

THEORY OF PHOTOSPHERIC MAGNETIC FIELDS

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In this review I discuss from a theoretical point of view the magnetic fields seen at the solar (stellar) surface. Since magnetic field lines have no ends, and the photospheric fields are mostly vertical, the discussion necessarily includes some of the properties of the fields above and below the photosphere. A more general discussion of the theory of solar magnetic fields can be found in Priest (1982).

1. GENERAL STRUCTURE

An important observed property is that at each length scale L the typical time scale τ on which the field changes is long compared with the Alfvén travel time L/v_A . This means that the field is close to magnetostatic equilibrium:

$$-\nabla p + \frac{1}{4\pi} (\nabla \wedge B) \wedge B + \rho g \approx 0 \quad (1)$$

where p, B, ρ, g are the gas pressure, field strength, density and gravity. Except for occasional more dramatic events the field evolves slowly in time. Superimposed on it there may be wave-like disturbances. The solution of (1) for a field in a stellar atmosphere, in which the density decreases rapidly with height, is a nonlinear problem for which no analytic methods seem to be available. Numerical solutions were obtained by Gabriel (1977), and Schmidt and Wegmann (1982). Nevertheless, a qualitative picture of the field configuration above the photosphere can be drawn easily (Figure 1, Spruit 1981a). The photospheric field, divided into isolated tubes, fans out with height. The fields merge at a height h_m , given approximately by

$$h_m = 2H \ln B_p / \bar{B}, \quad (2)$$

where H is the scale height of the atmosphere, B_p the field strength in the tube at the photospheric level, and \bar{B} the horizontally averaged field strength. Above the merging level, the plasma is everywhere magnetized, this is the "coronal" regime. In general, the magnetic field dominates over the gas pressure in this regime ($P \ll B^2/8\pi$). Below it is the "flux

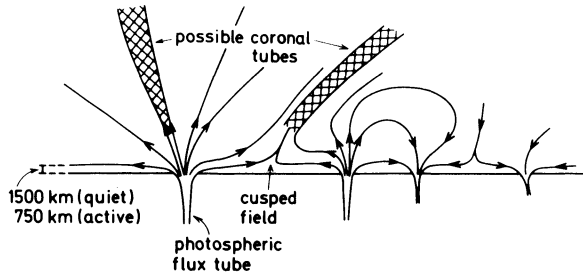


Fig. 1 Structure of the magnetic field near the photosphere (schematic)

tube" regime where the field is confined to flux tubes, separated by field free fluid. Here the gas pressure is of the same order as the magnetic energy density or larger. For the sun the merging height is expected to range from 700 to 1500 km. Observations (Giovanelli, 1980, Jones and Giovanelli, 1982, and this volume) indicate somewhat lower values. It is not quite clear what the cause of this discrepancy is (see however Anzer and Galloway, 1982).

2. FORMATION OF FLUX TUBES

The question naturally arises why there exists such a clear distinction, at least at the solar surface, between field free regions and concentrated patches of field. Secondly, why does the field in the flux tubes have the rather high value $B \approx 1500$ G (at $z = 0$)? To the first of these questions we can answer, on the most heuristic level, that this is the way in which the field tries to minimize its interaction with the turbulent flow in the convection zone. In the absence of any backreaction of the field on the turbulent flow, and in the absence of ohmic diffusion, the field would very rapidly become tangled-up. Both the magnetic energy density $B^2/8\pi$ and the intermittency ("patchiness") would then increase exponentially in time with a time scale on the order of a few turnover times. This was shown, for example, by Kraichnan's (1976) numerical studies. Still assuming that the effects of magnetic diffusion are small (more about this below), it is clear that the Lorentz forces become important and start interfering with the tangling-up process within a short period of time. Thus they have to be included in any realistic treatment of MHD turbulence. Numerical simulations of MHD turbulence were done by Pouquet and Patterson (1978), Meneguzzi, Frisch and Pouquet (1981) and, in the context of a realistic model for granulation, by Nordlund (this volume). Such simulations confirm the tendency of the magnetic field to form intermittent structures ("spongy" distribution of field lines between turbulent elements). The most extensive numerical calculations on the formation of flux tubes are those by Weiss and coworkers (Weiss, 1981a,b, Peckover and Weiss, 1978, Galloway and Moore, 1979, and references therein, see

also Proctor, this volume). In these studies, magnetoconvection in a Boussinesq fluid is calculated in a 2-dimensional (plane or cylindrical) geometry. The calculations follow the development of a single convective cell. For a given strength of the convective forces (measured by the Rayleigh number) the results depend mostly on the average field strength across the cell (measured by the Chandrasekhar number) and the magnetic Prandtl number $\sigma = \nu/\eta$ where ν and η are the viscosity and the magnetic diffusivity. Let us define the equipartition field strength B_e as the value at which the energy density in the field is the same as the energy density stored in the unstable thermal stratification. Equivalently, it is the value for which the Alfvén speed is equal to the typical convective flow speed in the absence of a field. If the initial field is stronger than about B_e , the convective forces are not strong enough to form an overturning cell (Cowling, 1976); instead, convection with a reduced efficiency occurs in the form of an oscillating flow, provided that the thermal diffusivity κ is larger than η . This mode of energy transport may be important in the strong field of a sunspot umbra (Meyer et al 1974). The average field strength in an active region is well below the critical value B_e (which is about 600G for the top of the convection zone). Overturning convection occurs easily. The numerical experiments then show the formation, in about one turnover time, of a flux rope in which the fluid is stagnant, and a convective cell containing a tangled field which vanishes on a diffusive time scale $\tau_d = d^2/\eta$, where d is the size of the cell. The field strength B_m in the tube is approximately (Galloway et. al 1978)

