dicular from P to the line

$$2\triangle = AB \cdot PN$$

$$= \pm \frac{c}{ab} \sqrt{a^2+b^2} . PN.$$

Equating these two expressions for $2\triangle$ we have at once

$$PN = \pm \frac{ax_1 + by_1 + c}{\sqrt{(a^2+b^2)}}.$$

(ii) The special case in which the straight line passes through the origin (when A and B coincide with O, and $\triangle = 0$) may obviously be regarded as the limiting case of (i) for $c \rightarrow 0$.

Or, if it is desired to avoid the limit-conception, we have only to note that the perpendicular from P to $ax + by = 0$ is equal to the perpendicular form O to the parallel through P, namely

$$a(x - x_1) + b(y - y_1) = 0,$$

and that, by (i), the length of that perpendicular is

$$\pm \frac{a(0 - x_1) + b(0 - y_1)}{\sqrt{(a^2+b^2)}} = \pm \frac{ax_1 + by_1}{\sqrt{(a^2+b^2)}}.$$

(iii) The reader may be interested to refer to a method of finding the length of the perpendicular by projections, given by Prof. R. J. T. Bell in *Mathematical Notes*, No. 18 (May 1915), pp. 206–207.

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Formulae for the Construction of Right-Angled Triangles.

Use the formulae

$$[a(a+2b)]^2 + [2b(a+b)^2]^2 = [2b(a+b) + a^2]^2 \qquad \text{(i)}$$

or

$$[2a(a+b)]^2 + [b(2a+b)]^2 = [b(2a+b) + 2a^2]^2. \qquad \text{(ii)}$$

In these a and b need not be integers, and may be positive or negative.

The formulae develop into two systems of sets of triangles.

For the first system the triangles are

$$[\overline{2N-1}\,.\,(2n-\overline{2N-1})],\ [2n\,(n-\overline{2N-1})],$$
$$[2n\,(n-\overline{2N-1})+(2N-1)^2], \qquad \text{(iii)}$$

where N is the ordinal number of the set of triangles in question, and n is any number, not necessarily an integer and not necessarily positive.

For formula (i), $a=2N-1$, an odd integer; $b=n-(2N-1)$.

For formula (ii), $a=(2N-1)/\sqrt{2}$, $b=\sqrt{2}\,(n-\overline{2N-1})$.

For the second system the triangles are

$$[2N(2n-\overline{N-1})],\ [4n\,(n-\overline{N-1})-\overline{2N-1}],$$
$$[4n\,(n-\overline{N-1})+2N(N-1)+1]. \qquad \text{(iv)}$$

For formula (i), $a=\sqrt{2}\,N$, $b=\{2\,(n-N)+1\}/\sqrt{2}$.

For formula (ii), $a=N$, an integer; $b=2\,(n-N)+1$.

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Note on Isogonal Conjugates.

If T, U are any pair of isogonal conjugates with respect to a triangle ABC (circumcentre O, orthocentre H), then

$$OU=(TH/T\Phi)\,.\,(\text{circumradius});$$

where Φ is the fourth point of intersection of the circumcircle with the rectangular hyperbola $ABCHT$ (whose centre Ω is the middle point of $H\Phi$).

It has been established by the method of isogonal transforma tion that if T is any point on a fixed rectangular hyperbola $ABCH\Phi$, then the point U (the isogonal conjugate of T) always lies on a fixed circumdiameter EOF.

Now AT, AU are equally inclined to the bisector of the angle A; hence the cross ratio of the pencil formed by joining A to any four positions of T is equal to the cross ratio of the four corresponding positions of U on EOF.