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Orbit computation of the TELECOM-2D satellite with a Genetic Algorithm

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Abstract. In order to test a preliminary orbit determination method, we fit an orbit of the geostationary satellite TELECOM-2D, as if we did not know any a priori information on its trajectory. The method is based on a genetic algorithm coupled to an analytical propagator of the trajectory, that is used over a couple of days, and that uses a whole set of altazimutal data that are acquired by the tracking network made up of the two TAROT telescopes. The adjusted orbit is then compared to a numerical reference. The method is described, and the results are analyzed, as a step towards an operational method of preliminary orbit determination for uncatalogued objects.

Keywords. Orbit computation, Genetic algorithm, Analytical Propagation

1. Introduction

We aim at fitting an orbit to tracking data, provided as a time series of angular coordinates on the sky, when no *a priori* information on the trajectory is available at all. In that case, classical methods such as least-squares can not be used any more, since the function to be minimized can not be linearized in the neighborhood of the *a priori* values of the parameters. Moreover, the usual methods of preliminary orbit determination may suffer from many drawbacks which can make them be unappropriate: the well-known Gauss, Laplace, Escobal... approaches are not valid for all dynamical configurations in case of singularities due to orbital planes alignments; additionally, they are often based on motion theories accounting only for the Keplerian motion, and can hence not be applied over time scales longer than a couple of hours, since in that case a propagator has to account for the main perturbations, at least for the secular ones. A general comparison between these methods could be the subject of a forthcoming paper.

On the contrary, even if other kinds of difficulties have to be managed, methods based on genetic algorithms are supposed to be valid for all dynamical configurations, since the algorithm itself is independent from the orbit propagator used to compute the cost function. With an efficient dynamical modeling, they can be used over different periods of time, from a couple of minutes (for Too-Short Arcs, TSA) up to a couple of days or weeks.

The starting point is the system of the equations of motion, that can be written in an usual way:

$$\frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}t^2} = \boldsymbol{F}(\boldsymbol{r}, \dot{\boldsymbol{r}}, t, \sigma) \qquad \text{with} \qquad \boldsymbol{r}(t_0) = \boldsymbol{r}_0 \qquad \dot{\boldsymbol{r}}(t_0) = \dot{\boldsymbol{r}}_0$$

and where the initial positions and velocities to be estimated at an epoch t_0 are denoted $\boldsymbol{r}(t_0)$ and $\dot{\boldsymbol{r}}(t_0)$. The right-hand side describes the force model through the vector $\boldsymbol{F}$, that is characterized with a set of parameters σ. Genetic algorithms allow a way to find satisfying initial conditions $\boldsymbol{r}(t_0)$ and $\dot{\boldsymbol{r}}(t_0)$, without testing all the possibilities in a space of dimension 6, once the frame is roughly defined.

Following (Deleflie *et al.*, (2013)), we provide the finalized results that we obtain after some refinements of the method.

2. Orbital modeling

To keep a reasonable computation time, since many iterations are tested, we use an analytical approach to get orbital element time series. Since the method is supposed to be valid in all dynamical configuration (whatever the values of the eccentricity and the inclination, in particular), the model is written in a set $\bar{\boldsymbol{E}}$ of equinoctial elements (Deleflie & Decosta (2013)), namely: a, $\xi = \Omega + \omega + M$, $e\cos(\Omega + \omega)$, $e\sin(\Omega + \omega)$, $\sin i/2 \cos\Omega$, $\sin i/2 \sin\Omega$, where a, e, i, Ω, ω, M stand for the classical Keplerian elements. The whole analytical modeling is governed by the set of mean initial conditions, whereas it is the corresponding osculating initial conditions that are adjusted by the genetic algorithm. The relation between mean (denoted $\bar{\boldsymbol{E}}(t_0)$) and osculating initial elements (denoted $\boldsymbol{E}(t_0)$) is merely obtained by setting the time t to the initial epoch t_0 in the equation defining the shape of the analytical solution, that is: $\boldsymbol{E}(t) = \bar{\boldsymbol{E}}(t) + \mathcal{L}(\bar{\boldsymbol{E}})\frac{\partial W}{\partial \bar{\boldsymbol{E}}}\big(\bar{\boldsymbol{E}}(t)\big)$, the matrix 6×6 $\mathcal{L}(\bar{\boldsymbol{E}})$ standing for the Lagrange Planetary Equations and W being a generating function of the short periodic terms, both depending of the mean equinoctial elements at a given epoch. Here, the force model is the central gravity field developed up to degree 10, to strike a balance between the accuracy and the total required CPU time.