$$B_m \sim B_e (\nu/\eta)^{1/2} (\ln R_m)^{-1/2} \quad (3)$$

where $R_m = Ud/\eta$ is the magnetic Reynolds number, and U the typical convective flow speed. Clearly, the field strength obtained depends rather critically on the values of ν and η used in the calculations. These are usually interpreted as turbulent diffusivities that mimic the effects of motion on small scales that are not resolved numerically. In order to get a field strength of the order 1500G, (3) shows that one needs $\nu/\eta \sim 25$, for $\ln R_m = 2$, and more for higher R_m . Estimates of ν and η are obtained using some statistical theory of turbulence. The simplest estimates (short-correlation time or first order smoothing approximations) yield $\nu = \eta$ (see e.g. Moffat, 1978). More elaborate theories yield widely diverging values and no convincing estimates seems to be available (see also section 7). Taking the simplistic value $\nu = \eta$ would $B_m \sim B_e$, too low to explain the observed fields. Thus, though the calculations demonstrate the process of flux tube formation and show that field strengths different from B_e can be obtained, they do not yield a value that can be compared directly with observations.

3. FLUX TUBES IN A STRATIFIED ATMOSPHERE

Accepting that flux tubes exist (from the above, or from observations) we can go a step further and study their properties under astrophysically more realistic conditions. This is possible in the so-called thin-tube-

approximation (Defouw, 1976, Roberts and Webb, 1978, Spruit, 1981b). In this approximation one assumes that the component of the field perpendicular to the tube axis is small compared with the parallel component. The tube is thus approximated by a strand of field lines whose thickness varies slowly along the length of the strand. In an exponential atmosphere with scale height H a vertical flux tube expands with height h approximately as $\exp(h/4H)$. For the thin tube approximation to be valid, it is therefore necessary that at all heights considered the radius of the tube is less than $4H$. For the small scale field of the sun this is satisfied roughly up to the temperature minimum or lower chromosphere. Details of the internal structure of the tube are ignored by characterizing the field strength by a single function of time t and length l along the tube:

$$B = B(l, t) \hat{l}, \quad (4)$$

where \hat{l} is the unit vector along the tube. Since the tube is thin we may assume that during the subsequent evolution of the tube lateral pressure balance is maintained at all times:

$$P_i(l, t) + B^2(l, t)/8\pi = p_e(z). \quad (5)$$

In this approximation the MHD equations become a set of equations in one spatial dimension (Spruit 1981b,c). Due to this simplification, compressibility and stratification can be taken into account realistically without invoking turbulent diffusivities.

3.1 Convective collapse

As an example, we first consider the stability of a tube, embedded vertically in a convective envelope. Suppose that in its equilibrium state the tube is vertical, and in temperature equilibrium with its surroundings:

$$T_{i0}(z) = T_e(z), \quad (6)$$

then the pressure scale height in the tube is at each level equal to that of the surroundings (except for a small difference due to the effect of pressure on the molecular weight μ). In the equilibrium state the ratio of gas to magnetic pressure, $\beta_0 = 8\pi P_{i0}/B_0^2$ is therefore a constant independent of z . If the superadiabaticity of the stratification $T_e(z)$ is sufficiently strong, the tube is unstable to flows along the tube. During this flow the tube expands and contracts so that its field strength changes, in such a way that (5) remains satisfied. This instability was proposed by Nordlund (1976) and Parker (1978) as a mechanism to increase the field strength of the tube to the observed values. It has been studied in detail by Roberts and Webb (1978), Spruit and Zweibel (1979), Spruit (1979) and Unno and Ando (1979). For a stratification corresponding to the solar convection zone, the tube is unstable to adiabatic perturbations if $\beta_0 > 1.8$ (Spruit and Zweibel 1979). This means that in a tube with a field strength less than 1200 G (at $\tau = 1$ inside the tube) a downflow will

occur that empties the tube and increases its field strength. Since at large field strengths tubes are stable, the process is self-limiting, and transforms the tube into a new equilibrium at a higher field strength (convective collapse). Starting with an initial β_0 of 6.7 (corresponding to a field strength equal to the equipartition value) the final field strength (again at $\tau = 1$ inside the tube) is 1800G (Spruit 1979). These calculations show that the convective concentration of an initially weak field does not stop at the equipartition field strength. Though at this value the tube is strong enough to resist the external flow, and thus is expelled from the convective cell by the mechanism described in the previous section, it is still unstable to a convective downflow inside the tube. This effect is due to compressibility and the strong stratification of the atmosphere; it would not occur in a Boussinesq fluid.

The evolution of a magnetic field in the solar granulation was demonstrated in a spectacular numerical simulation by Nordlund (report in this volume). His results show the formation of flux tube - like structures between granules with field strengths close to the observed values.

Calculations of the collapse process for convection zones other than the sun's are not available yet. We may speculate however that the ratio of the final field strength B_f in the tube to the equipartition value B_e is a constant, since both values are determined by the degree of super-adiabaticity in the convection zone. B_e , as estimated from mixing length models, is fairly constant along the main sequence. The field strength in the small scale fields at the surface of these stars is thus expected to be fairly constant as well, at about 1800G (at $\tau = 1$, corresponding to about 1200 - 1500G for observations in spectral lines). For stars with a different gravity, the field strength probably scales with the square root of the surface pressure.

