COMPARISON BETWEEN PRESENT REPRESENTATIONS OF THE MOTIONS OF THE EIGHT MAJOR SATELLITES OF SATURN

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Abstract. We compare the two current representations (TASS and that of Harper and Taylor) of the motion of Mimas, Enceladus, Tethys, Dione, Rhea, Titan, Hyperion and Japetus. Although both theories produce almost the same (o-c) residuals with the available observations, we show that they may give positions significantly different at dates corresponding to a gap in the distribution of these observations. In the interval 2000–2020 (corresponding to the CASSINI mission) these differences can reach 5 000 km for Mimas, 8 000 km for Hyperion, 6 000 km for Japetus. So, to test a theory of motions it is necessary to use other criteria among which the most important ones are its dynamical consistency and its internal accuracy.

1. Introduction

To compute the positions of the satellites Mimas, Enceladus, Tethys, Dione, Rhea, Titan, Hyperion and Japetus at a given date, we have considered two representations of their motions:

- H–T described in (Taylor, 1992) for Hyperion and in (Harper and Taylor, 1993) for the other satellites.
- TASS described in (Duriez and Vienne, 1997) for Hyperion and in (Vienne and Duriez, 1995) for the other satellites.

Hyperion is now included in TASS, because its updated theory has just been adjusted to observations. However, the complete result of this adjustment will be presented in a paper still in preparation. These two representations are based nearly on the same set of Earth-based observations. Most of them can be found in the catalogue of Strugnell and Taylor (1990) and

I. M. Wytrzyszczak, J. H. Lieske and R. A. Feldman (eds.),
Dynamics and Astrometry of Natural and Artificial Celestial Bodies, 177, 1997.

TABLE 1. Rms (root mean square) (o-c) residuals (in arcseconds), obtained for each satellite, with TASS and with H−T using all available observations from 1874 to 1985 with the same way to weight them for TASS and H−T and with a rejection level for (o-c) residuals larger than 2″.

satellites	1	2	3	4	5	6	7	8
TASS	0.19	0.15	0.14	0.14	0.14	0.15	0.22	0.21
H−T	0.20	0.15	0.14	0.14	0.14	0.15	0.24	0.22

cover more than one century, from 1874 to 1989. However, the theories from which these representations are issued, are different: Harper and Taylor use mainly as parameters for each satellite, an independent slowly precessing orbit, plus eventually some terms of libration in the mean longitude to represent the features of the resonance between two satellites. On the other side, TASS was constructed in a dynamically consistent way, in which the satellites are considered all together; its parameters are explicitly the initial conditions, the masses of the satellites and the oblateness coefficients of Saturn only. That leads to significant differences between both theories specially for Mimas, Hyperion and Japetus.

Note that Dourneau (1993) proposes a third representation, which was also compared to TASS. For Hyperion the differences are very important because of the lack of short-period perturbations in Dourneau. For the other satellites, its representation is very close to that of H–T. So, the results concerning these satellites are not significantly different, and in order to preserve space, the tables included in the present paper concern only the comparison between TASS and H–T.

The reader will find in the papers quoted above the details of these representations; a global view of the theories is given in (Duriez, 1996). The aim of the present work is to compare critically both representations. We show that, at least in the case of the Saturnian system, the traditional (o-c) residuals (difference between observed and computed position) are not the best criterion to test a representation. Let us remind that the first quality of a dynamical theory is to be able to predict future positions and to determine also the dynamical parameters.

2. Representations of the Motions and their (o-c) Residuals

Strugnell and Taylor (1990) have compiled about 51 000 observations of Saturn's satellites, made between 1874 and 1989, all put now in a consistent format, and with all times reduced to UTC. This catalogue contains most

TABLE 2. Rms (o-c) residuals (in units of 0.″01) obtained for Mimas, Rhea and Hyperion with TASS and with H–T, from some major data sets of observations (with a rejection level of (o-c) fixed at 2″). $N_{u/r}$ represents for each satellite the number of observations "used/rejected" in each data set with TASS (H–T gives almost the same numbers). The data sets (or observers) are numbered like in the catalogue of Strugnell and Taylor (1990) where their full reference may be found, with the meanings: "1 USNO" = USNO (1877–1887), "3 USNO" = USNO (1911), "9 Struve" = Struve (1898), "31 Pascu" = Pascu (1982), "47 V–D" = Veillet-Dourneau (1992) T1m, "48 V–D" = Veillet-Dourneau (1992) ESO T1.5m, "52 Dourn" = Dourneau *et al.* (1986) ESO T1.5m.

