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Structure of the Fi_{24}' maximal 2-local geometry point-line collinearity graph

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Abstract

The point-line collinearity graph $\mathcal{G}$ of the maximal 2-local geometry for the largest simple Fischer group, Fi_{24}', is extensively analysed. For an arbitrary vertex a of $\mathcal{G}$, the ith-disc of a is described in detail. As a consequence, it follows that $\mathcal{G}$ has diameter 5. The collapsed adjacency matrix of $\mathcal{G}$ is given as well as accompanying computer files which contain a wealth of data about $\mathcal{G}$.

Supplementary materials are available with this article.

1. *Introduction and main results*

The elegant theory of buildings, due to Tits [**41**], provides a conceptual framework for the groups of Lie type. Also, in many respects it captures the essence of the finite groups of Lie type as exemplified by, for example, the classification of irreducible spherical buildings of rank at least three (see [**41**] and [**42**]). This success in unifying the groups of Lie type led to attempts to widen the underlying ideas of buildings by studying more general geometries for groups. Early attempts in this direction were Buekenhout [**1**, **2**], Ronan and Smith [**19**] and Ronan and Stroth [**20**]. The latter two papers focused on geometries for the sporadic finite simple groups defined via p-local subgroups (p a prime). Such geometries are usually referred to as p-local geometries and, for the sporadic simple groups, there is now a considerable literature for this species of geometry. These papers range from studying specific properties of the geometry to establishing various characterization theorems. These two aims are frequently intertwined, and the so-called point-line collinearity graph of the geometry often features in some form or other. Some of the papers which have as their major aim the uncovering of the structure of the point-line collinearity graph are Rowley [**23**], Rowley and Walker [**25**–**32**] and Segev [**35**]. Those which have other aims are Buekenhout [**3**], Buekenhout *et al.* [**4**], Hall and Shpectorov [**8**], Ivanov [**9**–**11**], Ivanov and Shpectorov [**12**–**16**], Ivanov and Wiedorn [**17**], Mason and Smith [**18**], Rowley [**21**, **22**], Rowley and Walker [**24**], Shpectorov [**36**, **37**], Smith [**38**], Stroth [**39**, **40**], Weiss and Yoshiara [**44**], Weiss [**43**] and Yoshiara [**45**]. This is only a partial list; for further references, consult the bibliographies of the above-cited papers.

The aim of the present work is to obtain a very detailed description of $\mathcal{G}$, the point-line collinearity graph of the maximal 2-local geometry for the largest simple Fischer group Fi_{24}'. Throughout this paper we let G denote Fi_{24}' and Γ the maximal 2-local geometry for G. The geometry Γ was first introduced by Ronan and Smith in [**19**]. The diagram of Γ is shown in Figure 1, where we have put the type of each object of Γ and the stabilizer (in G) of the object, respectively, above and below the node of the diagram.

Objects of type 0 will be called points and those of type 1 lines. Thus, the vertices of $\mathcal{G}$ are the points of Γ (the objects of type 0) with two distinct vertices adjacent in $\mathcal{G}$ if they are incident with a common line of Γ. We remark that the number of vertices in $\mathcal{G}$ is

$$2\,503\,413\,946\,215.$$

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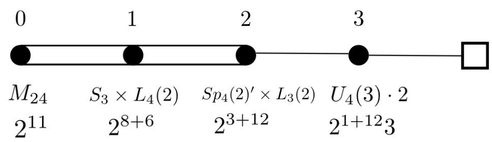

FIGURE 1. *Diagram of* Γ.

Let $d(,)$ denote the distance metric on $\mathcal{G}$. For x a vertex of $\mathcal{G}$ (that is, a point of Γ) and $i \in \mathbb{N} \cup \{0\}$, the ith disc of x is

$$\Delta_i(x) = \{y \in \mathcal{G} \mid d(x, y) = i\}.$$

Since G acts flag transitively on Γ, G induces graph automorphisms upon $\mathcal{G}$. This action is both faithful and transitive. From now on a denotes a fixed point of Γ. We can now be more specific about our goal: it is to give for each i the breakdown of $\Delta_i(a)$ into orbits under G_a and, for all $x \in \mathcal{G}$ the structure of G_{ax}, the stabilizer in G_a of x. Additionally, we will also determine the collapsed adjacency matrix of $\mathcal{G}$. An investigation along these lines into the first three discs of a (and some of the fourth disc) has been carried out in Rowley and Walker [**34**]. In this paper we shall complete this analysis for the remaining discs. In contrast to [**34**] (which is machine-free), here we shall make heavy use of the algebra package MAGMA [**5**], together with old-fashioned brain power and utilizing information from [**33**] and [**34**]. S. Linton (private communication), using structure constants and GAP [**7**], calculated that the permutation rank of G acting on the vertices of $\mathcal{G}$ is 120: so there is, be warned, a mountain of data here. The computer files containing the data from which our main theorems follow have been arranged so as to be compatible with the results in [**34**]. Moreover, these files, which are available as online supplementary material from the publisher's website, will also allow the user to navigate around $\mathcal{G}$. Further details on this will be given in § 4.

We have three main results, the first being the following theorem.

THEOREM 1.1. (i) *The diameter of* $\mathcal{G}$ *is* 5.

(ii) $|\Delta_1(a)| = 1518$ *and* $\Delta_1(a)$ *is a* G_a*-orbit.*

(iii) $|\Delta_2(a)| = 1\,560\,504$ *and* $\Delta_2(a)$ *consists of three* G_a*-orbits.*

(iv) $|\Delta_3(a)| = 1\,400\,874\,432$ *and* $\Delta_3(a)$ *consists of ten* G_a*-orbits.*

(v) $|\Delta_4(a)| = 656\,569\,113\,600$ *and* $\Delta_4(a)$ *consists of 46* G_a*-orbits.*

(vi) $|\Delta_5(a)| = 1\,845\,442\,396\,160$ *and* $\Delta_5(a)$ *consists of 59* G_a*-orbits.*

Our second theorem is the promised breakdown of each $\Delta_i(a)$ into G_a-orbits. For each representative of a G_a-orbit, say x, we present the structure of G_{ax} (in column 4 of Table 1 below), using the ATLAS [**6**] conventions in our descriptions of such groups (though we deviate in our use of $Sym(n)$, $Alt(n)$ and $Dih(n)$ for, respectively, the symmetric group of degree n, the alternating group of degree n and the dihedral group of order n). Recalling that for a vertex of $\mathcal{G}$, G_x has shape $2^{11}.M_{24}$, we use Q_x to denote the largest normal 2-subgroup of G_x. So, Q_x is elementary abelian of order 2^{11}. The fifth column of Table 1 lists the orders of $G_{ax} \cap Q_x$. One final point about Table 1 is that the transposition profile of a representative of a G_a-orbit, which appears in the third column, is with respect to a: an explanation of transposition profiles will be given in § 2. Suffice to say here that this idea plays an important role in our calculations.

THEOREM 1.2. *For* $i = 1, \ldots, 5$, $\Delta_i(a)$ *is the union of the* G_a*-orbits* $\Delta_i^j(a)$ *as detailed in Table* 1.

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TABLE 1. *The G_a-orbits.*

$\Delta_i^j(a)$	$\lvert\Delta_i^j(a)\rvert$	Transposition profile	Structure of G_{ax}	$\lvert G_{ax} \cap Q_x\rvert$
$\Delta_0^1(a)$	1	24|0|0	$2^{11}.M_{24}$	2048
$\Delta_1^1(a)$	1518	8|16|0	$2^{10}.2^4.Alt(8)$	1024
$\Delta_2^1(a)$	30 360	0|24|0	$2^9.2^6.(L_3(2) \times 3)$	512
$\Delta_2^2(a)$	170 016	4|20|0	$2^7.2^6.3.Sym(5)$	128
$\Delta_2^3(a)$	1 360 128	2|6|16	$2^5.2^4.Sym(6)$	32
$\Delta_3^1(a)$	282 624	2|0|22	$2.M_{22}.2$	2
$\Delta_3^2(a)$	566 720	0|24|0	$2^7.2^6.3.3^2.4$	128
$\Delta_3^3(a)$	1 036 288	3|21|0	$2^2.L_3(4).Sym(3)$	4
$\Delta_3^4(a)$	11 658 240	2|14|8	$2^4.2^3.(L_3(2) \times 2)$	16
$\Delta_3^5(a)$	21 762 048	2|16|6	$2.2^4.Sym(6)$	2
$\Delta_3^6(a)$	40 803 840	0|8|16	$2^3.2^2.2^4.Sym(4)$	8
$\Delta_3^7(a)$	40 803 840	0|8|16	$2^4.2^2.2^3.Sym(4)$	16
$\Delta_3^8(a)$	108 810 240	1|7|16	$2^2.2^2.2^2.3.Sym(4)$	4
$\Delta_3^9(a)$	522 289 152	1|1|22	$2^4.Alt(5)$	1
$\Delta_3^{10}(a)$	652 861 440	0|2|22	$2.2.2^3.Sym(4)$	2
$\Delta_4^1(a)$	11 658 240	0|16|8	$2^4.2^4.L_3(2)$	16
$\Delta_4^2(a)$	11 65 8240	0|16|8	$2^4.2^4.L_3(2)$	16
$\Delta_4^3(a)$	24 870 912	1|15|8	$Alt(8)$	1
$\Delta_4^4(a)$	65 286 144	0|0|24	$2.2^6.Alt(5)$	2
$\Delta_4^5(a)$	93 265 920	0|2|22	$2.2^4.L_3(2)$	2
$\Delta_4^6(a)$	93 265 920	0|2|22	$2.2^4.L_3(2)$	2
$\Delta_4^7(a)$	198 967 296	1|1|22	$Alt(7)$	1
$\Delta_4^8(a)$	217 620 480	0|8|16	$2^6.(Sym(3) \times Sym(3))$	1
$\Delta_4^9(a)$	217 620 480	0|8|16	$2^6.(Sym(3) \times Sym(3))$	1
$\Delta_4^{10}(a)$	217 620 480	0|8|16	$2^2.2^4.(Sym(3) \times Sym(3))$	4
$\Delta_4^{11}(a)$	217 620 480	0|8|16	$2^2.2^4.(Sym(3) \times Sym(3))$	4
$\Delta_4^{12}(a)$	244 823 040	0|8|16	$2^3.2^2.2^3.2^3$	8
$\Delta_4^{13}(a)$	326 430 720	0|0|24	$2.2^4.2^4.3$	2
$\Delta_4^{14}(a)$	652 861 440	0|10|14	$2.2^2.2^4.Sym(3)$	2
$\Delta_4^{15}(a)$	652 861 440	0|10|14	$2.2^2.2^4.Sym(3)$	2
$\Delta_4^{16}(a)$	746 127 360	1|9|14	$2.L_3(2).2$	2
$\Delta_4^{17}(a)$	759 693 312	1|11|12	$L_2(11)$	1
$\Delta_4^{18}(a)$	870 481 920	1|3|20	$2^6.3^2$	1
$\Delta_4^{19}(a)$	1 305 722 880	0|6|18	$2.2^5.Sym(3)$	2
$\Delta_4^{20}(a)$	1 305 722 880	0|6|18	$2.2^5.Sym(3)$	2
$\Delta_4^{21}(a)$	1 392 771 072	1|5|18	$(3 \times Alt(5)).2$	1
$\Delta_4^{22}(a)$	2 611 445 760	0|4|20	$2^2.2^3.Sym(3)$	1
$\Delta_4^{23}(a)$	2 611 445 760	0|4|20	$2^5.Sym(3)$	1
$\Delta_4^{24}(a)$	2 611 445 760	0|4|20	$2.2^4.Sym(3)$	2
$\Delta_4^{25}(a)$	3 917 168 640	0|0|24	$2^2.2^5$	1