Unfortunately, the above results cannot be extrapolated to large flux tubes like sunspots. The structure of spots may well be different from that of small tubes. In particular the reason why the field strength of spots has a value of about 3000G is essentially unknown. For details I refer to the proceedings of the recent sunspot workshop (Gram and Thomas, 1981).

3.2 Buoyancy

In a tube in temperature equilibrium with its surroundings the density is

$$\rho_i = \rho_e \beta / (1 + \beta) \quad (7)$$

For the strong photospheric fields ($\beta \approx 1$) the density is thus much less than the surrounding density ρ_e , and the tubes are strongly buoyant. This implies that as long as they are rooted somewhere deeper in the convection zone, they have a strong tendency to remain vertical. The turbulent convective flow is strong enough to shuffle them around between granules or supergranules and to bend them slightly, but not strong enough to pull them back below the surface (see also section 8).

3.3 Energy balance of tubes

The strong field of the tube allows only the less efficient oscillatory type of convection. At the photosphere, the tube therefore cools until its radiation losses are balanced by the reduced convective flux, augmented by the radiation that leaks in from the sides. For larger tubes the lateral influx of radiation is small so that they will be dark, like sunspots. In small tubes however the lateral influx is strong, at least near the photosphere, and the internal temperature is only slightly less than the surrounding temperature. Since the internal opacity is less (due to the reduced pressure) one sees a higher than photospheric temperature when looking down on such a tube. Small tubes are therefore brighter than their surroundings even though at each geometrical level the internal temperature is lower than the surroundings. Calculations of this effect were made by Spruit (1976, 1977). They predict that the tubes are bright structures when their radius is less than about 2 pressure scale heights (300 km). This agrees well with observations (Spruit and Zwaan, 1981). Small tubes thus act as net emitters of continuum radiation. One can say that due to their lower opacity they form leaks through which photons from the surrounding convection zone escape. The extra emission can be as high as 100% of the photospheric flux, averaged over the surface area occupied by the tube. The chromospheric emission in flux tubes has a different origin, probably the dissipation of mechanical energy from the convection zone carried along the tube in the form of MHD waves.

4. WAVES IN FLUX TUBES

The propagation of waves in an inhomogeneous field in the presence of gravity is in general an intractable problem. The wave modes of a cylindrical flux tube of finite diameter, in the absence of gravity, have been studied in detail (Wilson 1978, 1979, Wentzel, 1979b, Edwin and Roberts, 1981). The full spectrum of such a tube is rather complex consisting of surface type modes (concentrated at the tube boundary) and body waves, either of the slow or fast magnetoacoustic type. In addition there are simple Alfvén waves, consisting of torsional oscillations of the tube. If the boundary of the tube is not sharp, as assumed in these studies, but has a smoothly varying field strength, the spectrum is far more complicated. This is because the linearized MHD equations in general are singular, so that they have solutions which cannot be described in terms of eigenmodes of the system. For such a solution there are points in the plasma at which the local Alfvén speed matches the phase speed of the wave (the resonant points). Mathematically, the situation is similar to that of hydrodynamic waves in a shear flow (see, e.g. the book by Drazin and Reid, 1981). These singular wave problems have been studied in some detail in the tokamak context (see e.g. Sedláček, 1971, Tataronis and Grossmann, 1976, Goedbloed, 1975, Hasegawa and Chen, 1976, Hameiri, 1981). Applications to astrophysics were made by Ionson (1978), Wentzel (1979a), Rae and Roberts (1981) and Rae (1982).

Most of these complications can be avoided elegantly by considering again the thin tube limit. In doing so we concentrate on the behaviour of

the tube as a whole, skipping over the details of its internal dynamics. We now explore the consequences of this approach, bearing in mind that strictly speaking its validity should be checked in each particular application.

A thin tube has three wave modes, which are analogous to the waves in an elastic wire under tension (Spruit, 1982a). There is a torsional wave, a longitudinal tube wave and a transversal tube wave. The torsional wave is an Alfvén wave, satisfying the equation

$$\partial_{tt}^2 \phi = v_A^2 (1) \partial_{ll} \phi, \quad (8)$$

where ϕ is the torsion angle and l the arclength along the tube. We have assumed that the tube is untwisted in its equilibrium state; this assumption is implicit also in the following discussion. In the absence of gravity the tube is straight and of constant cross section. The longitudinal wave consists of fluid motions along the tube, accompanied expansions and contractions ("sausage mode"). Its propagation speed is v_1 :

$$v_1^2 = c^2 v_A^2 / (c^2 + v_A^2). \quad (9)$$

where c and v_A are the sound and Alfvén speeds inside the tube. The transversal wave consists of bends in the tube, without flow along the tube, resembling an Alfvén wave. Its propagation speed is, however, less than v_A :

$$v_t^2 = \rho_i v_A^2 / (\rho_i + \rho_e). \quad (10)$$

When gravity is present the fluid is stratified and the cross-section of the tube varies. The longitudinal and transversal motions of the tube are in general coupled. We consider the case of a vertical flux tube. In this special case the longitudinal and transversal motions are uncoupled (except when nonlinear effects are considered). The amplitude of the horizontal displacement ξ is governed by the equation (Spruit 1981b)

$$\partial_{tt}^2 \xi = -g \frac{\rho_i - \rho_e}{\rho_i + \rho_e} \partial_z \xi + \frac{\rho_i}{\rho_i + \rho_e} v_A^2 \partial_{zz} \xi, \quad (11)$$

where g is the acceleration of gravity and z the depth (positive into the sun). The second term on the RHS is the restoring force due to magnetic tension. The first term is the restoring force due to the buoyancy of the tube. It changes the character of the waves significantly: in contrast with Alfvén waves, transversal tube waves are dispersive. The transversal wave is incompressible, so that it is not affected by radiative damping.