	MIMAS			RHEA			HYPERION		
observers	$N_{u/r}$	TASS	H-T	$N_{u/r}$	TASS	H-T	$N_{u/r}$	TASS	H-T
1 USNO	65/6	52	52	108/2	39	39	464/16	64	80
3 USNO	232/40	51	50	1409/8	23	21	282/11	53	70
9 Struve	224/0	15	14	958/0	13	12	469/1	41	79
31 Pascu	535/5	22	23	1071/17	10	11	100/0	24	23
47 V–D	252/0	15	17	138/0	8	8	338/0	18	19
48 V–D	96/0	19	16	856/0	9	10	1776/0	13	19
52 Dourn	106/0	25	24	290/0	10	11	800/0	14	18

of the published observations made since 1966 (mainly photographic ones); the other data consist of a selection of older observations (mostly visual).

We have computed the (o-c) residuals of all these observations, once using TASS, and once using H–T (in this last case, the Saturnicentric positions are converted in the J2000 ecliptic system, as for TASS, so that the next transformations to geocentric positions are exactly the same in both cases). Table 1 gives a global result of these computations: the rms residuals for TASS are found between 0.″14 and 0.″22, depending on satellites, but above all, the residuals are practically the same for each satellite with both theories. Note that the way of weighting the observations is exactly the same when computing the residuals for TASS and for H–T. It appears that the satellites may be put in two groups: One (from Enceladus to Titan) with typical residuals in [0.″14–0.″15], and the other (Mimas, Hyperion and Japetus) with significantly larger residuals.

More precisely, these residuals are almost the same with both theories even if we consider each group of observations (defined mainly by the observed satellite and the observer). This is shown in Table 2 where an extract of the distribution of the r.m.s residuals is presented for Mimas, Rhea and Hyperion and for some significant observers.

Note that for the most ancient (visual) observations, the residuals are rather large (up to 0.″8 for Hyperion in H–T), despite a rejection level fixed at 2″; that shows how difficult it was to observe well the faint satellites as Mimas and Hyperion. However, when fitting TASS to observations, the weight of these ancient observations appears to be far less important than of that the recent ones. For example, the only "47 V−D" observations of Hyperion have a weight representing 42% of all observations. Note also that the differences in the residuals between TASS and H−T for ancient observations of Hyperion may be explained: Taylor (1992) says that he has not used the observations made before 1966 in the fit of his theory of Hyperion.

From these tables, it seems impossible to prefer one theory to another, because the distribution of the residuals does not depend on the theory (except for Hyperion for which TASS gives a more complete representation). Are we allowed to conclude that both representations of the motions of the satellites of Saturn are equivalent? The answer should be yes if we wanted only the best residuals on the past observations. But one of the aims of a theory is rather to be able to predict future positions of the satellites!

3. The Differences between both Representations of Motions

In this aim, we have compared the geocentric positions given by H−T and by TASS over the 1874–2020 period (from the date of the first Earth-based observations to the expected moment of the CASSINI mission). So, we have computed with a 10-days step the angular separation as seen from Earth, between the two positions of a given satellite computed with H−T and with TASS. We have found that, in the case of Mimas, Hyperion and Japetus, these differences are significantly correlated with the distribution of the observations used to fit both representations. This correlation is evident in Figure 1 where, with each plot of these differences, we show the corresponding distribution of the observations (to preserve space, this concerns here the satellites Mimas, Rhea and Hyperion only).

We note two major gaps in the distribution of the observations: The first, concerns the period 1947–1966, enlarged to 1930–1966 for Hyperion!

The second "gap" is after 1984; of course, there are no observations between 1997 to 2020; however, during the 1985–1996 period Saturn's satellites were observed but, unfortunately, these observations are not yet available.

The influence of these gaps on Rhea is negligible, because H−T and TASS are two rather complete representations for this satellite. This means that both theories compute nearly the same positions at the level of 400 km, even for dates far from the periods of the observations. But, we recall that for Rhea the internal accuracy of TASS reaches 10 km, so we think that the plotted differences represent in fact the accuracy of the best observations on which both theories have been fit. We have found that the same occurs