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Table 1. *Continued.*

$\Delta_4^{26}(a)$	3 917 168 640	0|6|18	$2.2^3.2^3$	2
$\Delta_4^{27}(a)$	5 222 891 520	0|2|22	$2.2^3.Sym(3)$	1
$\Delta_4^{28}(a)$	5 222 891 520	0|6|18	$2^4.Sym(3)$	1
$\Delta_4^{29}(a)$	5 222 891 520	0|6|18	$2^4.Sym(3)$	1
$\Delta_4^{30}(a)$	5 222 891 520	0|6|18	$2.2^3.Sym(3)$	2
$\Delta_4^{31}(a)$	5 222 891 520	0|6|18	$2.2^3.Sym(3)$	2
$\Delta_4^{32}(a)$	6 963 855 360	0|0|24	$2^2.(3 \times 3).2$	1
$\Delta_4^{33}(a)$	6 963 855 360	0|0|24	$2^2.(3 \times 3).2$	1
$\Delta_4^{34}(a)$	10 445 783 040	0|2|22	$2^3.Sym(3)$	1
$\Delta_4^{35}(a)$	10 445 783 040	0|2|22	$2^3.Sym(3)$	1
$\Delta_4^{36}(a)$	10 445 783 040	0|2|22	$2^3.Sym(3)$	1
$\Delta_4^{37}(a)$	10 445 783 040	0|2|22	$2^3.Sym(3)$	1
$\Delta_4^{38}(a)$	15 668 674 560	0|2|22	$2^3.2^2$	1
$\Delta_4^{39}(a)$	15 668 674 560	0|2|22	$2^3.2^2$	1
$\Delta_4^{40}(a)$	41 783 132 160	0|2|22	$Dih(12)$	1
$\Delta_4^{41}(a)$	50 139 758 592	0|1|23	$Dih(10)$	1
$\Delta_4^{42}(a)$	50 139 758 592	0|1|23	$Dih(10)$	1
$\Delta_4^{43}(a)$	62 674 698 240	0|2|22	2^3	1
$\Delta_4^{44}(a)$	62 674 698 240	0|2|22	2^3	1
$\Delta_4^{45}(a)$	125 349 396 480	0|1|23	2^2	1
$\Delta_4^{46}(a)$	125 349 396 480	0|1|23	2^2	1
$\Delta_5^{1}(a)$	24 870 912	0|16|8	$Alt(8)$	1
$\Delta_5^{2}(a)$	24 870 912	0|16|8	$Alt(8)$	1
$\Delta_5^{3}(a)$	232 128 512	0|6|18	$3.Sym(6)$	1
$\Delta_5^{4}(a)$	232 128 512	0|6|18	$3.Sym(6)$	1
$\Delta_5^{5}(a)$	870 481 920	0|4|20	$2^4.(Sym(3) \times Sym(3))$	1
$\Delta_5^{6}(a)$	870 481 920	0|4|20	$2^4.(Sym(3) \times Sym(3))$	1
$\Delta_5^{7}(a)$	2 611 445 760	0|4|20	$2^5.Sym(3)$	1
$\Delta_5^{8}(a)$	2 611 445 760	0|4|20	$2^5.Sym(3)$	1
$\Delta_5^{9}(a)$	2 611 445 760	0|4|20	$2^5.Sym(3)$	1
$\Delta_5^{10}(a)$	2 611 445 760	0|4|20	$2^5.Sym(3)$	1
$\Delta_5^{11}(a)$	2 984 509 440	0|2|22	$L_3(2)$	1
$\Delta_5^{12}(a)$	2 984 509 440	0|2|22	$L_3(2)$	1
$\Delta_5^{13}(a)$	3 481 927 680	0|2|22	$2^2.(Sym(3) \times Sym(3))$	1
$\Delta_5^{14}(a)$	3 481 927 680	0|2|22	$2^2.(Sym(3) \times Sym(3))$	1
$\Delta_5^{15}(a)$	3 917 168 640	0|0|24	$2^2.2^5$	1
$\Delta_5^{16}(a)$	4 642 570 240	0|3|21	$3_+^{1+2}.2^2$	1
$\Delta_5^{17}(a)$	4 642 570 240	0|3|21	$3_+^{1+2}.2^2$	1
$\Delta_5^{18}(a)$	4 642 570 240	0|9|15	$3_+^{1+2}.2^2$	1
$\Delta_5^{19}(a)$	4 642 570 240	0|9|15	$3_+^{1+2}.2^2$	1
$\Delta_5^{20}(a)$	7 958 691 840	0|3|21	3.7.3	1

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Table 1. *Continued.*

$\Delta_5^{21}(a)$	8 356 626 432	0|6|18	$Alt(5)$	1
$\Delta_5^{22}(a)$	8 356 626 432	0|6|18	$Alt(5)$	1
$\Delta_5^{23}(a)$	10 445 783 040	0|6|18	$2^3.Sym(3)$	1
$\Delta_5^{24}(a)$	10 445 783 040	0|6|18	$2^3.Sym(3)$	1
$\Delta_5^{25}(a)$	10 445 783 040	0|2|22	$2^3.Sym(3)$	1
$\Delta_5^{26}(a)$	10 445 783 040	0|2|22	$2^3.Sym(3)$	1
$\Delta_5^{27}(a)$	13 927 710 720	0|4|20	$Sym(3) \times Sym(3)$	1
$\Delta_5^{28}(a)$	13 927 710 720	0|4|20	$Sym(3) \times Sym(3)$	1
$\Delta_5^{29}(a)$	13 927 710 720	0|7|17	$Sym(3) \times Sym(3)$	1
$\Delta_5^{30}(a)$	15 668 674 560	0|4|20	2^{1+4}	1
$\Delta_5^{31}(a)$	15 668 674 560	0|2|22	$2^3.2^2$	1
$\Delta_5^{32}(a)$	15 668 674 560	0|2|22	$2^3.2^2$	1
$\Delta_5^{33}(a)$	20 891 566 080	0|2|22	$Sym(4)$	1
$\Delta_5^{34}(a)$	20 891 566 080	0|2|22	$Sym(4)$	1
$\Delta_5^{35}(a)$	25 069 879 296	0|0|24	$Dih(20)$	1
$\Delta_5^{36}(a)$	41 783 132 160	0|0|24	$Dih(12)$	1
$\Delta_5^{37}(a)$	41 783 132 160	0|3|21	$Dih(12)$	1
$\Delta_5^{38}(a)$	41 783 132 160	0|3|21	$Dih(12)$	1
$\Delta_5^{39}(a)$	41 783 132 160	0|3|21	$Dih(12)$	1
$\Delta_5^{40}(a)$	41 783 132 160	0|3|21	$Dih(12)$	1
$\Delta_5^{41}(a)$	41 783 132 160	0|3|21	$Dih(12)$	1
$\Delta_5^{42}(a)$	41 783 132 160	0|3|21	$Dih(12)$	1
$\Delta_5^{43}(a)$	41 783 132 160	0|1|23	$Dih(12)$	1
$\Delta_5^{44}(a)$	41 783 132 160	0|1|23	$Dih(12)$	1
$\Delta_5^{45}(a)$	41 783 132 160	0|1|23	$Dih(12)$	1
$\Delta_5^{46}(a)$	41 783 132 160	0|1|23	$Dih(12)$	1
$\Delta_5^{47}(a)$	50 139 758 592	0|1|23	$Dih(10)$	1
$\Delta_5^{48}(a)$	62 674 698 240	0|0|24	2×4	1
$\Delta_5^{49}(a)$	62 674 698 240	0|4|20	2×4	1
$\Delta_5^{50}(a)$	62 674 698 240	0|4|20	2×4	1
$\Delta_5^{51}(a)$	62 674 698 240	0|2|22	2×4	1
$\Delta_5^{52}(a)$	62 674 698 240	0|2|22	2×4	1
$\Delta_5^{53}(a)$	62 674 698 240	0|2|22	2×4	1
$\Delta_5^{54}(a)$	62 674 698 240	0|2|22	2×4	1
$\Delta_5^{55}(a)$	83 566 264 320	0|0|24	6	1
$\Delta_5^{56}(a)$	83 566 264 320	0|1|23	$Sym(3)$	1
$\Delta_5^{57}(a)$	83 566 264 320	0|1|23	$Sym(3)$	1
$\Delta_5^{58}(a)$	125 349 396 480	0|3|21	2^2	1
$\Delta_5^{59}(a)$	250 698 792 960	0|1|23	2	1

Our final main result is the collapsed adjacency matrix of $\mathcal{G}$. Since this is a 120×120 matrix, it has been given § 5 all to itself.

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In the course of compiling the information given above, we obtain specific representatives a_i^j for each G_a-orbit $\Delta_i^j(a)$ along with the set of neighbours of a_i^j in $\mathcal{G}$. As mentioned earlier, this data is gathered in files (reps_for_all_discs.m, NeighbourData.m and CollapsedAdjacencyMatrix.m) so as to facilitate further study of $\mathcal{G}$. The manner in which these files are arranged is described in § 4. Details of other files and routines used in our analysis of $\mathcal{G}$ are also to be found in § 4. In § 2, we introduce notation and elaborate upon our overall strategy in studying $\mathcal{G}$. Section 3 is devoted to discussing the specific details of how we uncovered the G_a-orbits and the adjacencies between these orbits, particularly giving an account of the machine–brain interface.

2. *Background results and notation*

We shall use F to denote the Fischer group Fi_{24}. Then $F' \cong G$ and $[F : G] = 2$. Now, as mentioned in § 1, for x a point in Γ, $G_x \sim 2^{11}M_{24}$ ($\sim$ means 'has the same shape as'). Famously, F possesses a conjugacy class of involutions, $\mathcal{T}$, usually referred to as a class of 3-transpositions. This means that for $t, s \in \mathcal{T}$, the order of ts is $1, 2$ or 3. A maximal set of mutually commuting transpositions is called a base. It is a fact that for any base $\mathcal{B}$, $|\mathcal{B}| = 24$ and any two bases are conjugate in F. Further, the stabilizer in F (respectively in G) of $\mathcal{B}$ has shape $2^{12}M_{24}$ (respectively $2^{11}M_{24}$). Since there is only one conjugacy class of subgroups in G of shape $2^{11}M_{24}$, we may identify each point of Γ with a base in F. (For the previously stated facts about Fi_{24}, see [**6**].) This identification of the points of Γ (vertices of $\mathcal{G}$) with bases is key to this work. In order to do this, of course, we must work in F. Since F, acting by conjugation on $\mathcal{T}$, has a permutation representation of degree 306 936, this enables us to carry out our calculations in $Sym(306\,936)$ rather than in $Sym(2\,503\,413\,946\,215)$. As preparation for this enterprise, in § 4 we assemble a file *Fi24perms.m* giving F as a subgroup of $Sym(306\,936)$, together with other relevant subgroups.

We must now introduce some further notation. First we shall employ the standard notation for geometries as found, for example, in Buekenhout [**2**]. So, $I = \{0, 1, 2, 3\}$ is the set of types for Γ and we use $\star$ to denote incidence in Γ. For $i \in I$, Γ_i is the set of all objects of Γ of type i. The residue of x, where $x \in \Gamma$, will be denoted by Γ_x and, we recall,

$$\Gamma_x = \{y \in \Gamma \mid x * y\}.$$

Now let x be a point of Γ (so $x \in \Gamma_0$). The base identified with x will, henceforth, be denoted by Ω_x. So, $|\Omega_x| = 24$ and $G_x \sim 2^{11}M_{24}$ acts upon Ω_x with the induced action being the standard action of M_{24} on a 24-element set. We remark here that Q_x is the kernel of this induced action as well as that of the induced action upon Γ_x. From this point of view, the lines incident with x may be identified with the octads of Ω_x. In fact, the octads of Ω_x are precisely the subsets of Ω_x of size 8 whose product in F is the identity (see [**6**]). So, $\mathcal{G}$ may be described as the graph whose vertices are $\{\Omega_x | x \in \Gamma_0\}$ with Ω_x and Ω_y adjacent if and only if $\Omega_x \cap \Omega_y$ is an octad of either Ω_x or Ω_y (and hence is an octad in both Ω_x and Ω_y). We will frequently think of $\mathcal{G}$ in this way.

