MEASUREMENTS OF MAGNETIC FIELDS

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Abstract. It is demonstrated that certain interrelations between the Stokes parameters are much less dependent on line formation than generally expected. Thus the dependence of these parameters on the magnetic field may be made more explicit and may lead to reliable calibration of polarimetric measurements in terms of magnetic field.

1. Introduction

Three preliminary hints indicate that such an independence on line formation is very likely to occur.

First, the Sears formula, for the case of weak fields, is easily generalized by treating the transfer equations (Semel, 1967). That means that only broadening mechanisms should be estimated to make this formula valid.

Second, for strong fields, that is when all Zeeman components are separated, the polarization in each component is a function of the magnetic field only.

Third, in the relation $V/Q=\eta_V/\eta_Q$ in the Unno theory (1956) only the broadening mechanism and magnetic field should be considered. This relation can be easily generalized by treating the transfer equations. The only condition necessary is that the broadening mechanism is kept constant in the observed element of the Sun. (However neglect is made of all parasitic effects as depolarization etc...).

The critical factor in all these cases is the broadening mechanism.

2. Numerical Computation

Now we shall extend our treatment with the help of numerical computations using the Schuster-Schwarzschild model (Michard, 1961) which for our purpose is quite equivalent to the Milne-Eddington model (Unno, 1956). For each model three parameters of line formation should be specified:

(1) For Doppler broadening, ξ the half Doppler width;

(2) τ_0 for the $S-S$ model or η_0 for $M-E$ model;

(3) $A_{SS}=(I_0-S)/I_0$ for $S-S$ model or $A_{ME}=\beta_0\cos\theta/(1+\beta_0\cos\theta)$ for $M-E$ model.

In the proposed mathematical method A_{SS} and A_{ME} are eliminated. The dependence on ξ and on τ_0 or η_0 is shown by the computed curves. The terms r_I, r_V and r_Q are used to represent the Stokes parameters I, V and Q in the depression representation.

The case of a Zeeman triplet observed with circular analysis is considered first. The quantity calculated is

$$\Delta\lambda_G=\frac{\int \Delta\lambda r_V\,\mathrm{d}\lambda}{\int r_I\,\mathrm{d}\lambda}$$

Howard (ed.), Solar Magnetic Fields, 37–43.

$\Delta\lambda = \lambda - \lambda_0$; λ is the wave-length and λ_0 is the origin (the wave-length of the undisturbed line).

The first results are given in Figure 1 where $\Delta\lambda_G$ is plotted against $\Delta\lambda_H$ both in units of ξ_0 the half Doppler width. $\Delta\lambda_H$ is related to the magnetic field H by the well known expression:

$$\Delta\lambda_H = 4.67 \times 10^{-13}\, g\lambda^2 H\,.$$

For practical reasons the values $-5\xi_0$ and $5\xi_0$ were taken as the lower and upper limits respectively in the integrals giving $\Delta\lambda_G$.

$\Delta\lambda_G$ approximates the longitudinal component ($\Delta\lambda_G \simeq \Delta\lambda_H \cos\psi$). The differences between the curves for $\tau_0 = 1.45$ and $\tau_0 = 3$ are very small. The curves are practically

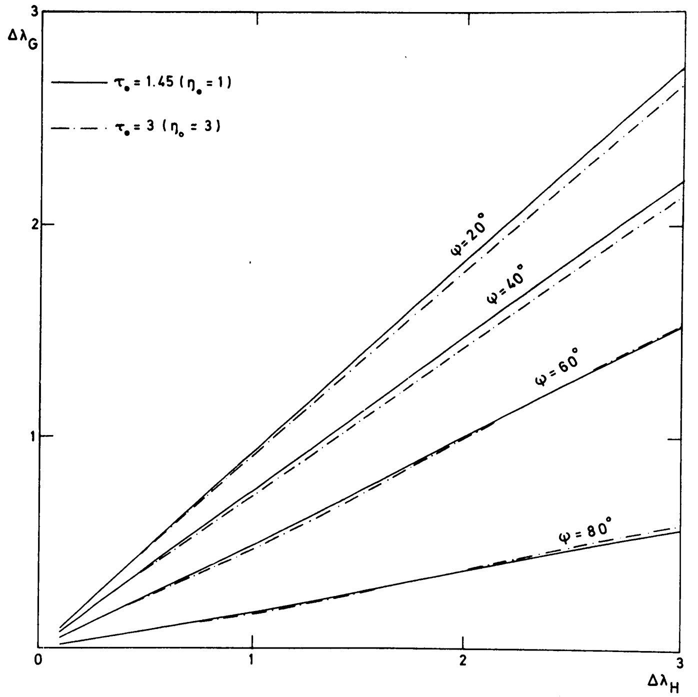

Fig. 1. For the case of Zeeman triplet observed with circular polarization. $\Delta\lambda_G$ is plotted against $\Delta\lambda_H$ in units of ξ_0 the half Doppler width.