The longitudinal tube wave is compressive, so that its propagation depends on the exchange of heat with the surroundings. If the motion is adiabatic, the wave equation for the vertical displacement ζ in a tube which is initially in temperature equilibrium with its surroundings, is (Defouw, 1976, Roberts and Webb, 1978, Spruit and Zweibel, 1979):

$$(2/\gamma + \beta) \partial_{tt}^2 \zeta + 2gH \partial_{zz} \zeta + g \partial_z \zeta + \delta(1 + \beta)g/H \zeta = 0. \quad (12)$$

where $H(z)$ is again the pressure scale height and $\delta = d \ln T / d \ln P - (1 - 1/\gamma)$ is the superadiabaticity of the stratification. If δ is sufficiently positive, (12) has unstable solutions. These correspond to the convective collapse instability described above. The effects of heat exchange with the surroundings on the longitudinal tube wave are discussed in Roberts and Webb (1980a,b).

In the simple case of an isothermal atmosphere we have

$$\frac{v_A^2}{c^2} = \frac{\gamma}{2} \beta \quad (13)$$

The transversal and the (adiabatic) longitudinal wave are then of the form (Spruit 1981b)

$$\exp(i\omega t + ikz - z/4H) . \quad (14)$$

The dispersion relation is of the form

$$\omega^2 = v^2 k^2 + \omega_c^2 , \quad (15)$$

where, for the transversal wave $v^2 = v_t^2$ (eq.10), and the cutoff frequency is given by

$$\omega_{ct}^2 = \frac{g}{8H} \frac{1}{2\beta+1} \quad (16)$$

The speed of the longitudinal wave is $v=v_l$ (eq.9), and ω_{cl} is given by

$$\omega_{cl}^2 = \frac{g}{8H} [1 + 8(1-1/\gamma)(1+\beta)] / (2/\gamma+\beta) \quad (17)$$

The torsional tube wave, described by equation (8) has solutions of the form

$$\phi \sim \exp(i\omega t + ikz) , \quad (18)$$

with

$$\omega/k = v_A$$

Since the radius r of the tube varies as

$$r \sim \exp(-z/4H) , \quad (19)$$

the torsional velocity amplitude $v_\phi = r \partial_t \phi$ varies as

$$v_\phi \sim \exp(i\omega t + ikz - z/4H) \quad (20)$$

The dispersion relations are shown in Figure 2 for solar conditions. For comparison, the dispersion relation for a vertically propagating adiabatic sound wave is shown. Its solution is of the form $\exp(i\omega t + ikz - z/2H)$, with $v=c$, $\omega_c^2 = \gamma g/4H$.

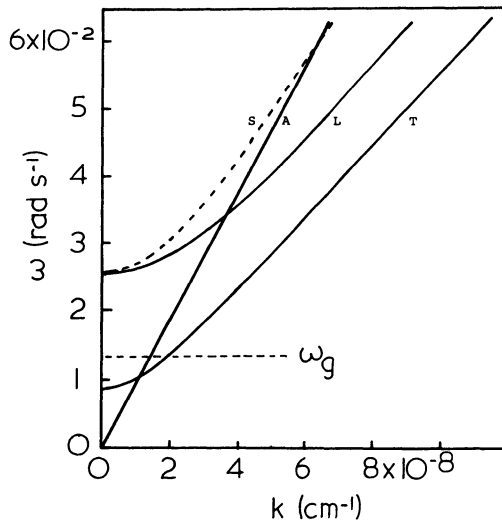


Fig.2. Diagnostic diagram for flux tubes in an exponential atmosphere with scale height 160 km, sound speed 8.3 km s^{-1} and Alfvén speed 9.4 km s^{-1} . A: torsional Alfvén wave, L: longitudinal tube wave, T: transversal tube wave. A vertically propagating sound wave (s) is shown for comparison. The characteristic frequency ω_g corresponding to granulation is shown.

4.1 Nonlinear tube waves

The amplitude of the motions observed in the chromosphere are of the same order as the sound speed, or larger. The waves are therefore nonlinear in these layers. In a nonlinear wave, transversal or torsional motions are always accompanied by longitudinal motions. This is because they produce "centrifugal" accelerations along the tube. Longitudinal motions can exist on their own at finite amplitude (though, at large amplitude such flows can be unstable to transversal motions, due to the "fire hose" instability). We consider finite amplitude longitudinal waves first. Roberts and Mangeney (1982) studied waves at small but finite amplitude in a flux sheet (rather than a tube) of small but finite thickness, in the absence of gravity and stratification. They show that such a sheet supports solitons. In this soliton the nonlinear effects, which tend to steepen the wave, just balance the dispersion of the wave introduced by the finite thickness of the sheet. As a result a soliton propagates, in the form of a single pulse of permanent shape. The importance of this result is that it shows that a nonlinear longitudinal wave need not form shocks. This contrasts with the case of a sound wave in one dimension, which forms shocks at arbitrarily low amplitude.