Let $x \in \Gamma_0$ and $s \in \mathcal{T}$. Then either $s \in \Omega_x$ or s centralizes (the transpositions) in an octad of Ω_x or s centralizes (exactly) a duad of transpositions in Ω_x (again consult [**6**]). If the second possibility, respectively third, holds we call s an octadic transposition (with respect to Ω_x or x), respectively a duadic transposition (with respect to Ω_x or x). Let $\mathcal{O}_x$ and $\mathcal{D}_x$ denote the sets of octadic and duadic transpositions (with respect to x). The orbits of G_x on $\mathcal{T}$ are Ω_x, $\mathcal{O}_x$ and $\mathcal{D}_x$ (again see [**6**]). These orbits will play an important role in our exploration of $\mathcal{G}$; of particular importance is the idea of a transposition profile. For $y \in \Gamma$, put $\ell_1 = |\Omega_y \cap \Omega_x|$, $\ell_2 = |\Omega_y \cap \mathcal{O}_x|$ and $\ell_3 = |\Omega_y \cap \mathcal{D}_x|$. Then $\ell_1|\ell_2|\ell_3$ will be referred to as the transposition profile of y (or Ω_y) with respect to x (or Ω_x). Clearly, if two points y_1 and y_2 are in the same G_x-orbit,

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then Ω_{y_1} and Ω_{y_2} will have the same transposition profile with respect to x (or Ω_x). However, the converse is very far from being true, as a perusal of Table 1 will reveal. For example, points in $\Delta_3^9(x)$ and $\Delta_4^7(x)$ both have transposition profile $1|1|22$ with respect to x.

For the moment, fix $t \in \mathcal{T}$. Let Γ_0^t denote the set of all points x in Γ_0 for which $t \in \Omega_x$. The set Γ_0^t turns out to be the points of a geometry Γ^t for Fi_{23} (the second largest Fischer group). For more information on Γ^t, such as the other objects in Γ^t, consult [**34**, § 2]. The point-line collinearity graph, $\mathcal{G}^t$, of Γ^t has been extensively analysed in [**32**]: this information resource will feed into the current work as two vertices of $\mathcal{G}$ which are in $\mathcal{G}^t$ are adjacent in $\mathcal{G}$ if and only if they are adjacent in $\mathcal{G}^t$. One specific fact we note here is the following theorem.

THEOREM 2.1. *Let $t \in \mathcal{T}$. Then the diameter of $\mathcal{G}^t$ is* 4.

Proof. See [**32**, Theorem 1]. □

As an immediate consequence of Theorem 2.1, we have the following lemma.

LEMMA 2.2. *Let $x \in \Gamma_0$. If $\Omega_a \cap \Omega_x \neq \emptyset$, then $d(a, x) \leqslant 4$.*

LEMMA 2.3. (i) *If $x \in \Delta_1(a)$, then Ω_x has transposition profile with respect to a of* $8|16|0$.
(ii) *If $x \in \Delta_2^2(a)$, then Ω_x has transposition profile with respect to a of* $2|6|16$.

Proof. See [**34**] or Table 1. □

LEMMA 2.4. (i) *Let $x \in \Gamma_0$ be such that $\Omega_x \cap \mathcal{O}_a \neq \emptyset$. Then $d(a, x) \leqslant 5$.*
(ii) *Let $x \in \Gamma_0$ be such that $\Omega_x \cap \mathcal{D}_a \neq \emptyset$. Then $d(a, x) \leqslant 6$.*

Proof. Suppose that $x \in \Gamma_0$ and $\Omega_x \cap \mathcal{O}_a \neq \emptyset$. Let $t \in \Omega_x \cap \mathcal{O}_a$. Select $b \in \Delta_1(a)$. Then, by Lemma 2.3(i), there exists $s \in \Omega_b \cap \mathcal{O}_a$. Since $\mathcal{O}_a$ is a G_a-orbit, there exists $g \in G_a$ such that $s^g = t$. So, $b^g \in \Delta_1(a)$ and $t \in \Omega_b^g \cap \Omega_x = \Omega_{b^g} \cap \Omega_x$. Hence, $d(b^g, x) \leqslant 4$ by Lemma 2.2. Consequently, $d(a, x) \leqslant 5$, so proving part (i). Part (ii) may be proved similarly, but using Lemma 2.3(ii) in place of Lemma 2.3(i). □

Together Lemmas 2.2 and 2.4 imply that the diameter of $\mathcal{G}$ is at most 6. A quick calculation, using the sizes of $\Delta_1(a), \Delta_2(a)$ and $\Delta_3(a)$ given in [**34**], shows that the diameter is greater than 4. So, at an early stage of our campaign we already know that $\mathcal{G}$ has diameter 5 or 6.

3. *Determining the discs of a*

We will work in the permutation representation for $F \cong Fi_{24}$ of degree 306 936, which arises from the conjugation action of F on its transpositions. This representation is given explicitly in the following section together with subgroups of F, respectively G, of shape $2^{12}.M_{24}$, respectively $2^{11}.M_{24}$, which are named Fa, respectively Ga, though we shall use F_a and G_a in this section. Since there is only one conjugacy class of subgroups of these shapes, they must be the stabilizer of some base in F, respectively G. By asking MAGMA for the orbits of G_a upon the transpositions, we can determine the base Ω_a, the orbit of length 24, and $\mathcal{O}_a$, $\mathcal{D}_a$, the octadic and duadic transpositions (the octadic transpositions being the second smallest of the orbits).

Within our representation we have an element, called a_{10} (the tenth generator of our Fi_{24}), which takes the base Ω_a to the base Ω_b, where a and b are adjacent in $\mathcal{G}$. Now, for any $x \in \Gamma_0$

and octad X of the base Ω_x, there are (exactly) two further points of Γ, say y, y', such that

$$X = \Omega_x \cap \Omega_y = \Omega_x \cap \Omega_{y'} = \Omega_y \cap \Omega_{y'}.$$

(The octad X corresponds to a line m of Γ with x, y, y' being the three points of Γ collinear with m, and any two of x, y and y' determines m uniquely.) Let a, b, b' be the three points of the line determined by a and b. Set $O = \Omega_a \cap \Omega_b$ $(= \Omega_a \cap \Omega_{b'} = \Omega_b \cap \Omega_{b'})$, and let ℓ be the line corresponding to O. In the file Fi24perms.m, we define a group element, going by the sobriquet, 'twiddle', which stabilizes Ω_a and interchanges Ω_b and $\Omega_{b'}$. (So, twiddle corresponds to $\tau(Z)$, Z a hyperplane; see [**34**, § 4].) We have also defined subgroups of shape $2^{12}.2^4 Alt(8)$ (respectively $2^{11}.2^4 Alt(8)$), named $Fa\ell$ (respectively $Ga\ell$) which are the stabilizer in F (respectively G) of both Ω_a and this octad O. We have created a sequence (stored as words in the generators) giving a transversal for $Ga\ell = G_{a\ell}$ in G_a (which is also a transversal for $Fa\ell = F_{a\ell}$ in F_a), and is called *Tran*.

Using these objects, we can calculate all the neighbours of Ω_a, that is, $\Delta_1(a)$, and all the octads of Ω_a, which we will call *Octadsa*. Thus, all the octads of Ω_a are given by

$$\mathit{Octadsa} = \{O^h \mid h = \mathit{Tran}[i], 1 \leqslant i \leqslant 759\}$$

and, for O^h, where $h = \mathit{Tran}[i]$, we refer to i as the octad number of O^h. We also have

$$\begin{aligned}\Delta_1(a) &= \{\Omega_b^h \mid h \in \mathit{Tran}\} \cup \{\Omega_b^{(twiddle*h)} \mid h \in \mathit{Tran}\} \\ &= \{\Omega_a^{(a_{10}*h)} \mid h \in \mathit{Tran}\} \cup \{\Omega_a^{(a_{10}*twiddle*h)} \mid h \in \mathit{Tran}\}.\end{aligned}$$

As we create new G_a-orbits in $\mathcal{G}$, we will want to store a representative Ω_x of these orbits. We will do this by storing the group element that takes us from the base Ω_a to Ω_x. Since storing these as permutations would take a lot of memory, we store them as words in the generators of Fi_{24}. These words will simply be stored as an array in Magma, and there are functions given in the next section which can be used to convert these words into actual permutations. For example, for the first disc $\Delta_1(a)$, which we know from [**34**] is a G_a orbit, we just store the word $[a10]$.

As it is our aim to make the results here compatible with those in [**34**], we proceed as follows in determining the second and third discs of a. From [**34**], we know that $\Delta_2(a)$ is made up of three G_a-orbits while $\Delta_3(a)$ consists of ten G_a-orbits. Now $Neighboursa^{a10}$ gives all the 1518 neighbours of $b \in \Delta_1(a)$ (see § 4). Knowing (as we can deduce from [**34**]) the transposition profiles for points in $\Delta_2^1(a), \Delta_2^2(a)$ and $\Delta_2^3(a)$, we can choose representatives for these orbits. Now one feature of the results in [**34**] is the determination of the so-called point-line distribution for a point x in Γ_0 and ℓ a line in Γ_x. Clearly, it suffices to provide this for ℓ running through orbit representatives of lines in Γ_x (or octads in Ω_x) under G_{ax}. So, for each of the three G_a-orbit representatives, say a_{21}, a_{22} and a_{23}, we repeat the above routine. That is, if $a_{2i} = a^{g_i}$ $(g_i \in G)$, $i = 1, 2, 3$, we investigate $Neighboursa^{g_i}$, fishing out representatives for the ten G_a-orbits in $\Delta_3(a)$. As we select representatives x for $\Delta_2(a)$ and $\Delta_3(a)$, we keep track of the G_{ax}-orbits on the lines in Γ_x. In examining $\Delta_3(a)$, we have two G_a-orbits which have the same transposition profile (both $\Delta_3^6(a)$ and $\Delta_3^7(a)$ have profile 0|8|16) and one G_a-orbit has the same profile as a G_a-orbit in $\Delta_2(a)$. The latter can be easily settled, as we can distinguish a third disc point from a second disc point by checking whether it is a neighbour of a point in $\Delta_1(a)$. As $\Delta_1(a)$ is relatively small, this is computationally easy. To differentiate between the two G_a-orbits in $\Delta_3(a)$ with profile 0|8|16, we use the fact that for $x_1 \in \Delta_3^6(a)$ and $x_2 \in \Delta_3^7(a)$, there exists $x_3 \in \Delta_1(a)$ for which $|\Omega_{x_1} \cap \Omega_{x_3}| = 2$ whereas for all $y \in \Delta_1(a)$, $|\Omega_{x_1} \cap \Omega_y| \neq 2$. The correspondence between the G_a-orbits in $\Delta_2(a)$ and $\Delta_3(a)$ in [**34**] and here is tabulated in an Appendix.

Moving out from $\Delta_3(a)$ to $\Delta_4(a)$, we use the combinatorial data in [33] to find octad numbers for each G_{ax}-orbit representative $x \in \Delta_3(a)$ and each octad orbit of G_{ax} upon the octads of Ω_x. For such an octad orbit representative X, there will exist $y_1, y_2 \in \Gamma_0$ such that $\Omega_x \cap \Omega_{y_1} = \Omega_x \cap \Omega_{y_2} = \Omega_{y_1} \cap \Omega_{y_2} = X$. Now

$$\Omega_{y_1} = \Omega_a^{(a_{10}*h*g)} \text{ and}$$
$$\Omega_{y_2} = \Omega_a^{(a_{10}*twiddle*h*g)},$$

where $g \in G$ is such that $a^g = x$ and $h = Tran[i]$ for some $i \in \{1, \ldots, 759\}$. (So i is the octad number of X.) As we run through all G_{ax} representatives for $\Delta_3(a)$ and all G_{ax}-orbits of octads of Ω_x, accumulating the bases Ω_{y_1} and Ω_{y_2}, we obtain G_a representatives for all the points of $\mathcal{G}$ which are distance 1 from some point in $\Delta_3(a)$. Of course, some of these will be in $\Delta_2(a) \cup \Delta_3(a)$. From [33], we know that, up to a few easy exceptions, the profiles in $\Delta_2(a) \cup \Delta_3(a)$ are unique and so may be crossed off our list, leaving only points in $\Delta_4(a)$. Now, of those that remain, some may be in the same G_a-orbit. Our next step is to use transposition profiles as an initial sieve: and this is where transposition profiles are very important in speeding up our calculations. Then the MAGMA command `IsConjugate`$(G_a, \Omega_y, \Omega_z)$ is deployed to settle matters; we note that G_a is small enough for this command to produce an answer in approximately 7 seconds (on a 3.2GHz 8 GB memory machine). By removing duplicates in this manner, we end up with a complete list of G_a-orbits for $\Delta_4(a)$ together with representatives.