$$\Delta\lambda_G = \frac{\int_{-5\xi_0}^{5\xi_0} \Delta\lambda\, r_V\, \mathrm{d}\lambda}{\int_{-5\xi_0}^{5\xi_0} r_I\, \mathrm{d}\lambda}$$

ψ is the angle between the magnetic field and the line of sight. $\Delta\lambda_G \approx \Delta\lambda_H \cos\psi$.

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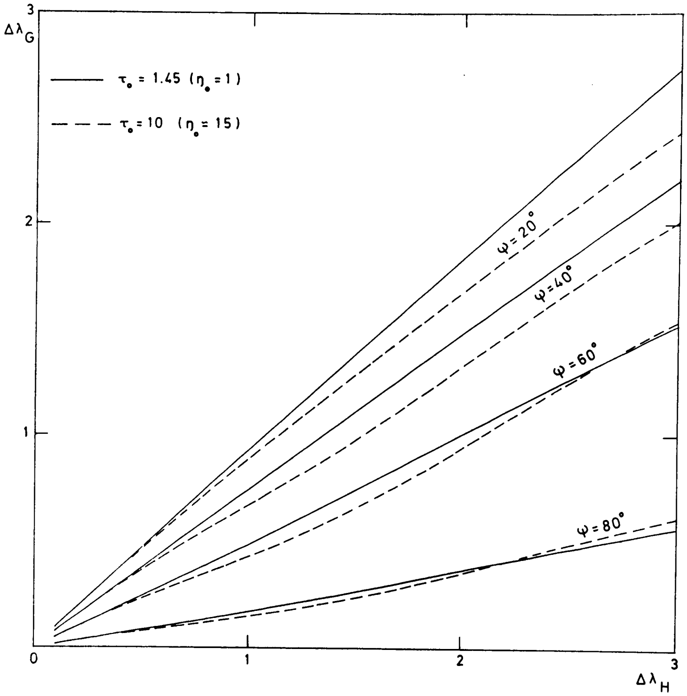

Fig. 2. The same as in Figure 1. Comparison between high saturation and low saturation.

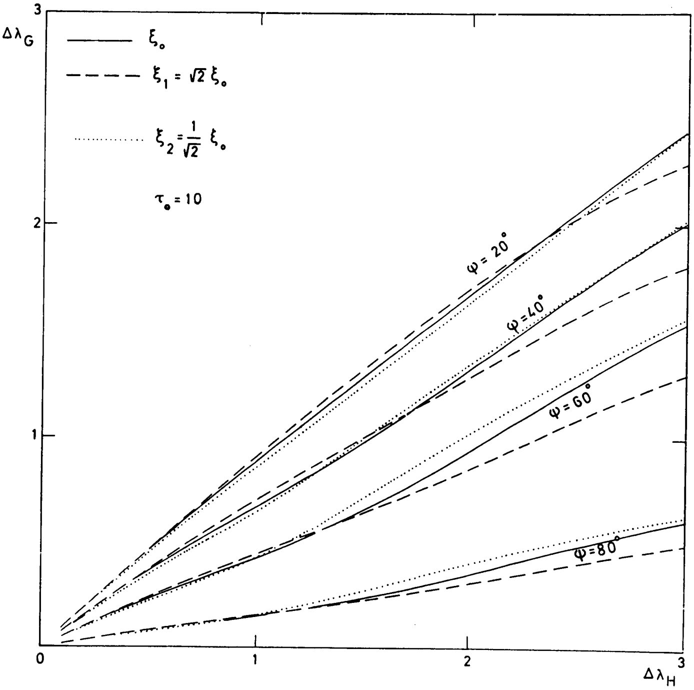

Fig. 3. The same as in Figure 1. $\Delta\lambda_G$ is plotted against $\Delta\lambda_H$ in units of ξ_0, but three different values for the Doppler broadening are considered.