3. Multi-Objective Genetic Algorithm (MOGA) used

3.1. *Description*

The criteria to be optimized (maximized or minimized) are defined as functions of the initial conditions, and they are optimized through a large number of iterations that make the process converge to a set of optima. For the computation of the TELECOM-2D orbit, the double-objective of getting a minimum on the two components of the data, independently, has been set up. Though more time consuming than a standard Genetic Algorithm, a multi-objective Genetic Algorithm does not require any weighting choice as it appears when using a solely aggregated objective.

The Multi-Objective Genetic Algorithm (MOGA) used here is ϵ-MOEA (Deb *et al.* (2003). In the course of successive iterations, some vectors of initial conditions are replaced by other ones and the best ones are archived. The evolution through the iterations is governed by mutations (random small changes in vectors of possible initial conditions) and by crossover (mix two vectors of possible initial conditions). At the end of the iteration procedure, a set of solutions is supplied (Coello Coello *et al.* (2007)).

As in many orbit determination algorithms, an evaluation is made up of several steps:

- ϵ-MOEA provides a vector of initial conditions, randomly chosen among a large set of possible (osculating) initial values;

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• These initial conditions are used to propagate an analytical orbit over the period when tracking or astrometric data are available;

• The analytical orbit, as time series of orbital elements, is used to compute predicted measurements, that can be compared to the available data sets, at the same epochs of the observations;

• These predicted measurements are compared to the true data;

• The cost functions are computed, with the n observations (Obs) and the corresponding theoretical quantities carried out by the Genetic Algorithm (GA), for the two angles elevation (el) and azimut (az): $\sqrt{\frac{1}{n}\sum_{i=1}^{n}(\mathrm{el}_i^{\mathrm{obs}}-\mathrm{el}_i^{\mathrm{GA}})^2}$, and $\sqrt{\frac{1}{n}\sum_{i=1}^{n}(\mathrm{az}_i^{\mathrm{obs}}-\mathrm{az}_i^{\mathrm{GA}})^2}$.

The process is then iterated until optimal values of initial conditions are found, optimal values being defined as set of values (not necessary unique) providing the minimum distance in the objective space.

3.2. *Parameterization*

The chromosomes represent the initial state of the solution: each one is then made up of a vector with six components, which determines an unique orbit.

We choose here an initial population of 400 chromosomes, with fixed intervals for each of the initial components of the state vector, natively written through the set of equinoctial elements:

• semi–major axis $a \in [40000; 45000]$ km for Telecom-2D

• eccentricity vector assuming $e \in [0; 0.1]$, i.e. the two components of the eccentricity vector $\in [-0.1; 0.1]$

• inclination $i \in [0; 180°[$, i.e. the two components of the eccentricity vector $\in [-1; 1]$

• the angle $\xi = \Omega + \omega + M \in [0; 360°$

Let us note that to reduce computation time, the search for the initial eccentricity has been reduced to an interval with a wideness of 0.1, and the search of the initial semi major axis to intervals large of a few thousands of kilometers. But, the results that are shown hereafter would not have been worse if we have kept all the possibilities ($e \in [0; 1]$, and $a \in [6\,500; 45\,000]$ km for instance) for these two elements as well. But the computation time would have been significantly larger.