For each of the orbits in the fourth disc there are far too many octad orbits to give their combinatorial structure, so we use the following algorithm to get the required octad numbers for each $\Delta_4^j(a)$.

(1) For y, the representative for $\Delta_4^j(a)$, calculate $\mathcal{O}_y$, the octads of y.
(2) Choose $O \in \mathcal{O}_y$, note its octad number and calculate $H = Stab_{G_{ay}}(O)$.
(3) Calculate T, the transversal for H in $Stab_{ay}$; then $\{O^t \mid t \in T\}$ will be an octad orbit for $\mathcal{O}_y$.
(4) Let $\mathcal{O}_y = \mathcal{O}_y \setminus \{O^t \mid t \in T\}$ and go to (2).

Repeating this process for all orbits in $\Delta_4(a)$ creates orbit representatives (just as we did for the third disc), by crossing off any repetitions and anything in the third and fourth discs using transposition profiles (which again are crucial in making the calculations manageable) and `IsConjugate`(G, Ω_x, Ω_y). This gives a full list of representatives for the orbits of $\Delta_5(a)$, at which point we observe that all points of Γ_0 are accounted for (and we also have 120 G_a-orbits). Thus, we have finished.

A function called `WhereAmI` was created to determine which orbit a particular base Ω_x was in. This was done in the obvious way, by first cutting down the possibilities using transposition profiles, and then using `IsConjugate`$(G_a, \Omega_x, \Omega_y)$ (y running through the remaining G_a-orbit representatives). By using this command on each of the 1518 neighbours of each orbit representative, we can obtain the complete neighbour data for $\mathcal{G}$, and thus the collapsed adjacency matrix for $\mathcal{G}$, which is given in § 5.

4. *Computer files*

This section is a roll call of the files involved in our investigation of $\mathcal{G}$, and those which record the various structural features of $\mathcal{G}$ that are uncovered. Before itemizing and discussing these files, we give part of the code (in the file Fi24perms.m) which produces F as a subgroup of $Sym(306\,936)$ (we note that the presentation used here is based on a Y-type diagram; see [6]).

```
F<a,b1,c1,d1,e1,f1,b2,c2,d2,e2,b3,c3> := FreeGroup(12);
Rels:={a^2=Id(F),b1^2=Id(F),c1^2=Id(F),d1^2=Id(F),e1^2=Id(F),
      f1^2=Id(F),b2^2=Id(F),c2^2=Id(F),d2^2=Id(F),e2^2=Id(F),
      b3^2=Id(F),c3^2=Id(F),
```

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```
        (a*b1)^3=Id(F),(a*c1)^2=Id(F),(a*d1)^2=Id(F),(a*e1)^2=Id(F),
        (a*b2)^3=Id(F),(a*c2)^2=Id(F),(a*d2)^2=Id(F),(a*e2)^2=Id(F),
        (a*b3)^3=Id(F),(a*c3)^2=Id(F),(b1*c1)^3=Id(F),(b1*d1)^2=Id(F),
        (b1*e1)^2=Id(F),(b1*b2)^2=Id(F),(b1*c2)^2=Id(F),(b1*d2)^2=Id(F),
        (b1*e2)^2=Id(F),(b1*b3)^2=Id(F),(b1*c3)^2=Id(F),(c1*d1)^3=Id(F),
        (c1*e1)^2=Id(F),(c1*b2)^2=Id(F),(c1*c2)^2=Id(F),(c1*d2)^2=Id(F),
        (c1*e2)^2=Id(F),(c1*b3)^2=Id(F),(c1*c3)^2=Id(F),(d1*e1)^3=Id(F),
        (d1*b2)^2=Id(F),(d1*c2)^2=Id(F),(d1*d2)^2=Id(F),(d1*e2)^2=Id(F),
        (d1*b3)^2=Id(F),(d1*c3)^2=Id(F),(e1*b2)^2=Id(F),(e1*c2)^2=Id(F),
        (e1*d2)^2=Id(F),(e1*e2)^2=Id(F),(e1*b3)^2=Id(F),(e1*c3)^2=Id(F),
        (b2*c2)^3=Id(F),(b2*d2)^2=Id(F),(b2*e2)^2=Id(F),(b2*b3)^2=Id(F),
        (b2*c3)^2=Id(F),(c2*d2)^3=Id(F),(c2*e2)^2=Id(F),(c2*b3)^2=Id(F),
        (c2*c3)^2=Id(F),(d2*e2)^3=Id(F),(d2*b3)^2=Id(F),(d2*c3)^2=Id(F),
        (e2*b3)^2=Id(F),(e2*c3)^2=Id(F),(b3*c3)^3=Id(F),
        (a*b1*c1*a*b2*c2*a*b3*c3)^10=Id(F),
        (f1*e1)^3=Id(F),(f1*d1)^2=Id(F),(f1*c1)^2=Id(F),(f1*b1)^2=Id(F),
        (f1*a)^2=Id(F),(f1*b2)^2=Id(F),(f1*c2)^2=Id(F),(f1*d2)^2=Id(F),
        (f1*e2)^2=Id(F),(f1*b3)^2=Id(F),(f1*c3)^2=Id(F),
        f1=(a*b1*c1*d1*b2*c2*b3)^9,f1=(a*b1*c1*d1*b2*b3*c3)^9};
Y442 := quo<Fr|Rels>;
S:={a,b1,c1,d1,e1,f1,b2,c2,d2,b3,c3,
   (a*b1*c1*d1*e1*f1*a*b2*c2*d2*e2*a*b3*c3)^17};
H:=sub<Y442|S>;
m, F := CosetAction(Y442,H);
g1 := m(f1);
g2 := m((f1*d1)^e1);
g3 := m((d1*b1)^c1);
g4 := m((b1*b2)^a);
g5 := m((b2*d2)^c2);
f2 := (a*b2*c2*d2*b1*c1*b3)^9;
g6 := m((d2*f2)^e2);
g7 := m((b1*b3)^a);
g8 := m((b2*b3)^a);
g9 := m((b1*a*b2*b3*c3)^4);
Fa := sub<F|g1,g2,g3,g4,g5,g6,g7,g8,g9>;