In a stratified atmosphere the nonlinear propagation of tube waves can only be studied numerically. Hollweg (1982) studied the longitudinal wave in a rigid tube of varying cross section, for solar conditions. In such a tube, the wave is equivalent to a sound wave. When driving occurs

in the form of a sufficiently strong (1.4 km s^{-1}) and short pulse (90sec duration), a shock easily forms, followed at some distance by a "wake" which lifts the chromosphere upwards. This wake has many of the properties shown by solar spicules. In a related calculation, Hollweg et.al. (1981) followed the nonlinear development of a torsional wave, again in a tube with a given cross section (but varying with height). The results are a bit more complex in this case, but also show a strong upward flow resembling a spicule. The flow is initiated by the longitudinal acceleration in the strongly nonlinear torsional pulse.

In reality a flux tube can expand and contract in radius. In the usual thin tube approximation this is taken into account using Eq.(5). Solutions of the full set of thin tube equations for longitudinal flow, taking these changes into account, are given by Ulmschneider et.al. (1983). These solutions also show a tendency to form shocks. The concept of a shock in an expandable thin tube leads to certain philosophical problems. It seems likely however that some form of shock wave, and the associated heating, can indeed occur in such tubes (Ulmschneider et.al. 1983).

No calculations of nonlinear transversal tube waves are available at present.

5. ENERGY TRANSPORT FROM THE PHOTOSPHERE TO HIGHER LAYERS

It is generally realized that most of the mechanical energy that heats the chromosphere and corona is apparently transmitted from the convection zone to the atmosphere via the magnetic field. There are two conceptually different mechanisms for this transmission. The first is steady reconnection releasing in a quasicontinuous fashion the energy fed into the atmospheric magnetic field by motions of its photospheric footpoints. I refer to Chiuderi (this volume) Parker (1979,1981a,b) and Rosner et. al. (1978) for discussions of this mechanism. The detailed structure of the photospheric field is not of great importance in this mechanism, all that is needed is some knowledge about the motions of the foot points of the coronal field. Another mechanism is transport by MHD waves. Here knowledge of the field structure and the atmospheric stratification near the photospheric level is crucial since they determine the propagation characteristics in an essential way. Typically the photosphere is the level which is hardest to pass by a wave; an example is a pure acoustic wave which becomes evanescent at the photosphere for wave periods comparable with the time scale of convection. We now discuss the various kinds of waves from this perspective. For nonmagnetic atmospheric waves I refer to Stein and Leibacher(1981).

The efficiency with which magnetic waves are generated by turbulence in a homogeneous unstratified fluid was estimated by Kato (1968) and Stein (1981). The energy flux F according to these estimates is of the form $F = \rho u^3 f$; where u is the typical convective flow speed, and $f=(u/c)^5$ for acoustic waves, $f=(u/v_A)^5$ for fast mode waves, $f=u/c$ for slow modes and $f=u/v_A$ for Alfvén waves. Slow modes and Alfvén waves are therefore produced most effectively. Inserting typical numbers however, one finds that F is very large, of the order of a few percent of the total solar energy flux. This is evidently unrealistic, it is due to the neglect of

stratification on the waves. Though it can be argued that the estimates still may give an impression of the variation of the heating rate with spectral type (Ulmschneider and Stein, 1982), this may equally well be a rather optimistic point of view.

Waves in a stratified atmosphere permeated by a magnetic field are called magneto-atmospheric waves. A case which has been studied in some detail (Thomas, 1982a,b, Schwartz and Leroy, 1982, Leroy and Schwartz, 1982) is a horizontally homogeneous atmosphere with either a completely uniform field or a horizontal field which varies with height in magnitude and/or direction. Thomas (1982) showed that in the case of a uniform horizontal field the magnetoatmospheric waves have a cutoff frequency $\omega_c = c/2H$, equal to the acoustic cutoff of the corresponding nonmagnetic atmosphere.

Homogeneous vertical fields have been considered by Leroy and Schwartz (1982) and Schwartz and Leroy (1982). They calculated the propagation of Alfvén waves and compressive waves in a stratification simulating solar conditions. Alfvén waves are strongly reflected by the strong density drop between the photosphere and the transition zone. Nevertheless, an energy flux of a magnitude which is potentially interesting for coronal heating can be obtained. The behavior of the compressive magnetoatmospheric waves depends strongly on their frequency. Below the acoustic cutoff frequency the waves tend to be evanescent so that little energy penetrates into the corona. If driving occurs above the cutoff large energy fluxes can be produced. Like in the case of heating by acoustic waves the main question is therefore which part of the spectrum of driving motions in the convection zone lies above the acoustic cutoff. The observational upper limits (Athay and White, 1978) on the level of acoustic waves in the atmosphere suggest that this fraction is small. This is consistent with the fact that the typical time scale of granulation is three times as long as that corresponding to the cutoff. Schwartz and Leroy (1982) conclude that the Alfvén mode, rather than one of the compressive waves is the most promising wave for heating the corona. This does not rule out the possibility that compressive waves, including sound waves, could be important for heating the lower atmosphere (Ulmschneider and Stein, 1981).

Because of the concentrated nature of the photospheric field, wave propagation calculated in a homogeneous field gives a misleading picture. In a photospheric flux tube for example the Alfvén speed is around 8 km s^{-1} and varies slowly with height. In a homogeneous field of 20 G on the other hand the Alfvén speed increases rapidly with height and is only 9 m s^{-1} at the photosphere. Let us consider the flux tube waves described above as a means to transport energy to the corona. Of course the flux tube picture breaks down at the merging level h_m ($\sim 1200 \text{ km}$). Above this level it is more appropriate to think in terms of a more or less uniform field.