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straight lines passing through the origin of the axes and consequently are independent of ξ. Figure 2 compares the curves for $\tau_0 = 1.45$ and $\tau_0 = 10$ (high saturation). The differences are of the order of only 10%. In Figure 3 a comparison is made between curves all corresponding to the same saturation $\tau_0 = 10$ but with different values for $\xi: \xi_0, \xi_1 = \sqrt{2}\xi_0$ and $\xi_2 = \xi_0/\sqrt{2}$. The dependence on ξ seems to be small. The deviation for ξ_1 is due to the fact that integrations were limited to the range $-5\xi_0 < \Delta\lambda < 5\xi_0$.

$\Delta\lambda_G$ is the displacement of the center of gravity of a line profile observed with circular polarization, and can be measured by a magnetograph. A proposal for such a method is described by Semel (1970a).

3. Observations

I would like now to sketch very briefly how progress in this direction was achieved with photographic observations. At the beginning we knew only the Sears formula for weak field. For the purpose of quantitative analysis Michard established a method which was the analog of a standard magnetograph. We soon felt the response is saturated for strong fields. Then Michard had the project of the Lambdameter (Rayrole *et al.*, 1962). The advantages were evident. However we had as a first problem: the choice of slits for the Lambdameter, we made the most intuitive choice of slits from technical point of view only, Rayrole established a programme for computing the response of the Lambdameter using the Unno theory. He expected to measure the vector field by using different lines (Rayrole, 1964).

When I proceeded to test the longitudinal component approximation by comparison of observation with different lines, I was able to show that the approximation is fairly good even for strong fields. I was more and more convinced that the longitudinal component approximation should not be limited to weak fields and I had the conviction that this approximation is not due to pure hazard. This conviction led Rayrole and me to have many animated discussions and also to many tests by numerical computation and by experiments with the Lambdameter using different slits. For this purpose the photographic method proved to be very useful. Thus Rayrole was able to establish a method for measuring the vector field by simultaneously observing different lines and using the Lambdameter with different slits (Rayrole, 1967).

Now our first happy conclusion is that: By the analysis of the circularly polarized component of spectral lines formed in the presence of a magnetic field, magnetic data can easily be deduced. The displacements of the center of gravity of the profiles $r_I \pm r_V$ can be interpreted as the measurements of the longitudinal component of the magnetic field. In many cases this procedure is very slightly dependent on the unknown parameters of the line formation.

4. The Analysis in the Case of Linear Polarization

The interpretation of linear polarization in terms of magnetic fields cannot be made independent of the parameters of line formation. A dependence on the broadening mechanism is expected. However for many lines the broadening mechanisms are likely to be identical. Therefore it would be interesting to develop a method where the calibration of polarimetric measurements depends only on the broadening parameters. In such a case the disadvantage due to this dependence might have its counterpart, namely the determination of the broadening mechanisms, which has an important physical meaning.

Using the $S-S$ model I calculated:

$$\Delta\lambda_G = \frac{\int_0^{5\xi_0} \Delta\lambda r_Q \, \mathrm{d}\lambda}{\int_0^{5\xi_0} r_I \, \mathrm{d}\lambda}.$$

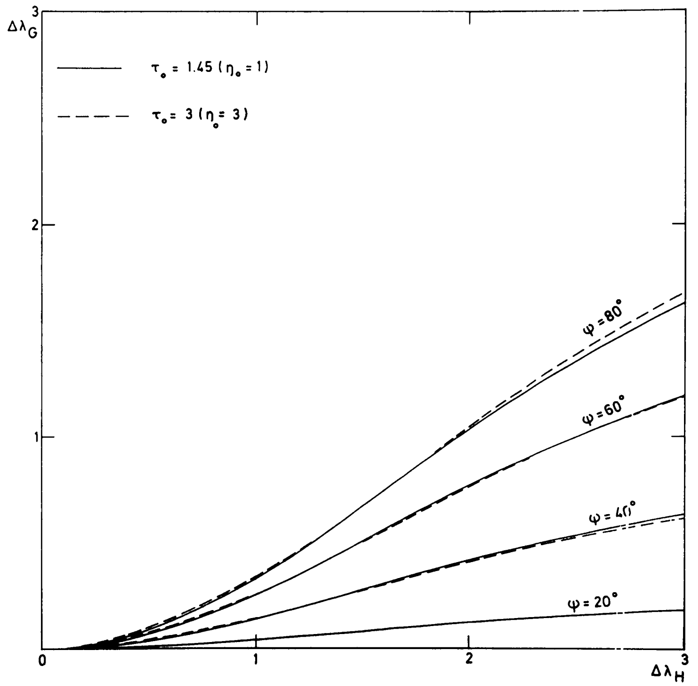

Fig. 4. For the case of Zeeman triplet observed with linear polarization. $\Delta\lambda_G$ is plotted against $\Delta\lambda_H$ in units of ξ_0 the half Doppler width.

$$\Delta\lambda_G = \frac{\int_0^{5\xi_0} \Delta\lambda \, r_Q \, \mathrm{d}\lambda}{\int_0^{5\xi_0} r_I \, \mathrm{d}\lambda}.$$

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The results are given in Figure 4. Again the variations of τ_0 are unimportant.