a := Orbit(Fa,1);
b := a^F.10;
Fal := sub<F|g1,g2,g3,g4,g6,g7,g8,g9,g1^g5,g2^g5,g3^g5,g7^g5,
          g1^(g2*g5),g1^(g2*g3*g5),g1^(g3*g5),g1^(g4,g5),
          g1^(g2*g4*g5),g1^(g2*g3*g4*g5),g1^(g3*g4*g5)>;
```

We recall that $Y_{542} = Y_{442} \cong 3^{\cdot}Fi_{24}$ (so $Y422 \cong 3^{\cdot}Fi_{24}$); see [6]. The above code was run only once so as to produce permutation generators for F. The easiest way to use the computer files when investigating $\mathcal{G}$ is to load the file Fi24load.m into MAGMA. This file will load all the relevant files that are needed; a synopsis of these files is given below.

Fi24perms.m

This file contains the following.

(1) Generators $a1, \ldots, a12$ of $F \cong Fi_{24}$ stored as permutations in $Sym(306\,936)$.
(2) Commands to define $G \cong Fi'_{24}$.

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(3) Generators $g1, \ldots, g9$, again stored as permutations in $Sym(306\,936)$, which generate Fa of shape $2^{12\cdot}M_{24}$. It is the stabilizer of the base Ω_a in F, which corresponds to the fixed vertex a of $\mathcal{G}$. Also, there are commands to calculate Ga of shape $2^{11\cdot}M_{24}$, which is the stabilizer of Ω_a in G.

(4) In addition to Ω_a, we store $\mathcal{O}_a$ (the octadic transpositions with respect to a) as $OctTran$. When calculating the transposition profile of Ω_x (with respect to a), we determine $\ell_1 = |\Omega_a \cap \Omega_x|$ and $\ell_2 = |\mathcal{O}_a \cap \Omega_x|$, and then Ω_x has profile $(\ell_1, \ell_2, (24 - \ell_1 - \ell_2))$. Therefore, there is no need to store $\mathcal{D}_a$.

(5) Contains words in the generators $g1, \ldots, g9$, which generate $Fa\ell$, respectively $Ga\ell$, of shape $2^{12}.2^4 Alt(8)$, respectively $2^{11}.2^4 Alt(8)$. These subgroups are the stabilizer in Fa, respectively Ga, of the line ℓ (ℓ corresponding to the octad O, as described in § 3).

(6) An array called $Neighboursa$, giving all 1518 neighbours of the fixed point a, corresponding to $\Delta_1(a) = \Delta_1^1(a)$. For a base Ω_x such that $\Omega_x = \Omega_a^g$ for some $g \in F$, if we wish to have the neighbours of x, we simply calculate $Neighboursa^g$.

(7) Contains a word for the element twiddle. This element is in $Ga\ell$ and interchanges Ω_b and Ω_b', where a, b, b' are the points incident with the line ℓ.

reps_for_all_discs.m

(1) Contains all 120 words (in generators of F) for the representatives of the discs around a. The representatives are stored as arrays named $DisciOrbitj$, where $\Delta_i^j(a)$ is the orbit in question. One must use the procedure `MultiplyRandomWord`, described below, to convert these arrays into usable group elements.

(2) Contains arrays $Disci$ for $i = 0, \ldots, 5$ giving all words in that disc. These arrays are useful if you need to run a loop over all representatives for a certain disc.

(3) Also contains an array $Orbits$, which is a concatenation of the arrays described in (2).

MultiplyRandomWord.m

Contains a procedure used to convert a word into a usable permutation of degree 306 936. To use, type

`MultiplyRandomWord(~z, Disc4Orbit23, F)`

to convert (for example) the word for the representative of $\Delta_4^{23}(a)$ into a permutation labelled z. This is a procedure, so $\sim$ is necessary and the output of the procedure is assigned to z.

Tran.m

Contains a transversal in the form of an array named *Tran* (stored as words in generators for F) for $Ga\ell$ in Ga. Again one needs to use `MultiplyRandomWord` to convert these words into usable permutations.

We remark on the fact that we did not just directly use the Magma function `Transversal(`Ga`,`$Ga\ell$`)` to determine a transversal for $Ga\ell$ in Ga (as permutations in $Sym(306\,936)$). The reasons behind this are:

(1) to make sure that we get the same coset representatives every time, so we can reproduce results;

(2) to make the transversal easier to store. Storing it as words in generators instead of permutations reduced the space needed to store the transversal from about 1.5 GB to 70 KB;

(3) so, we know exactly what element we are looking at. Instead of having to print a huge permutation to the screen, we can just look at a short(ish) word in at most nine generators.

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We now look at how the transversal in Tran.m was constructed.

(1) Recall that the base Ω_a is the 24-element orbit of Ga on Ω, where $\Omega := \{1 \dots 306\,936\}$.
(2) We calculate the action of each of the generators g_i upon Ω_a. These permutations $\overline{g1}, \dots, \overline{g9}$ will generate a subgroup, $\overline{Ga}$, of $Sym(24)$ with $\overline{Ga} \cong M_{24}$.
(3) Now consider the generators for $Ga\ell$; these are words in our old generators $g1, \dots, g9$. If we convert these words into words in our new generators $\overline{g1}, \dots, \overline{g9}$, we can construct a subgroup of $\overline{Ga}$ equal to the image of $Ga\ell$ and of shape $2^4 Alt(8)$. Call this subgroup $\overline{Ga\ell}$.
(4) By generating random words in $\overline{Ga}$, find elements in each of the 759 cosets of $\overline{Ga\ell}$ in $\overline{Ga}$. The easiest way to do this is to generate a transversal using the Magma function `Transversal(`$\overline{Ga}$`,`$\overline{Ga\ell}$`)` and then check which coset each of the random words is in. In that way every time we find an element in a new coset, we can strike that coset off a list and we do not need to recheck it, speeding up the process.
(5) Now, converting the words in our transversal, which are words in the generators $\overline{g1}, \dots, \overline{g9}$, back into words in our old generators $g1 \dots g9$, we have a transversal for $Ga\ell$ in Ga, as required.

TransProfile.m

Contains a function $TransProfile(x)$ outputting the transposition profile for a base Ω_x.

Octadsa.m

Gives all 759 octads for the base Ω_a stored in an array named $Octadsa$. To get octads for a base Ω_x such that $\Omega_x = \Omega_a^g$ for some $g \in F$, we calculate $Octadsx = Octadsa^g$.

IsDistance3.m

Contains a function $IsDistance3(g)$ which determines whether a base $\Omega_x = \Omega_a^g$ is in $\Delta_1(a) \cup \Delta_2(a) \cup \Delta_3(a)$. In this case it will output the exact orbit the base Ω_x is in as an array $[i, j]$ corresponding to the orbit $\Delta_i^j(a)$. If the base is not in $\Delta_1(a) \cup \Delta_2(a) \cup \Delta_3(a)$, then the function will output $[0, 0]$.

WhereAmI.m

Contains a function `WhereAmI(g)` which determines which orbit, as an ordered pair $[i, j]$ corresponding to the orbit $\Delta_i^j(a)$, the base $\Omega_x = \Omega_a^g$ lies in.

CollapsedAdjacencyMatrix.m

(1) Gives the collapsed adjacency matrix for $\mathcal{G}$ stored as an array (of arrays) called CollapsedAdjacencyMatrix. To find the number of elements of the jth disc joined to a given element in the ith orbit (where $1 \leqslant i, j \leqslant 120$), type

`CollapsedAdjacencyMatrix[i][j].`

(2) This file also contains functions $NumberToName$ and $NameToNumber$. The first converts an orbit number into its name (given as an array $[i, j]$ corresponding to $\Delta_i^j(a)$)

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and the other converts a orbit name to its number. Hence (for example), if you want the number of elements in $\Delta_3^6(a)$ joined to a given element of $\Delta_2^3(a)$, you would type

$$\texttt{CollapsedAdjacencyMatrix}[\texttt{NameToNumber}([2,3])][\texttt{NameToNumber}([3,6])]$$

and you should get 60.

NeighbourData.m

Contains an array named $NeighbourData$ which gives information on the neighbours of each of the 120 representatives of the orbits of $\mathcal{G}$. For the kth representative (use $NameToNumber$ to determine what k is), $NeighbourData[k]$ gives an array of length 1518 listing the location of each of the 1518 neighbours. For example,

$$\texttt{NeighbourData}[4][500]$$

tells us in which orbit the 500th neighbour of $\Delta_2^2(a)$ belongs, and it is $\Delta_3^8(a)$. The function outputs an array $[i,j]$ corresponding to $\Delta_i^j(a)$.

Qa.m

Contains generators for Q_a, the normal elementary abelian subgroup of G_a of order 2^{11}.

5. *The collapsed adjacency matrix of $\mathcal{G}$*

In this section we display the collapsed adjacency matrix for $\mathcal{G}$. As this matrix is rather large, it is spread across a number of pages. Hence, we first give a grid to enable the matrix to be reconstructed. The one page for which all entries are equal to 0 is, of course, omitted. The entry, say d, of the collapsed adjacency matrix whose row is indexed by $\Delta_j^i(a)$ and column by $\Delta_m^\ell(a)$ tells us that a fixed point in the G_a-orbit $\Delta_j^i(a)$ is joined to exactly d points in the G_a-orbit $\Delta_m^\ell(a)$. So, for example, if $x \in \Delta_5^{20}(a)$, then x joins to three points in $\Delta_4^{16}(a)$, 21 points in $\Delta_4^{24}(a)$, 63 points in $\Delta_4^{40}(a)$ and 126 points in $\Delta_4^{45}(a)$ (information gleaned from pages 141–144). Continuing (on pages 144–148), x joins to 126 points in $\Delta_4^{46}(a)$, 24 points in $\Delta_5^{20}(a)$, 42 points in each of $\Delta_5^{21}(a)$ and $\Delta_5^{22}(a)$, 21 points in each of $\Delta_5^{33}(a)$ and $\Delta_5^{34}(a)$, 84 points in each of $\Delta_5^{37}(a)$ and $\Delta_5^{38}(a)$, 21 points in each of $\Delta_5^{39}(a)$ and $\Delta_5^{40}(a)$, 63 points in each of $\Delta_5^{41}(a)$, $\Delta_5^{42}(a)$, $\Delta_5^{49}(a)$, $\Delta_5^{50}(a)$, $\Delta_5^{51}(a)$, $\Delta_5^{52}(a)$, $\Delta_5^{56}(a)$, $\Delta_5^{57}(a)$ and $\Delta_5^{58}(a)$ and 252 points in $\Delta_5^{59}(a)$.