The torsional tube wave propagates at all frequencies (Figure 2). In an isothermal atmosphere the Alfvén speed in the tube is constant so that no reflection occurs until the wave reaches the merging level. Once generated, the wave therefore easily reaches the corona. The generation of such waves by convective motions presents some conceptual difficulties. It would require, first, a substantial amount of vertical vorticity at a scale compared to that of the flux tube (100 km, say) and, second, a way

to couple this motion to the tube. Neither of these conditions is easily satisfied. If torsional waves are present, they are more likely due to the gradual unwinding of twists stored in the tube before its eruption at the surface, possibly by a dynamo-related process (Parker, 1974a).

The longitudinal tube wave is excited by pressure fluctuations in the convection zone. Since its cutoff frequency is close to the acoustic cutoff (Figure 2), the amplitude of such waves depends critically on the amount of power present in the high frequency tail of the convective spectrum. Yet longitudinal waves have some advantage over acoustic waves because they are channeled along the field. Like ordinary slow mode waves this leads to a higher generating efficiency (Kato, 1968, Stein 1981).

Transversal tube waves are probably the easiest waves to excite. Their cutoff frequency lies just below the typical frequency of convective motions, so they respond to the bulk of the convective spectrum. The waves are excited by horizontal motions in the granulation (Spruit, 1982a). Observations show ample evidence for transversal tube waves. They correspond with the "swaying" motion that is typical for H_{α} mottles (e.g. Bray and Loughhead, 1974). The wave motions seen in H_{α} fibrils (Giovannelli, this volume) could represent the response of the chromospheric magnetic field to such waves (i.e. the field near the merging level). The expected periods and wave amplitudes agree well with these observations (Spruit, 1981b). Hollweg (1981) has calculated the propagation of these waves in open as well as closed field regions by assuming that they behave in the same way as torsional Alfvén waves. He finds large amplitudes in the corona, especially in closed field regions. A model for the propagation of transversal tube waves in the solar atmosphere is given in Spruit (1983). The tubes are matched to a homogeneous field above the merging level in this model. The tube waves continue as Alfvén waves in this homogeneous field. The wave amplitude at the base of the corona is of the order of 12 km s^{-1} for an average field strength of 20 G. At such amplitudes the waves are no longer linear. In particular, they will generate strong longitudinal flows, like in the case of the torsional waves considered by Hollweg et.al. (see previous section), that could be the cause of spicules.

The discussion in this section is not meant to suggest that coronal heating by MHD waves is the most viable mechanism. The mechanisms of the "DC" type are in the present stage of the theory at least equally attractive.

6. MISSING FLUX

Since sunspots are dark, they reduce the local heat flux at the surface. The question where the blocked heat flux goes is known as the "missing flux" problem (e.g. Sweet 1955, Wilson, 1971, Parker, 1974b, Foukal and Vernazza, 1979). The blocked flux is either redistributed over the remaining surface, or temporarily stored elsewhere inside the star. The answer to the question is of obvious importance when one tries to interpret the light variations of RS CVn and BY Dra stars in terms of star spot models (Hartmann and Rosner, 1979). A similar question arises concerning the small tubes in the small scale magnetic field since theory

predicts that they are net emitters of radiation (section 3.3).

To calculate the effect which a spot or a thin tube has on the temperature field in its surroundings, one needs a model for the transport of heat in the convection zone. The only model used so far is a "turbulent diffusion" model but this is probably adequate for the main conclusions reached below.

To specify the problem more precisely, let us "switch on" a sunspot at $t=t_0$ and ask how the thermal structure of the convective envelope will develop in time. Detailed numerical calculations of this problem were done by Foukal et.al. (1983), Fowler et.al. (1983). If we wait long enough, the star returns to thermal equilibrium, at the same luminosity as before. The blocked heat flux then reappears elsewhere at the surface. Numerical calculations for this case by Spruit (1977, cf. also Parker, 1974b, Clark, 1979, Eschrich and Krause, 1977), showed that the blocked flux spreads out over a large area, and does not cause a bright ring around the spot. Since the timescale on which spots appear and disappear is short compared with the thermal timescale of the envelope, spots may cause deviations from thermal equilibrium, i.e. some of the blocked heat flux may be temporarily stored in the convective envelope. Let f be the fraction of the surface covered by spots, and express the change δL in the surface luminosity L of the star as

$$\delta L/L = -(1 - \alpha)f . \quad (21)$$

The quantity α represents that fraction of the heat flux blocked by spots that reappears at the surface; $1-\alpha$ is the fraction that is stored in the envelope. At $t=t_0$ we have $\alpha=0$, and for $t \rightarrow \infty$ $\alpha \rightarrow 1$. It was shown by Spruit (1982b,c) that for changes in spot area that occur on a timescale which is short compared with the thermal (Kelvin-Helmholtz) timescale of the envelope, α is at most of the order

$$\alpha \sim [1 + d/(3 H_0)]^{-2} , \quad (22)$$

where d is the depth at which the blocking of the heat flux takes place and H_0 the pressure scale height at the surface. Since d/H_0 is large for any reasonable sunspot structure, $\alpha \ll 1$, and $\delta L/L \approx -f$. The blocked heat flux is therefore effectively stored in the convection zone. Note that this holds for the variable part of the sunspot area f ; the average of f on timescales long compared with the thermal timescale causes a change in radius and surface temperature, but no change in luminosity. The effect of sunspots on the solar luminosity has been observed (Willson et. al. 1981; Willson, 1982) and shows a small value of α ($\alpha \lesssim 0.1$).

