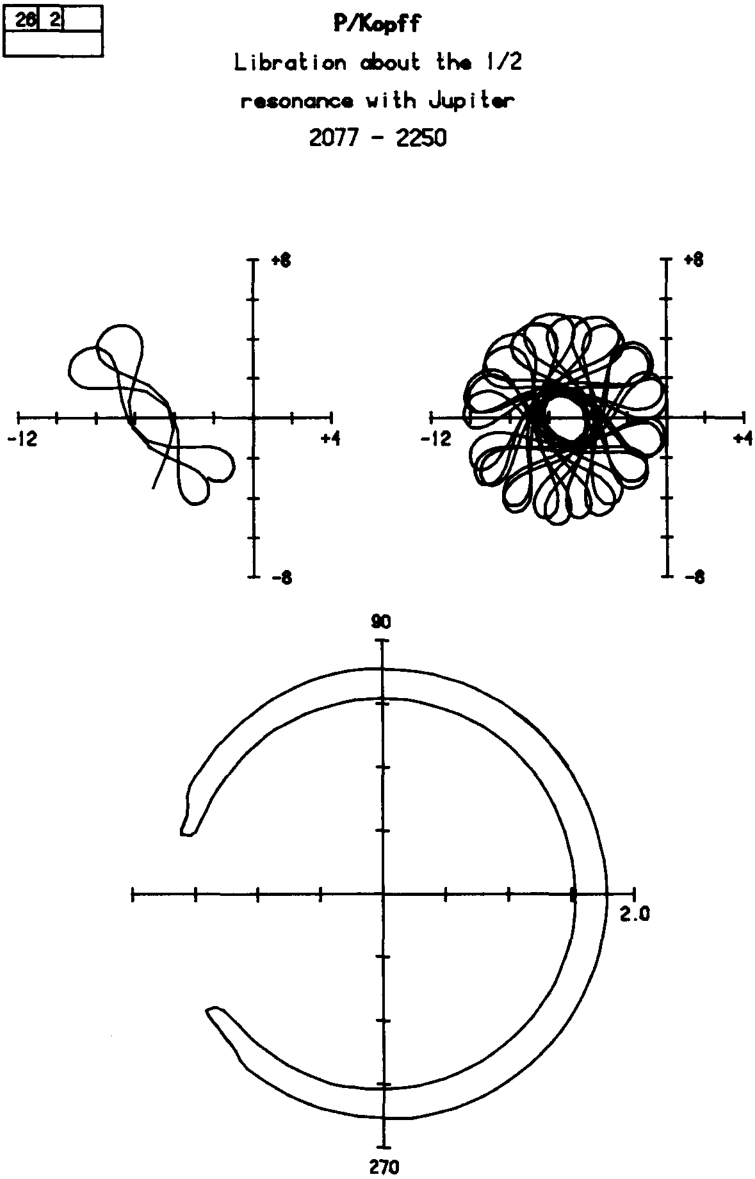


Figure 4. An example of the Special Section - Resonances: the libration of P/Kopff around the 1/2 resonance with Jupiter. Usually only one libration cycle is shown: the upper left plot contains the jovicentric trajectory of the comet for the first revolutions, in order to show its shape, while the right-hand plot follows the orbital path over the whole time span indicated. Below is the horseshoe-shaped libration pattern with respect to Jupiter, during one libration cycle. The position angle is the libration argument, counted counter-clockwise; the radius vector measures the mean motion of the comet, in tenth of a degree per day.

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and to the numerical approach used, the LTEP has created a homogeneous data-base on the dynamics of short-period comets. It allowed the publication of a book in which the history of each member of this population is shortly described, and the presentation of some of the results obtained after a first survey of the output data (Carusi et al., 1985a).

The project is planned to continue in the future to ensure both a regular updating and a refinement of the computations. Possible future developments of the LTEP will include the influence of nongravitational forces on the motion of the comets; taking into account the presence of satellite systems or of planetary oblateness during close encounters; a uniform treatment of asteroidal objects moving in comet-like orbits, and extension of the time span covered by integrations for individual interesting objects.