We remark here on the ordering chosen for the G_a-orbits. Within each disc of a, the G_a-orbits are ordered according to their size, the smallest coming earlier in the ordering. In the cases where we have, say, $\Delta_i^j(a)$ and $\Delta_i^{j+1}(a)$ with $|\Delta_i^j(a)| = |\Delta_i^{j+1}(a)|$, then the order of the superscripts indicates that for representatives x and y of, respectively, $\Delta_i^j(a)$ and $\Delta_i^{j+1}(a)$, we have

$$|G_{ax}Q_x/Q_x| \geqslant |G_{ay}Q_y/Q_y|.$$

That is, G_{ax} induces a group upon Γ_x of order greater than or equal to that which G_{ay} induces upon Γ_y. This accords in spirit with the ordering for conjugacy classes used in the ATLAS [6].

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	Δ_0^1 to Δ_3^7		Δ_3^8 to Δ_4^9		Δ_4^{10} to Δ_4^{21}		Δ_4^{22} to Δ_4^{33}		Δ_4^{34} to Δ_4^{45}
Δ_0^1 to Δ_4^{25}	Page 119	Δ_0^1 to Δ_4^{25}	Page 120	Δ_0^1 to Δ_4^{25}	Page 121	Δ_0^1 to Δ_4^{25}	Page 122	Δ_0^1 to Δ_4^{25}	Page 123
	Δ_0^1 to Δ_3^7		Δ_3^8 to Δ_4^9		Δ_4^{10} to Δ_4^{21}		Δ_4^{22} to Δ_4^{33}		Δ_4^{34} to Δ_4^{45}
Δ_4^{26} to Δ_5^{19}	Page 129	Δ_4^{26} to Δ_5^{19}	Page 130	Δ_4^{26} to Δ_5^{19}	Page 131	Δ_4^{26} to Δ_5^{19}	Page 132	Δ_4^{26} to Δ_5^{19}	Page 133
	Δ_0^1 to Δ_3^7		Δ_3^8 to Δ_4^9		Δ_4^{10} to Δ_4^{21}		Δ_4^{22} to Δ_4^{33}		Δ_4^{34} to Δ_4^{45}
Δ_5^{20} to Δ_5^{59}	All zero	Δ_5^{20} to Δ_5^{59}	Page 140	Δ_5^{20} to Δ_5^{59}	Page 141	Δ_5^{20} to Δ_5^{59}	Page 142	Δ_5^{20} to Δ_5^{59}	Page 143
	Δ_4^{46} to Δ_5^{11}		Δ_5^{12} to Δ_5^{23}		Δ_5^{24} to Δ_5^{35}		Δ_5^{36} to Δ_5^{47}		Δ_5^{48} to Δ_5^{59}
Δ_0^1 to Δ_4^{25}	Page 124	Δ_0^1 to Δ_4^{25}	Page 125	Δ_0^1 to Δ_4^{25}	Page 126	Δ_0^1 to Δ_4^{25}	Page 127	Δ_0^1 to Δ_4^{25}	Page 128
	Δ_4^{46} to Δ_5^{11}		Δ_5^{12} to Δ_5^{23}		Δ_5^{24} to Δ_5^{35}		Δ_5^{36} to Δ_5^{47}		Δ_5^{48} to Δ_5^{59}
Δ_4^{26} to Δ_5^{19}	Page 134	Δ_4^{26} to Δ_5^{19}	Page 135	Δ_4^{26} to Δ_5^{19}	Page 136	Δ_4^{26} to Δ_5^{19}	Page 137	Δ_4^{26} to Δ_5^{19}	Page 138
	Δ_4^{46} to Δ_5^{11}		Δ_5^{12} to Δ_5^{23}		Δ_5^{24} to Δ_5^{35}		Δ_5^{36} to Δ_5^{47}		Δ_5^{48} to Δ_5^{59}
Δ_5^{20} to Δ_5^{59}	Page 144	Δ_5^{20} to Δ_5^{59}	Page 145	Δ_5^{20} to Δ_5^{59}	Page 146	Δ_5^{20} to Δ_5^{59}	Page 147	Δ_5^{20} to Δ_5^{59}	Page 148

	Δ_0^1	Δ_1^1	Δ_2^1	Δ_2^2	Δ_2^3	Δ_3^1	Δ_3^2	Δ_3^3	Δ_3^4	Δ_3^5	Δ_3^6	Δ_3^7
Δ_0^1	0	1518	0	0	0	0	0	0	0	0	0	0
Δ_1^1	1	1	60	560	896	0	0	0	0	0	0	0
Δ_2^1	0	3	3	0	0	0	168	0	0	0	0	1344
Δ_2^2	0	5	0	5	0	0	20	128	480	0	240	0
Δ_2^3	0	1	0	0	1	16	0	0	120	16	60	120
Δ_3^1	0	0	0	0	77	0	0	0	0	77	0	0
Δ_3^2	0	0	9	6	0	0	15	0	0	0	0	0
Δ_3^3	0	0	0	21	0	0	0	21	0	336	0	0
Δ_3^4	0	0	0	7	14	0	0	0	21	112	0	0
Δ_3^5	0	0	0	0	1	1	0	16	60	76	0	0
Δ_3^6	0	0	0	1	2	0	0	0	0	0	3	0
Δ_3^7	0	0	1	0	4	0	0	0	0	0	0	5
Δ_3^8	0	0	0	1	4	0	0	0	0	0	0	0
Δ_3^9	0	0	0	0	1	0	0	0	0	0	0	0
Δ_3^{10}	0	0	0	0	1	0	0	0	0	0	0	0
Δ_4^1	0	0	0	0	0	0	7	0	8	0	42	14
Δ_4^2	0	0	0	0	0	0	7	0	8	0	42	14
Δ_4^3	0	0	0	0	0	0	0	0	15	0	0	0
Δ_4^4	0	0	0	0	0	0	0	0	0	0	0	15
Δ_4^5	0	0	0	0	0	1	0	0	0	0	14	7
Δ_4^6	0	0	0	0	0	1	0	0	0	0	14	7
Δ_4^7	0	0	0	0	0	1	0	0	0	0	0	0
Δ_4^8	0	0	0	0	0	0	0	1	0	0	9	0
Δ_4^9	0	0	0	0	0	0	0	1	0	0	9	0
Δ_4^{10}	0	0	0	0	0	0	1	0	0	0	0	12
Δ_4^{11}	0	0	0	0	0	0	1	0	0	0	0	12
Δ_4^{12}	0	0	0	0	0	0	1	0	4	0	8	18
Δ_4^{13}	0	0	0	0	0	0	0	0	0	0	0	3
Δ_4^{14}	0	0	0	0	0	0	0	0	0	1	0	1
Δ_4^{15}	0	0	0	0	0	0	0	0	0	1	0	1
Δ_4^{16}	0	0	0	0	0	0	0	1	7	0	0	0
Δ_4^{17}	0	0	0	0	0	0	0	0	0	11	0	0
Δ_4^{18}	0	0	0	0	0	0	0	0	3	0	0	0
Δ_4^{19}	0	0	0	0	0	0	0	0	0	1	1	0
Δ_4^{20}	0	0	0	0	0	0	0	0	0	1	1	0
Δ_4^{21}	0	0	0	0	0	0	0	0	0	5	0	0
Δ_4^{22}	0	0	0	0	0	0	0	0	0	0	1	0
Δ_4^{23}	0	0	0	0	0	0	0	0	1	0	0	0
Δ_4^{24}	0	0	0	0	0	0	0	0	0	0	0	5
Δ_4^{25}	0	0	0	0	0	0	0	0	0	0	0	0

	Δ_3^8	Δ_3^9	Δ_3^{10}	Δ_4^1	Δ_4^2	Δ_4^3	Δ_4^4	Δ_4^5	Δ_4^6	Δ_4^7	Δ_4^8	Δ_4^9
Δ_0^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_1^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^2	640	0	0	0	0	0	0	0	0	0	0	0
Δ_2^3	320	384	480	0	0	0	0	0	0	0	0	0
Δ_3^1	0	0	0	0	0	0	0	330	330	704	0	0
Δ_3^2	0	0	0	144	144	0	0	0	0	0	0	0
Δ_3^3	0	0	0	0	0	0	0	0	0	0	210	210
Δ_3^4	0	0	0	8	8	32	0	0	0	0	0	0
Δ_3^5	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^6	0	0	0	12	12	0	0	32	32	0	48	48
Δ_3^7	0	0	0	4	4	0	24	16	16	0	0	0
Δ_3^8	5	0	0	0	0	16	0	0	0	64	10	10
Δ_3^9	0	97	0	0	0	0	0	0	0	56	0	0
Δ_3^{10}	0	0	1	0	0	0	0	15	15	0	0	0
Δ_4^1	0	0	0	7	22	0	0	0	0	0	0	0
Δ_4^2	0	0	0	22	7	0	0	0	0	0	0	0
Δ_4^3	70	0	0	0	0	15	0	0	0	0	0	0
Δ_4^4	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^5	0	0	105	0	0	0	0	0	1	0	0	0
Δ_4^6	0	0	105	0	0	0	0	1	0	0	0	0
Δ_4^7	35	147	0	0	0	0	0	0	0	1	0	0
Δ_4^8	5	0	0	0	0	0	0	0	0	0	6	9
Δ_4^9	5	0	0	0	0	0	0	0	0	0	9	6
Δ_4^{10}	8	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{11}	8	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{12}	0	0	32	2	2	0	0	0	0	0	0	0
Δ_4^{13}	0	0	0	0	0	0	3	0	0	0	0	0
Δ_4^{14}	0	0	1	6	0	0	0	2	0	0	0	24
Δ_4^{15}	0	0	1	0	6	0	0	0	2	0	24	0
Δ_4^{16}	35	42	0	0	0	0	0	0	0	0	0	0
Δ_4^{17}	0	11	0	0	0	11	0	0	0	11	0	0
Δ_4^{18}	14	84	0	0	0	2	0	0	0	32	0	0
Δ_4^{19}	0	0	3	0	1	0	0	1	3	0	4	0
Δ_4^{20}	0	0	3	1	0	0	0	3	1	0	0	4
Δ_4^{21}	5	51	0	0	0	0	0	0	0	20	0	0
Δ_4^{22}	4	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{23}	2	0	16	0	0	0	0	0	0	0	0	0
Δ_4^{24}	4	0	12	0	0	0	0	0	0	0	0	0
Δ_4^{25}	0	0	8	0	0	0	4	0	0	0	0	0

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	Δ_4^{10}	Δ_4^{11}	Δ_4^{12}	Δ_4^{13}	Δ_4^{14}	Δ_4^{15}	Δ_4^{16}	Δ_4^{17}	Δ_4^{18}	Δ_4^{19}	Δ_4^{20}	Δ_4^{21}
Δ_0^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_1^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^2	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^2	384	384	432	0	0	0	0	0	0	0	0	0
Δ_3^3	0	0	0	0	0	0	720	0	0	0	0	0
Δ_3^4	0	0	84	0	0	0	448	0	224	0	0	0
Δ_3^5	0	0	0	0	30	30	0	384	0	60	60	320
Δ_3^6	0	0	48	0	0	0	0	0	0	32	32	0
Δ_3^7	64	64	108	24	16	16	0	0	0	0	0	0
Δ_3^8	16	16	0	0	0	0	240	0	112	0	0	64
Δ_3^9	0	0	0	0	0	0	60	16	140	0	0	136
Δ_3^{10}	0	0	12	0	1	1	0	0	0	6	6	0
Δ_4^1	0	0	42	0	336	0	0	0	0	0	112	0
Δ_4^2	0	0	42	0	0	336	0	0	0	112	0	0
Δ_4^3	0	0	0	0	0	0	0	336	70	0	0	0
Δ_4^4	0	0	0	15	0	0	0	0	0	0	0	0
Δ_4^5	0	0	0	0	14	0	0	0	0	14	42	0
Δ_4^6	0	0	0	0	0	14	0	0	0	42	14	0
Δ_4^7	0	0	0	0	0	0	0	42	140	0	0	140
Δ_4^8	0	0	0	0	0	72	0	0	0	24	0	0
Δ_4^9	0	0	0	0	72	0	0	0	0	0	24	0
Δ_4^{10}	13	8	0	0	24	0	0	0	0	72	0	0
Δ_4^{11}	8	13	0	0	0	24	0	0	0	0	72	0
Δ_4^{12}	0	0	27	16	16	16	0	0	0	80	80	0
Δ_4^{13}	0	0	12	12	0	0	0	0	0	0	0	0
Δ_4^{14}	8	0	6	0	14	27	8	0	0	12	0	0
Δ_4^{15}	0	8	6	0	27	14	8	0	0	0	12	0
Δ_4^{16}	0	0	0	0	7	7	43	168	42	7	7	168
Δ_4^{17}	0	0	0	0	0	0	165	132	55	0	0	110
Δ_4^{18}	0	0	0	0	0	0	36	48	95	0	0	192
Δ_4^{19}	12	0	15	0	6	0	4	0	0	25	20	0
Δ_4^{20}	0	12	15	0	0	6	4	0	0	20	25	0
Δ_4^{21}	0	0	0	0	0	0	90	60	120	0	0	155
Δ_4^{22}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{23}	0	0	0	0	0	0	0	0	0	12	12	0
Δ_4^{24}	0	0	0	0	0	0	0	0	0	16	16	0
Δ_4^{25}	1	1	1	4	0	0	0	0	0	4	4	0

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	Δ_4^{22}	Δ_4^{23}	Δ_4^{24}	Δ_4^{25}	Δ_4^{26}	Δ_4^{27}	Δ_4^{28}	Δ_4^{29}	Δ_4^{30}	Δ_4^{31}	Δ_4^{32}	Δ_4^{33}
Δ_0^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_1^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^2	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^2	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^4	0	224	0	0	336	0	0	0	0	0	0	0
Δ_3^5	0	0	0	0	0	0	240	240	0	0	0	0
Δ_3^6	64	0	0	0	384	0	0	0	0	0	0	0
Δ_3^7	0	0	320	0	192	128	0	0	0	0	0	0
Δ_3^8	96	48	96	0	144	0	0	0	0	0	0	0
Δ_3^9	0	0	0	0	0	20	0	0	0	0	0	0
Δ_3^{10}	0	64	48	48	84	0	0	0	8	8	96	96
Δ_4^1	0	0	0	0	0	0	0	0	448	0	0	0
Δ_4^2	0	0	0	0	0	0	0	0	0	448	0	0
Δ_4^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^4	0	0	0	240	0	0	0	0	0	0	0	0
Δ_4^5	0	0	0	0	0	0	0	0	0	56	0	0
Δ_4^6	0	0	0	0	0	0	0	0	56	0	0	0
Δ_4^7	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^8	0	0	0	0	0	0	72	168	0	0	0	0
Δ_4^9	0	0	0	0	0	0	168	72	0	0	0	0
Δ_4^{10}	0	0	0	18	0	0	0	0	216	120	0	0
Δ_4^{11}	0	0	0	18	0	0	0	0	120	216	0	0
Δ_4^{12}	0	0	0	16	32	0	64	64	128	128	0	0
Δ_4^{13}	0	0	0	48	0	0	0	0	0	0	0	0
Δ_4^{14}	0	0	0	0	72	0	0	24	0	48	0	0
Δ_4^{15}	0	0	0	0	72	0	24	0	48	0	0	0
Δ_4^{16}	0	0	0	0	0	0	28	28	42	42	0	0
Δ_4^{17}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{18}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{19}	0	24	32	12	30	0	24	4	32	12	0	0
Δ_4^{20}	0	24	32	12	30	0	4	24	12	32	0	0
Δ_4^{21}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{22}	5	0	0	24	0	0	4	4	12	12	0	0
Δ_4^{23}	0	19	0	0	0	0	52	52	0	0	0	0
Δ_4^{24}	0	0	9	6	12	0	0	0	52	52	0	0
Δ_4^{25}	16	0	4	4	0	0	0	0	0	0	0	0

	Δ_4^{34}	Δ_4^{35}	Δ_4^{36}	Δ_4^{37}	Δ_4^{38}	Δ_4^{39}	Δ_4^{40}	Δ_4^{41}	Δ_4^{42}	Δ_4^{43}	Δ_4^{44}	Δ_4^{45}
Δ_0^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_1^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^2	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^2	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^4	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^5	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^6	0	0	0	0	384	384	0	0	0	0	0	0
Δ_3^7	256	256	0	0	0	0	0	0	0	0	0	0
Δ_3^8	0	0	96	96	0	0	384	0	0	0	0	0
Δ_3^9	40	40	20	20	60	60	80	96	96	0	0	240
Δ_3^{10}	32	32	16	16	72	72	192	0	0	96	96	192