Since thin flux tubes are net emitters of heat, facular areas have an effect similar to that of spots but of opposite sign. The precise magnitude of this effect is much harder to estimate than in the case of spots, since it depends on the details of the convective energy transport process in the top of the convection zone, as well as on the energy balance of a thin tube in these layers. For estimates see Spruit (1977).

7. ERUPTION OF NEW FLUX

The rise of a flux tube through the supergranulation layer, under the influence of magnetic buoyancy, aerodynamic drag, and an external flow, was modeled numerically by Meyer et al. (1979). The results show that after a section of the tube crosses the surface, its "ends" very quickly become vertical due to the strong buoyancy force. The ends move apart under the influence of magnetic tension; the rate of separation depends rather strongly on the size of the flux tube, but is in broad agreement with the rate of separation observed in emerging flux regions. More detailed calculations of the motion of flux tubes in the convection zone, taking into account also the flow along the tubes, were done by Moreno Insertis (1983).

The segment of the tube which has crossed the photosphere finds itself in a vacuum, and quickly expands. This expansion stops when the field above the surface has assumed a potential (or more generally, a force-free) configuration. Since the Alfvén speed in the atmosphere is high, the time needed to establish this equilibrium configuration is short. Observations indicate that the eruption of new flux is a gradual process. Instead of one big flux tube, a series small tubes surfaces in the same place (for a discussion see Zwaan, 1978). The eruption of the entire tube typically takes an hour or so; the process is seen in H_{α} as an "arch filament system". The tubes therefore do not expand in a vacuum but into the potential field set up by their predecessors.

Since the process is slow compared with the typical Alfvén travel time, it can be described elegantly in a model given by Shibata (1980). In this model, the field configuration in the atmosphere is a potential field defined by two point sources of opposite polarities at the surface. The strength of the field increases slowly as new flux emerges. Since the geometry of the field is known at all times, the hydrodynamics of the process can be calculated relatively easily. The results of this calculation agree with physical intuition as well as with observations. These observations (Tarbell and Title, 1983, Brands and Zwaan 1983, see also references in Shibata, 1980) show an upward motion of relatively dense and cool matter in the middle of the emerging loops, and a downward flow along their legs.

The process of flux eruption can therefore be described as slow and steady rather than as impulsive. The mass loss from the sun associated directly with the eruption process is likely to be negligible, the main reason being that the matter carried up through the photosphere finds itself contained in a strong quasi-static field ($P \ll B^2/8\pi$). Rather than expanding into interplanetary space, it drains back to the surface, along the field lines. Secondary effects, like the unwinding of twist stored in the emerging flux tubes (Parker, 1974a) and the interaction of the new flux with a preexisting field (Priest and Heyvaerts, 1974) are more likely to be responsible for the coronal activity seen in young active regions.

8. DISTRIBUTION OF FIELDS OVER THE SURFACE

The distribution of magnetic fields over the surface and its evolution in time is typically described in terms of a turbulent diffusion process (see for example Sheeley, this volume). A satisfactory theoretical description of the statistical behavior of a magnetic field in a turbulent environment like the solar convection zone is absent and not likely to be obtained in the near future. The three major obstacles are the high magnetic Reynolds number (R_m) of the flow, the stratification of the convection zone, which makes the turbulence extremely inhomogeneous in the vertical direction, and the strong magnetic forces (buoyancy and tension). For the kinematic problem (no magnetic forces) for homogeneous unstratified turbulence exact results exist only in the first order smoothing approximation, valid for small magnetic Reynolds numbers (Moffat, 1978, Krause and Rädler, 1981, Parker, 1979). Theories for arbitrary R_m have been developed by Kraichnan (1976), Knobloch (1977, 1978a,b, 1980), but the results of these theories are not consistent with each other. The possibility exists (Kraichnan, 1976, Parker, 1979, Knobloch, 1977) that the turbulent diffusivity is negative in some cases. It has been argued that this would explain the observed concentration of the magnetic field into small tubes. Even if this is a correct interpretation, however, the turbulent diffusion can surely not be represented by a single value, since on larger scales the diffusion of fields appears to be positive (spreading of active regions).

Some progress is being reported with theories based on closure approximations. Homogeneous dynamic MHD turbulence (i.e. with magnetic forces but in the absence of gravity or stratification) has been treated numerically in the so-called EDQNM approximation (Leorat et.al., 1981, Grappin et.al., 1982, Frisch, this volume, see these references for a translation of the acronym). These calculations yield the evolution in time of the magnetic and kinetic fluctuation spectra. They cannot yet be applied to the convection zone since they do not include thermal and magnetic buoyancy effects.

Some insight might be obtained by combining observations with a judicious choice of simplified turbulence models. This approach was taken by Knobloch (1981) and Knobloch and Rosner (1981). These authors conclude that the observed spectrum is formed at a depth of at least 15000 km, that the fluid motions are three dimensional, and that the field diffuses with negative diffusivity. In an alternative approach, Knobloch (1982) used the steady convection cells of Weiss and coworkers as building blocks for a turbulent magnetic cascade, and calculated a theoretical spectrum of the magnetic field fluctuations.