The choice of ξ is critical as expected. Thus the reduction of linear polarization measurements to magnetic data should simultaneously incorporate a determination of ξ. An attempt in this direction can be made by simultaneously observing several lines having different values of the Landé factor g. Assuming that ξ is the same for all lines used, then, the simultaneously measured values for $\Delta\lambda_G$ will only lie on the same curve of calibration if ζ is correctly evaluated.

A proposal for the 'realization' of this approach by a magnetograph was described in another paper and cannot be discussed in more detail here (Semel, 1970b).

5. Computation of Line Profiles

We now proceed to another aspect of the problem that is the shape of spectral line profiles, because in last analysis any observation is the analysis of line profiles either explicitly or implicitly. The following is the result of numerical computation made by Rayrole and myself.

Our first question was whether an observed profile in absence of magnetic field may allow to determine the model to be used and its parameters. We found that numerically speaking the $S-S$ and $M-E$ models are equivalent. That is we found that for each of the three parameters of one model we can find a choice of three parameters for the other model to obtain practically the same profile. Then we introduced the magnetic field but we find again that the two models give similar profiles when using for each model the parameters adjusted as above for the zero field profile. Thus we could continue our numerical exercises with one model only, the $M-E$ one. It became evident that the choice of the parameters might be critical. In the presence of a magnetic field the profiles may be quite different. As a result of high saturation, $\eta_0>4$, a new peak or a pseudo Zeeman component appears (Henoux, 1968; Göhring, 1969). We expected that such a peak would be easily distinguished and could be used as a criterion for the choice of η_0*. However we never observed such a pseudo component in our films. We were not content to conclude that η_0 should always be small for all lines. We realized soon that the occurrence of such a pseudo component is highly related to the choice of ξ and that in fact ξ is likely to be smaller than was usually considered ($\xi=40$ mÅ for Fe I). If ξ is made smaller by a factor of two the pseudo component may appear again (for weaker fields) but might be no more distinguished. Thus we came to the conclusion that the introduction of microturbulence to render high values for ξ should be highly criticized and that the analysis of line profiles in the presence of a magnetic field may be a key to the problem. High values for η_0 ($\eta_0>4$) are permissible.

Our next step was a demonstration of the equivalence of different arithmetic expressions to describe the profile of a bell and eventually also a spectral line profile. The $S-S$ and the $M-E$ models are examples. By considering quite different values

* The pseudo component should not be confused with other peculiarities. See Beckers and Schröter (1969).

for microturbulence, macroturbulence, finite width slits, instrumental profiles etc., we could get practically indistinguishable profiles with quite different values for the source function, η_0, and for ξ.

Finally we could confirm the results obtained from the simplified models by a series of computations I had made with empirical models: the BCA for the photosphere and the Henoux model for sunspots. The appearance of a pseudo component is highly related to the saturation and to the choice of the microturbulence in each model. Thus we conclude that our simple analysis with the Unno model is much more general than can be expected from such a simplified model.

We may conclude that the 'undisturbed profile' should not be considered for calibration of magnetic field measurements. But the Unno theory can be used when it is considered that:

(1) β_0 is unknown; (2) η_0 is unknown; and last but not least (3) ξ is unknown.

This theory can be used because the Stokes parameters are correctly introduced, and calibration of magnetic field measurements can be made without ad hoc fixing the line formation parameters.

A tentative attempt in this direction, observing simultaneously different lines, is described by Rayrole in this Symposium.

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Discussion

Brueckner: The 5250 line can be calculated with Henoux' spot model in a sunspot without changing η_0, ξ or g. Calculations by Olaf Moe at the Naval Research Laboratory have shown, that it is necessary to take into account the numerous molecular lines and include them into the continuous absorption coefficient. If one does so, the calculated line profile matches fairly well the observed one.

Semel: The necessity to introduce the molecular line in the computation of the line 5250.2 is evident. However your success is not in disagreement with my paper. In this paper the problem is not the possibility of a solution but the uniqueness of the solution. The profile of 5250.2 presented in this paper is merely a part of the 'mathematical exercise'. I have never tried to match observed profiles with calculated ones.