Δ_4^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^2	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^4	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^5	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^6	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^7	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^8	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^9	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{10}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{11}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{12}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{13}	0	0	0	0	96	96	0	0	0	0	0	0
Δ_4^{14}	0	0	0	96	48	24	0	0	0	0	0	0
Δ_4^{15}	0	0	96	0	24	48	0	0	0	0	0	0
Δ_4^{16}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{17}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{18}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{19}	0	0	0	0	36	24	0	0	0	144	0	96
Δ_4^{20}	0	0	0	0	24	36	0	0	0	0	144	0
Δ_4^{21}	0	0	0	0	0	0	0	0	0	90	90	0
Δ_4^{22}	0	0	0	0	0	0	0	0	0	96	96	0
Δ_4^{23}	0	0	0	0	0	0	0	0	0	48	48	0
Δ_4^{24}	0	0	0	0	0	0	0	0	0	24	24	0
Δ_4^{25}	32	32	0	0	20	20	0	64	64	48	48	0

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	Δ_4^{46}	Δ_5^1	Δ_5^2	Δ_5^3	Δ_5^4	Δ_5^5	Δ_5^6	Δ_5^7	Δ_5^8	Δ_5^9	Δ_5^{10}	Δ_5^{11}
Δ_0^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_1^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^2	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^2	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^4	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^5	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^6	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^7	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^8	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^9	240	0	0	0	0	0	0	0	0	0	0	0
Δ_3^{10}	192	0	0	0	0	0	0	0	0	0	0	0
Δ_4^1	0	0	32	0	0	224	0	0	0	0	224	0
Δ_4^2	0	32	0	0	0	0	224	0	0	224	0	0
Δ_4^3	0	16	16	0	0	0	0	0	0	210	210	0
Δ_4^4	0	0	0	0	0	0	0	120	120	0	0	0
Δ_4^5	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^6	0	0	0	0	0	0	0	0	0	0	0	32
Δ_4^7	0	0	0	0	0	0	0	0	0	0	0	30
Δ_4^8	0	0	0	0	0	0	48	0	48	0	0	0
Δ_4^9	0	0	0	0	0	48	0	48	0	0	0	0
Δ_4^{10}	0	8	0	0	16	0	8	12	12	24	0	0
Δ_4^{11}	0	0	8	16	0	8	0	12	12	0	24	96
Δ_4^{12}	0	0	0	0	0	0	0	32	32	0	0	0
Δ_4^{13}	0	0	0	0	0	0	0	24	24	0	0	0
Δ_4^{14}	0	0	0	16	0	0	0	48	0	0	0	0
Δ_4^{15}	0	0	0	0	16	0	0	0	48	0	0	0
Δ_4^{16}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{17}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{18}	0	0	0	0	0	4	4	12	12	6	6	0
Δ_4^{19}	0	0	0	0	0	0	0	0	0	0	24	0
Δ_4^{20}	96	0	0	0	0	0	0	0	0	24	0	0
Δ_4^{21}	0	0	0	1	1	0	0	0	0	0	0	0
Δ_4^{22}	0	0	0	0	0	0	0	32	32	12	12	0
Δ_4^{23}	0	2	2	0	0	2	2	0	0	8	8	0
Δ_4^{24}	0	0	0	0	0	4	4	20	20	0	0	0
Δ_4^{25}	0	0	0	0	0	0	0	2	2	0	0	0

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	Δ_5^{12}	Δ_5^{13}	Δ_5^{14}	Δ_5^{15}	Δ_5^{16}	Δ_5^{17}	Δ_5^{18}	Δ_5^{19}	Δ_5^{20}	Δ_5^{21}	Δ_5^{22}	Δ_5^{23}
Δ_0^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_1^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^2	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^2	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^4	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^5	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^6	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^7	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^8	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^9	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^{10}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^2	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^4	0	0	0	240	0	0	0	0	0	0	0	0
Δ_4^5	32	0	112	168	0	0	0	0	0	0	0	0
Δ_4^6	0	112	0	168	0	0	0	0	0	0	0	0
Δ_4^7	30	35	35	0	0	0	0	0	0	0	0	0
Δ_4^8	0	0	64	0	0	64	0	0	0	0	0	0
Δ_4^9	0	64	0	0	64	0	0	0	0	0	0	0
Δ_4^{10}	96	16	0	18	0	0	0	0	0	0	0	48
Δ_4^{11}	0	0	16	18	0	0	0	0	0	0	0	0
Δ_4^{12}	0	0	0	16	0	0	0	0	0	0	0	0
Δ_4^{13}	0	0	0	48	0	0	0	0	0	0	0	0
Δ_4^{14}	0	0	0	0	0	64	0	192	0	0	0	128
Δ_4^{15}	0	0	0	0	64	0	192	0	0	0	0	16
Δ_4^{16}	0	0	0	0	0	0	0	0	32	0	0	0
Δ_4^{17}	0	0	0	0	0	0	55	55	0	66	66	55
Δ_4^{18}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{19}	0	0	0	0	0	32	32	0	0	96	0	0
Δ_4^{20}	0	0	0	0	32	0	0	32	0	0	96	0
Δ_4^{21}	0	0	0	0	10	10	0	0	0	0	0	15
Δ_4^{22}	0	0	0	0	16	16	0	0	0	0	0	0
Δ_4^{23}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{24}	0	0	0	6	16	16	0	0	64	0	0	0
Δ_4^{25}	0	16	16	7	0	0	0	0	0	0	0	0

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	Δ_5^{24}	Δ_5^{25}	Δ_5^{26}	Δ_5^{27}	Δ_5^{28}	Δ_5^{29}	Δ_5^{30}	Δ_5^{31}	Δ_5^{32}	Δ_5^{33}	Δ_5^{34}	Δ_5^{35}
Δ_0^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_1^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^2	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^2	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^4	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^5	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^6	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^7	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^8	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^9	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^{10}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^2	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^3	0	0	0	0	0	560	0	0	0	0	0	0
Δ_4^4	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^5	0	224	112	0	0	0	0	0	168	0	0	0
Δ_4^6	0	112	224	0	0	0	0	168	0	0	0	0
Δ_4^7	0	105	105	0	0	0	0	0	0	0	0	0
Δ_4^8	0	192	0	0	128	128	0	0	0	0	0	0
Δ_4^9	0	0	192	128	0	128	0	0	0	0	0	0
Δ_4^{10}	0	48	0	192	0	0	144	0	0	96	0	0
Δ_4^{11}	48	0	48	0	192	0	144	0	0	0	96	0
Δ_4^{12}	0	0	0	0	0	0	64	64	64	0	0	0
Δ_4^{13}	0	0	0	0	0	0	0	96	96	0	0	0
Δ_4^{14}	16	0	0	0	0	0	0	0	24	32	0	0
Δ_4^{15}	128	0	0	0	0	0	0	24	0	0	32	0
Δ_4^{16}	0	0	0	56	56	112	84	0	0	0	0	0
Δ_4^{17}	55	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{18}	0	0	0	16	16	0	0	36	36	0	0	0
Δ_4^{19}	0	0	0	0	0	0	0	36	12	64	0	0
Δ_4^{20}	0	0	0	0	0	0	0	12	36	0	64	0
Δ_4^{21}	15	0	0	0	0	0	0	0	0	30	30	0
Δ_4^{22}	0	0	0	80	80	112	36	12	12	32	32	0
Δ_4^{23}	0	48	48	0	0	48	48	0	0	32	32	0
Δ_4^{24}	0	24	24	0	0	32	96	0	0	0	0	48
Δ_4^{25}	0	0	0	0	0	0	0	28	28	0	0	128

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	Δ_5^{36}	Δ_5^{37}	Δ_5^{38}	Δ_5^{39}	Δ_5^{40}	Δ_5^{41}	Δ_5^{42}	Δ_5^{43}	Δ_5^{44}	Δ_5^{45}	Δ_5^{46}	Δ_5^{47}
Δ_0^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_1^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^2	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^2	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^4	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^5	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^6	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^7	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^8	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^9	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^{10}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^2	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^4	0	0	0	0	0	0	0	0	0	0	0	768
Δ_4^5	0	0	0	0	0	0	0	0	0	448	0	0
Δ_4^6	0	0	0	0	0	0	0	0	0	0	448	0
Δ_4^7	0	0	0	0	0	0	0	0	0	210	210	252
Δ_4^8	0	0	192	0	0	0	0	0	0	0	0	0
Δ_4^9	0	192	0	0	0	0	0	0	0	0	0	0
Δ_4^{10}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{11}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{12}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{13}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{14}	0	0	0	0	192	0	0	0	0	0	0	0
Δ_4^{15}	0	0	0	192	0	0	0	0	0	0	0	0
Δ_4^{16}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{17}	0	55	55	110	110	0	0	0	0	0	0	0
Δ_4^{18}	0	0	0	0	0	0	0	48	48	0	0	0
Δ_4^{19}	0	128	64	32	0	192	96	96	0	0	0	0
Δ_4^{20}	0	64	128	0	32	96	192	0	96	0	0	0
Δ_4^{21}	0	60	60	30	30	90	90	0	0	0	0	0
Δ_4^{22}	0	0	0	0	0	0	0	0	0	48	48	192
Δ_4^{23}	0	0	0	48	48	0	0	96	96	80	80	0
Δ_4^{24}	48	0	0	48	48	0	0	16	16	64	64	0
Δ_4^{25}	128	0	0	0	0	64	64	96	96	32	32	0

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	Δ_5^{48}	Δ_5^{49}	Δ_5^{50}	Δ_5^{51}	Δ_5^{52}	Δ_5^{53}	Δ_5^{54}	Δ_5^{55}	Δ_5^{56}	Δ_5^{57}	Δ_5^{58}	Δ_5^{59}
Δ_0^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_1^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^2	0	0	0	0	0	0	0	0	0	0	0	0
Δ_2^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^2	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^4	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^5	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^6	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^7	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^8	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^9	0	0	0	0	0	0	0	0	0	0	0	0
Δ_3^{10}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^1	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^2	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^4	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^5	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^6	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^7	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^8	0	0	0	0	288	0	0	0	0	0	0	0
Δ_4^9	0	0	0	288	0	0	0	0	0	0	0	0
Δ_4^{10}	0	0	0	0	0	288	0	0	0	0	0	0
Δ_4^{11}	0	0	0	0	0	0	288	0	0	0	0	0
Δ_4^{12}	0	0	0	0	0	256	256	0	0	0	0	0
Δ_4^{13}	192	0	0	0	0	0	0	0	0	0	0	768
Δ_4^{14}	0	288	0	0	96	0	0	0	0	0	0	0
Δ_4^{15}	0	0	288	96	0	0	0	0	0	0	0	0
Δ_4^{16}	0	0	0	84	84	0	0	0	0	0	336	0
Δ_4^{17}	0	165	165	0	0	0	0	0	0	0	0	0