REFERENCES

Carusi, A., Kresák, L., Valsecchi, G.B.: 1981, Astron. Astrophys., **99**, 262-269.

Carusi, A., Kresák, L., Perozzi, E., Valsecchi, G.B.: 1984, IAS-Internal Report 12.

Carusi, A., Kresák, L., Perozzi, E., Valsecchi, G.B.: 1985a, this volume.

Carusi, A., Perozzi, E., Pittich, E., Valsecchi, G.B.: 1985b, this volume.

Everhart, E.: 1974a, Celest. Mech., **10**, 35-55.

Everhart, E.: 1974b, Denver Res. Inst. Techn. Report, 1 July 1974.

Everhart, E.: 1985, this volume.

Krogh, F.T.: 1970, JPL Technology Utilization Document n. CP-2308.

Krogh, F.T.: 1973, J. Ass. Comp. Mach., **20**, 545-562.

Lapidus, L., Seinfeld, J.H.: 1971, Numerical Solution of Ordinary Differential equations, Academic Press, New York and London.

Marsden, B.G.: 1982, Catalogue of Cometary Orbits, IAU Central Bureau for Astron. Telegrams, Cambridge, Mass.

Standish, E.M., Keesey, M.S.W., Newhall, X X: 1976, JPL Techn. Report n. 32-1603.

Zadunaisky, P.E.: 1979, Celest. Mech., **20**, 209-230.

DISCUSSION

H. Scholl: How did you handle the problem of close encounters between comets and Jupiter? Comment: it does not appear very fair to me to compare DVDQ and RADAU, since DVDQ is not specially designed for solving

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Newton's equations of motion but is designed for integrating ordinary differential equations in general, whereas RADAU is specially made for solving Newton's equations of motion.

E. Perozzi: The numerical procedure used was to integrate always solar system barycentric equations of motion; no change in the force modelling has been performed during the integrations, even when close encounters occurred. This choice came from the following considerations:

- a regularization of the equations of motion is difficult where, as in our case, only the motion of the object is integrated. First of all the change of independent variable, needed in a regularization procedure, is inconsistent with the format of the JPL Ephemeris (which uses Julian Dates) requesting an additional integration in order to go back to the usual time at each integration step. Furthermore, the structure of the integrator used (RADAU 19) implies a substep sampling of the perturbation within every step when the distance of the integrated object from the perturbing body, needed to regularize the problem, is still not computed;
- Encke's formulation of the equations of motion did not show the expected advantages with respect to the barycentric formulation, especially in computational speed. Additional refinements of the integration method are, nevertheless, possible in connection with the numerical instabilities arising when strong gravitational interactions occur.

For what concern the comment on the comparison between DVDQ and RADAU, it must be stressed that, Although RADAU has been developed by E. Everhart, who works in cometary dynamics, it has been presented as a general method for solving ordinary differential equations in an unfortunately still unpublished paper (Everhart, 1973; see also this volume). In that work the author discussed the efficiency of RADAU in a number of test cases, not only concerning celestial mechanics: the appendix listing of the program is the one used in our work. Furthermore, it is worthwhile to remark that, even if strong structural differences do, in fact, exist between the two integrators (DVDQ is a quite sophysticated routine, while RADAU is more straightforward), our testing procedure was specifically directed toward an optimal choice for solving a specific dynamical problem: the integration of short-period comet orbits. RADAU and DVDQ appeared, in the end, the most efficient integrators at our disposal, in the sense that they could really compete in accuracy and computational speed. Our final result, however, has no aim of generality, as the efficiency of the single integrator will depend on the problem to be solved.

P.E. Zadunaisky: The closure test may be useless if symmetries are involved in the numerical integrator.

E. Perozzi: The closure test was useful in comparing some different methods. Everhart's method does not involve symmetries.