A simple picture of the diffusion of fields results when we assume that the field is vertical down to a certain depth z_0 . Observations, as well as the theoretical calculations using a flux tube approximation indicate that this is at least a good approximation near the surface (section 3.3,6). The evolution of the field is then governed by a) some vertical average (down to z_0) of the horizontal velocity field, and b) magnetic tension forces at $z=z_0$. Under the influence of a) alone the field evolves passively and two-dimensionally. It will quickly seek the vertices of granules and supergranules and tends to stay there. This behavior

agrees well with observations (Tarbell, this symposium). There are some indications however (Zwaan, 1978) that the expansion of ageing active regions is faster than expected on the basis of process a) alone. It was shown by Van Ballegoijen (1982) that magnetic tension effects are indeed important, and could even be the main cause of the expansion of active regions. In his model, the magnetic field is stored at the base of the convection zone, and the fields seen in active regions are connected with this field by nearly vertical flux tubes extending through the convection zone. This model is related more closely to Leighton's (1969) classical model of the dynamo than to the models based on mean field electrodynamics. More on this subject is to be found in Schüßler's review (this volume).

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DISCUSSION

ROBERTS: I would like to make two comments on wave cut-off. Firstly, the behaviour of an impulsively generated disturbance is different from that of a fixed frequency disturbance. In an impulsively generated disturbance, all frequencies are effectively involved, and the final disturbance is a Fourier integrated sum. The result is the creation of a wake, which oscillates at the mode's cut-off, trailing behind a wavefront (which propagates at the tube speed of the mode). This behaviour, which has been described in greater detail in Rae and Roberts (*Astrophys.J.*, 1982), is also evident in Hollweg's spicule calculations. The second comment concerns the modifications in the concept of cut-off that are necessary if conditions are no longer adiabatic — which is the case in the sun's photosphere and low chromosphere. There is no longer a clear division between propagation and non-propagation: all frequencies have a propagating component. In deciding on the relative efficiency of a particular mode, both the above aspects have to be borne in mind.

FOING: Two questions about the thermal structure inside the fluxtubes: (1) Due to the net excess of emission from the walls, is the thermal structure inside the tubes affected by the radiation (heating by the walls). (2) What is the time scale of thermal relaxation compared to the dynamic time of downfall of material inside the tube to allow an isothermal equilibrium?

SPRUIT: (1) Yes. (2) The thermal relaxation time scale depends strongly on depth, since the opacity in the solar atmosphere increases so enormously with depth. Near the surface, tubes with say a width of one sec of arc can relax on a time scale of a minute or less. If you go deeper, although the fluxtube itself shrinks in size the opacity increases so rapidly that these time scales become of the order of days or months or longer. So you may assume that near the surface there may be something like an energy equilibrium state, while in the deeper layers there will in general be thermal disequilibrium.

ROXBURGH: When describing your tube and uniform field model you stated that the energy density is still an increasing function of the field strength. Can you give us some idea of the dependence of energy flux on field strength?

SPRUIT: For this particular model the energy flux is roughly proportional to the average field strength. But this result depends critically on one of its assumptions, namely that the base of the corona coincides in height with the merging level of fluxtubes. This is about right for the sun, but the height of the coronal base is itself a function of the coronal heating mechanism. One therefore needs to include a coronal heating mechanism in order to make a real prediction.

RIBES: (1) If the magnetic tubes are very thin, say 100 km or less below the photosphere, I would expect that convection would ignore them and not be much reduced. So, I do not see how you can get a large excess of flux from deep convective layers, in order to explain the center-to-limb variation of the continuum faculae. (2) If the downflow exists in the magnetic fluxtubes, which seems to be the case, then the hydrodynamical solutions will be quite different from the hydrostatic ones through the energy equation (the term of entropy is added). One characteristic of the hydromagnetic solutions is the decrease of the field strength at the bottom of the photosphere ($B_0 \lesssim 1 \text{ kG}$) and the rapid fanning out of the lines of force.

SPRUIT: (1) The center-to-limb variation is determined by the geometry of the depression in the visible surface created by the tube. The net heat flux excess produced by a tube is a consequence of the thermal properties of a convective envelope. If you deal with the diffusion of heat in the convection zone as a turbulent diffusion process, you will find that

the thermal conductivity is small near the surface and very large in the deeper layers. Near the surface one thus requires large temperature fluctuations to get significant fluctuations of the heat flux; in the deeper layers one needs much smaller fluctuations, about one degree or so, to get large flux variations. So in this model you can look at the surface layers as insulating, the deeper layers as superconducting for heat. A fraction of the heat flux entering the tube sideways comes in through the deeper layer and thereby from a very large part of the convection zone. How large this fraction is relative to the contribution near the surface depends critically on how deep the fluxtube is compared with the superadiabatic surface layer (which is about 500 km deep). (2) Of course the downdrafts will have a significant effect because they have large amplitudes and carry a lot of heat. Certainly for the thermal effects, static models would be somewhat inappropriate. For the lateral pressure balance it may not be too bad, because the flows are still subsonic. A problem in dealing with this question is that the nature of the flow is not known, though it is most likely not a steady flow. The reason for this statement is simply that you cannot get enough mass from the corona. If you take one single fluxtube, you will find that it drains the entire mass of the corona in a few minutes. This is just because the density at the photospheric levels is so high, and the velocities are also high. You have to get this mass from somewhere. You may assume that there is an anomalous diffusion process into the fluxtube, as has been done by Giovanelli. I personally tend to think that this is not a real steady flow at all, but an oscillating or stochastic flow. An apparent net downflow would then be the result of a correlation between brightness and velocity in the fluxtubes.