The crossover probability has been set up, classically, to $p_C = 0.9$, and the mutation probability to $p_m = 1/6 \simeq 0.16667$. The stop condition is the total number of iterations (here set up to 100 000), corresponding to a total CPU time of the order of 10 hours.

4. Genetic algorithm handled with angular data: computation of the TELECOM-2D satellite orbit

This example is based on the assimilation of altazimutal data obtained after astrometric reductions from images acquired by the two TAROT telescopes, respectively located in France and Chile, on the geostationary satellite Telecom-2D. We use data provided by CNES, which has an agreement to benefit from 15% of the available time each night, for space debris activities. The upper reachable magnitude is of the order of 15 within the GEO region, and the measurement accuracy is of the order of 700 m in GEO. The data set includes nine days of angular data, acquired in Sept. 2012 from the two TAROT-telescopes. The total number of measurements is 235 (97 for la Silla, Chile, and 138 for Calern, France).

The reference orbit was computed with the CNES s/w Romance.

Figure 1 shows how the G. A. coupled with the analytical propagation converges to the numerical solution seen as a reference.

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Figure 1. Convergence of the iterations for the TELECOM-2D orbit computation by the G. A. Left: global objective; Right: convergence of the initial semi-major axis

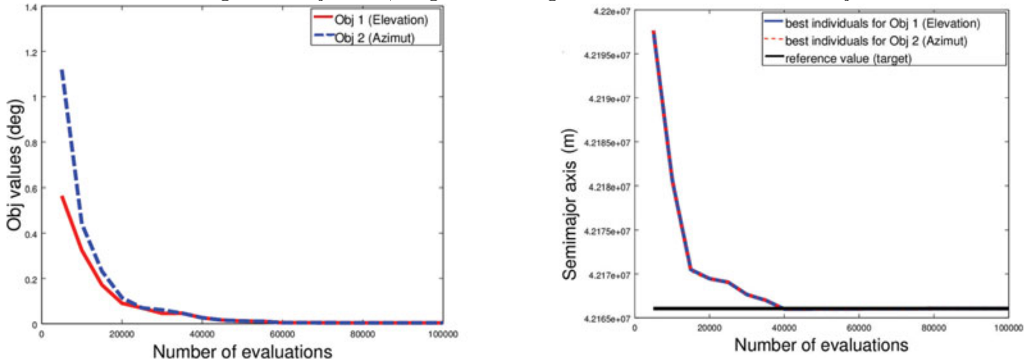

Table 1. Numerical references for the initial state vector, expressed in classical keplerian elements, and best candidates found by the G. A.. Let us remind that, once projected into classical keplerian elements, the values found by the G. A. are not that accurate for the argument of perigee ω and the mean anomaly M, because of a poor geometry defining these two angles. It is only an effect of projection, with no consequence on the computation.

	num. reference	**ref. elements - G. A. best candidate**
s.m.a.	$a = 42\,165\,286$ m	$\Delta a = 773$ m
eccentricity	$e = 0.000\,14$	$\Delta e = 0.000\,147$
inclination (deg)	$i = 5.705°$	$\Delta I = 0.003\,4°$
R. A. A. N. (deg)	$\Omega = 62.066°$	$\Delta\Omega = 0.049°$
arg. perigee (deg)	$\omega = 220.266°$	$\Delta\omega = 15.106°$
mean anomaly (deg)	$M = 29.129°$	$\Delta M = 15.111°$
orbital longitude (deg)	$\omega + M = 249.396°$	$\Delta(\omega + M) = 0.005°$

5. Conclusions

We combine a MOGA and an analytical satellite motion theory to adjust an orbit on tracking data, without any *a priori* knowledge of the initial conditions to be retrieved. The next steps lie (i) in a test of the capabilities of the algorithm in downgraded conditions (data sparse in time, very few number of data), (ii) in a reduction of the required CPU time, by reducing the width of the intervals of research with *a priori* more realistic values.

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