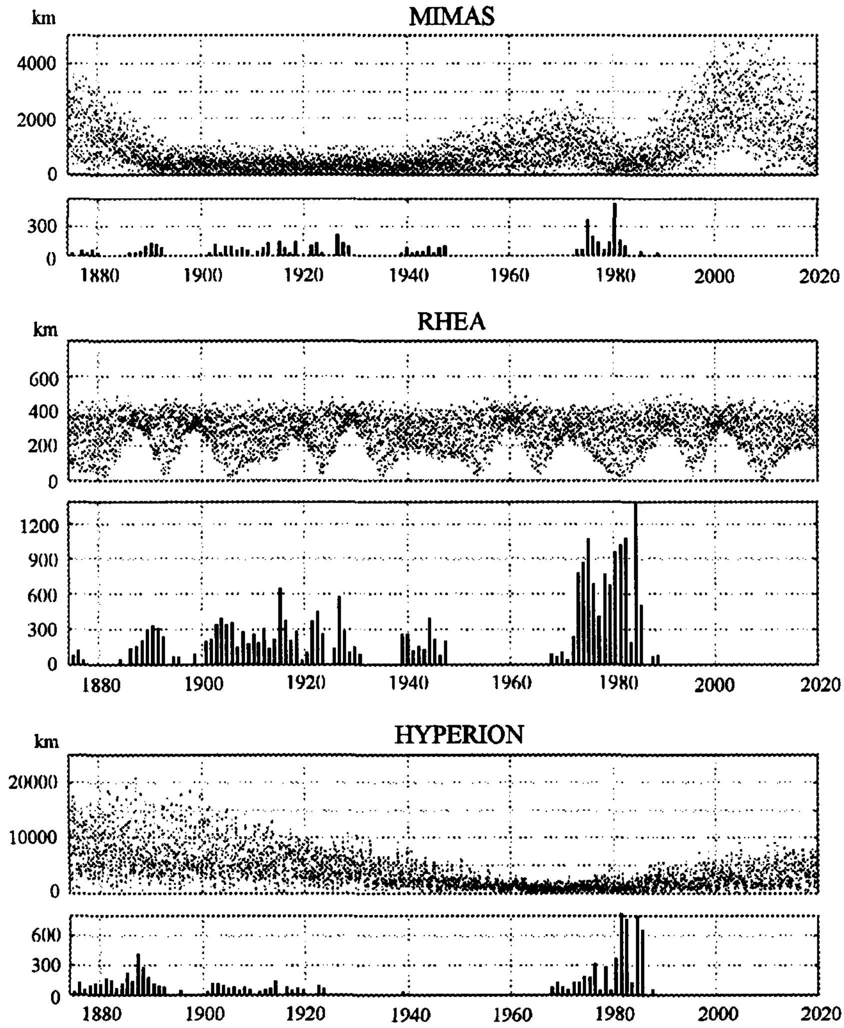

Figure 1. Distance, in kilometers, between TASS and H–T from 1874 to 2020 for Mimas, Rhea and Hyperion, and correspondance with the distribution of the number of available observations at each opposition for each satellite (at Saturn, 0.″1 represent 700 km as seen from the Earth).

for Enceladus, Tethys and Titan. On the contrary, for Mimas and Japetus, the forms of the two representations of motions used in each fit are very different. So, for these satellites the large differences in positions are well correlated to the gaps in the observations. In the interval 2000–2020 (corresponding to the CASSINI mission) these differences can reach 5 000 km for Mimas and 6 000 km for Japetus.

For Hyperion, it appears that H–T and TASS are close only in the interval 1966–1985 corresponding to the recent observations, which are in fact the only ones used by Taylor (1992) in its fit. However, even in this interval, the distance may reach 4 000 km, showing there the difference of completeness of both representations. Elsewhere, the maximum difference in positions given by H–T and TASS is far larger: 21 000 km in the interval 1874–1910 (that is about $3''$ as seen from the Earth), and 8 000 km in the interval 2000–2020.

4. Conclusion

This study shows that in spite of intrinsic differences between TASS and H–T, the rms residuals with observations are comparable (between $0\overset{''}{.}14$ for Rhea and $0\overset{''}{.}22$ for Hyperion), giving in fact the real accuracy of the available terrestrial observations, and not the accuracy of the theories.

However, for Mimas, Hyperion and Japetus, TASS and H–T may give positions significantly different at dates corresponding to a gap in the distribution of these observations. In the interval 2000–2020 (corresponding to the CASSINI mission) these differences can reach 5 000 km for Mimas, 8 000 km for Hyperion, 6 000 km for Japetus. So, to test a theory of motion it is necessary to use other criteria among which the most important are the dynamical consistency of the theory and its internal accuracy. In that respect, TASS will be able to give a new determination of all physical parameters as soon as new very accurate observations will be available.

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