Δ_4^{18}	0	0	0	72	72	144	144	0	96	96	144	0
Δ_4^{19}	0	48	0	0	0	0	0	0	0	0	0	0
Δ_4^{20}	0	0	48	0	0	0	0	0	0	0	0	0
Δ_4^{21}	0	90	90	0	0	0	0	0	0	0	0	180
Δ_4^{22}	24	0	0	0	0	0	0	0	96	96	240	0
Δ_4^{23}	0	0	0	0	0	144	144	0	0	0	240	0
Δ_4^{24}	0	0	0	24	24	120	120	0	64	64	192	0
Δ_4^{25}	112	0	0	32	32	16	16	64	64	64	0	0

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	Δ_0^1	Δ_1^1	Δ_2^1	Δ_2^2	Δ_2^3	Δ_3^1	Δ_3^2	Δ_3^3	Δ_3^4	Δ_3^5	Δ_3^6	Δ_3^7
Δ_4^{26}	0	0	0	0	0	0	0	0	1	0	4	2
Δ_4^{27}	0	0	0	0	0	0	0	0	0	0	0	1
Δ_4^{28}	0	0	0	0	0	0	0	0	0	1	0	0
Δ_4^{29}	0	0	0	0	0	0	0	0	0	1	0	0
Δ_4^{30}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{31}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{32}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{33}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{34}	0	0	0	0	0	0	0	0	0	0	0	1
Δ_4^{35}	0	0	0	0	0	0	0	0	0	0	0	1
Δ_4^{36}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{37}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{38}	0	0	0	0	0	0	0	0	0	0	1	0
Δ_4^{39}	0	0	0	0	0	0	0	0	0	0	1	0
Δ_4^{40}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{41}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{42}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{43}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{44}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{45}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{46}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{1}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{2}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{3}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{4}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{5}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{6}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{7}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{8}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{9}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{10}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{11}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{12}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{13}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{14}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{15}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{16}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{17}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{18}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{19}	0	0	0	0	0	0	0	0	0	0	0	0

	Δ_3^8	Δ_3^9	Δ_3^{10}	Δ_4^1	Δ_4^2	Δ_4^3	Δ_4^4	Δ_4^5	Δ_4^6	Δ_4^7	Δ_4^8	Δ_4^9
Δ_4^{26}	4	0	14	0	0	0	0	0	0	0	0	0
Δ_4^{27}	0	2	0	0	0	0	0	0	0	0	0	0
Δ_4^{28}	0	0	0	0	0	0	0	0	0	0	3	7
Δ_4^{29}	0	0	0	0	0	0	0	0	0	0	7	3
Δ_4^{30}	0	0	1	1	0	0	0	0	1	0	0	0
Δ_4^{31}	0	0	1	0	1	0	0	1	0	0	0	0
Δ_4^{32}	0	0	9	0	0	0	0	0	0	0	0	0
Δ_4^{33}	0	0	9	0	0	0	0	0	0	0	0	0
Δ_4^{34}	0	2	2	0	0	0	0	0	0	0	0	0
Δ_4^{35}	0	2	2	0	0	0	0	0	0	0	0	0
Δ_4^{36}	1	1	1	0	0	0	0	0	0	0	0	0
Δ_4^{37}	1	1	1	0	0	0	0	0	0	0	0	0
Δ_4^{38}	0	2	3	0	0	0	0	0	0	0	0	0
Δ_4^{39}	0	2	3	0	0	0	0	0	0	0	0	0
Δ_4^{40}	1	1	3	0	0	0	0	0	0	0	0	0
Δ_4^{41}	0	1	0	0	0	0	0	0	0	0	0	0
Δ_4^{42}	0	1	0	0	0	0	0	0	0	0	0	0
Δ_4^{43}	0	0	1	0	0	0	0	0	0	0	0	0
Δ_4^{44}	0	0	1	0	0	0	0	0	0	0	0	0
Δ_4^{45}	0	1	1	0	0	0	0	0	0	0	0	0
Δ_4^{46}	0	1	1	0	0	0	0	0	0	0	0	0
Δ_5^1	0	0	0	0	15	16	0	0	0	0	0	0
Δ_5^2	0	0	0	15	0	16	0	0	0	0	0	0
Δ_5^3	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^4	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^5	0	0	0	3	0	0	0	0	0	0	0	12
Δ_5^6	0	0	0	0	3	0	0	0	0	0	12	0
Δ_5^7	0	0	0	0	0	0	3	0	0	0	0	4
Δ_5^8	0	0	0	0	0	0	3	0	0	0	4	0
Δ_5^9	0	0	0	0	1	2	0	0	0	0	0	0
Δ_5^{10}	0	0	0	1	0	2	0	0	0	0	0	0
Δ_5^{11}	0	0	0	0	0	0	0	0	1	2	0	0
Δ_5^{12}	0	0	0	0	0	0	0	1	0	2	0	0
Δ_5^{13}	0	0	0	0	0	0	0	0	3	2	0	4
Δ_5^{14}	0	0	0	0	0	0	0	3	0	2	4	0
Δ_5^{15}	0	0	0	0	0	0	4	4	4	0	0	0
Δ_5^{16}	0	0	0	0	0	0	0	0	0	0	0	3
Δ_5^{17}	0	0	0	0	0	0	0	0	0	0	3	0
Δ_5^{18}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{19}	0	0	0	0	0	0	0	0	0	0	0	0

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	Δ_4^{10}	Δ_4^{11}	Δ_4^{12}	Δ_4^{13}	Δ_4^{14}	Δ_4^{15}	Δ_4^{16}	Δ_4^{17}	Δ_4^{18}	Δ_4^{19}	Δ_4^{20}	Δ_4^{21}
Δ_4^{26}	0	0	2	0	12	12	0	0	0	10	10	0
Δ_4^{27}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{28}	0	0	3	0	0	3	4	0	0	6	1	0
Δ_4^{29}	0	0	3	0	3	0	4	0	0	1	6	0
Δ_4^{30}	9	5	6	0	0	6	6	0	0	8	3	0
Δ_4^{31}	5	9	6	0	6	0	6	0	0	3	8	0
Δ_4^{32}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{33}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{34}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{35}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{36}	0	0	0	0	0	6	0	0	0	0	0	0
Δ_4^{37}	0	0	0	0	6	0	0	0	0	0	0	0
Δ_4^{38}	0	0	0	2	2	1	0	0	0	3	2	0
Δ_4^{39}	0	0	0	2	1	2	0	0	0	2	3	0
Δ_4^{40}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{41}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{42}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{43}	0	0	0	0	0	0	0	0	0	3	0	2
Δ_4^{44}	0	0	0	0	0	0	0	0	0	0	3	2
Δ_4^{45}	0	0	0	0	0	0	0	0	0	1	0	0
Δ_4^{46}	0	0	0	0	0	0	0	0	0	0	1	0
Δ_5^{1}	70	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{2}	0	70	0	0	0	0	0	0	0	0	0	0
Δ_5^{3}	0	15	0	0	45	0	0	0	0	0	0	6
Δ_5^{4}	15	0	0	0	0	45	0	0	0	0	0	6
Δ_5^{5}	0	2	0	0	0	0	0	0	4	0	0	0
Δ_5^{6}	2	0	0	0	0	0	0	0	4	0	0	0
Δ_5^{7}	1	1	3	3	12	0	0	0	4	0	0	0
Δ_5^{8}	1	1	3	3	0	12	0	0	4	0	0	0
Δ_5^{9}	2	0	0	0	0	0	0	0	2	0	12	0
Δ_5^{10}	0	2	0	0	0	0	0	0	2	12	0	0
Δ_5^{11}	0	7	0	0	0	0	0	0	0	0	0	0
Δ_5^{12}	7	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{13}	1	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{14}	0	1	0	0	0	0	0	0	0	0	0	0
Δ_5^{15}	1	1	1	4	0	0	0	0	0	0	0	0
Δ_5^{16}	0	0	0	0	0	9	0	0	0	0	9	3
Δ_5^{17}	0	0	0	0	9	0	0	0	0	9	0	3
Δ_5^{18}	0	0	0	0	0	27	0	9	0	9	0	0
Δ_5^{19}	0	0	0	0	27	0	0	9	0	0	9	0

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	Δ_4^{22}	Δ_4^{23}	Δ_4^{24}	Δ_4^{25}	Δ_4^{26}	Δ_4^{27}	Δ_4^{28}	Δ_4^{29}	Δ_4^{30}	Δ_4^{31}	Δ_4^{32}	Δ_4^{33}
Δ_4^{26}	0	0	8	0	15	0	44	44	44	44	0	0
Δ_4^{27}	0	0	0	0	0	3	6	6	6	6	0	0
Δ_4^{28}	2	26	0	0	33	6	33	30	0	0	8	0
Δ_4^{29}	2	26	0	0	33	6	30	33	0	0	0	8
Δ_4^{30}	6	0	26	0	33	6	0	0	33	36	0	4
Δ_4^{31}	6	0	26	0	33	6	0	0	36	33	4	0
Δ_4^{32}	0	0	0	0	0	0	6	0	0	3	0	0
Δ_4^{33}	0	0	0	0	0	0	0	6	3	0	0	0
Δ_4^{34}	0	0	0	12	0	0	0	12	6	6	0	0
Δ_4^{35}	0	0	0	12	0	0	12	0	6	6	0	0
Δ_4^{36}	0	0	0	0	0	0	0	6	0	6	0	0
Δ_4^{37}	0	0	0	0	0	0	6	0	6	0	0	0
Δ_4^{38}	0	0	0	5	1	0	0	18	2	2	0	0
Δ_4^{39}	0	0	0	5	1	0	18	0	2	2	0	0
Δ_4^{40}	0	0	0	0	0	12	6	6	3	3	0	0
Δ_4^{41}	0	0	0	5	0	0	0	0	5	0	0	10
Δ_4^{42}	0	0	0	5	0	0	0	0	0	5	10	0
Δ_4^{43}	4	2	1	3	1	0	4	2	1	5	5	0
Δ_4^{44}	4	2	1	3	1	0	2	4	5	1	0	5
Δ_4^{45}	0	0	0	0	0	6	2	1	0	2	6	4
Δ_4^{46}	0	0	0	0	0	6	1	2	2	0	4	6
Δ_5^{1}	0	210	0	0	0	0	0	0	0	0	0	0
Δ_5^{2}	0	210	0	0	0	0	0	0	0	0	0	0
Δ_5^{3}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{4}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{5}	0	6	12	0	0	36	0	36	0	36	0	0
Δ_5^{6}	0	6	12	0	0	36	36	0	36	0	0	0
Δ_5^{7}	32	0	20	3	12	0	0	0	0	0	0	0
Δ_5^{8}	32	0	20	3	12	0	0	0	0	0	0	0
Δ_5^{9}	12	8	0	0	12	0	0	0	0	0	0	0
Δ_5^{10}	12	8	0	0	12	0	0	0	0	0	0	0
Δ_5^{11}	0	0	0	0	0	42	0	0	0	0	0	0
Δ_5^{12}	0	0	0	0	0	42	0	0	0	0	0	0
Δ_5^{13}	0	0	0	18	18	0	0	0	0	6	24	0
Δ_5^{14}	0	0	0	18	18	0	0	0	6	0	0	24
Δ_5^{15}	0	0	4	7	8	0	0	0	0	0	48	48
Δ_5^{16}	9	0	9	0	0	0	0	27	0	9	18	0
Δ_5^{17}	9	0	9	0	0	0	27	0	9	0	0	18
Δ_5^{18}	0	0	0	0	0	0	45	27	0	36	0	0
Δ_5^{19}	0	0	0	0	0	0	27	45	36	0	0	0

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	Δ_4^{34}	Δ_4^{35}	Δ_4^{36}	Δ_4^{37}	Δ_4^{38}	Δ_4^{39}	Δ_4^{40}	Δ_4^{41}	Δ_4^{42}	Δ_4^{43}	Δ_4^{44}	Δ_4^{45}
Δ_4^{26}	0	0	0	0	4	4	0	0	0	16	16	0
Δ_4^{27}	0	0	0	0	0	0	96	0	0	0	0	144
Δ_4^{28}	0	24	0	12	0	54	48	0	0	48	24	48
Δ_4^{29}	24	0	12	0	54	0	48	0	0	24	48	24
Δ_4^{30}	12	12	0	12	6	6	24	48	0	12	60	0
Δ_4^{31}	12	12	12	0	6	6	24	0	48	60	12	48
Δ_4^{32}	0	0	0	0	0	0	0	0	72	45	0	108
Δ_4^{33}	0	0	0	0	0	0	0	72	0	0	45	72
Δ_4^{34}	3	26	0	24	0	30	0	48	0	6	72	24
Δ_4^{35}	26	3	24	0	30	0	0	0	48	72	6	72
Δ_4^{36}	0	24	24	3	0	24	0	72	24	24	108	0
Δ_4^{37}	24	0	3	24	24	0	0	24	72	108	24	156
Δ_4^{38}	0	20	0	16	3	20	16	48	16	16	76	48
Δ_4^{39}	20	0	16	0	20	3	16	16	48	76	16	96
Δ_4^{40}	0	0	0	0	6	6	5	48	48	33	33	42
Δ_4^{41}	10	0	15	5	15	5	40	26	26	40	40	125
Δ_4^{42}	0	10	5	15	5	15	40	26	26	40	40	105
Δ_4^{43}	1	12	4	18	4	19	22	32	32	57	40	70
Δ_4^{44}	12	1	18	4	19	4	22	32	32	40	57	56
Δ_4^{45}	2	6	0	13	6	12	14	50	42	35	28	83
Δ_4^{46}	6	2	13	0	12	6	14	42	50	28	35	91
Δ_5^{1}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{2}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{3}	0	45	0	90	0	0	0	0	216	0	0	0
Δ_5^{4}	45	0	90	0	0	0	0	216	0	0	0	0
Δ_5^{5}	0	48	0	72	36	144	0	0	0	0	0	0
Δ_5^{6}	48	0	72	0	144	36	0	0	0	0	0	0
Δ_5^{7}	48	24	24	0	36	0	32	0	0	24	0	96
Δ_5^{8}	24	48	0	24	0	36	32	0	0	0	24	0
Δ_5^{9}	0	16	80	0	84	0	96	0	0	0	0	48
Δ_5^{10}	16	0	0	80	0	84	96	0	0	0	0	0
Δ_5^{11}	42	35	7	21	0	0	0	84	84	0	84	0
Δ_5^{12}	35	42	21	7	0	0	0	84	84	84	0	126
Δ_5^{13}	3	0	0	90	0	72	84	72	0	36	36	0
Δ_5^{14}	0	3	90	0	72	0	84	0	72	36	36	108
Δ_5^{15}	16	16	32	32	28	28	0	64	64	64	64	96
Δ_5^{16}	0	27	0	0	0	27	18	0	0	27	81	54
Δ_5^{17}	27	0	0	0	27	0	18	0	0	81	27	54
Δ_5^{18}	0	0	0	9	0	27	0	0	0	27	27	0
Δ_5^{19}	0	0	9	0	27	0	0	0	0	27	27	27

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	Δ_4^{46}	Δ_5^{1}	Δ_5^{2}	Δ_5^{3}	Δ_5^{4}	Δ_5^{5}	Δ_5^{6}	Δ_5^{7}	Δ_5^{8}	Δ_5^{9}	Δ_5^{10}	Δ_5^{11}
Δ_4^{26}	0	0	0	0	0	0	0	8	8	8	8	0
Δ_4^{27}	144	0	0	0	0	6	6	0	0	0	0	24
Δ_4^{28}	24	0	0	0	0	0	6	0	0	0	0	0
Δ_4^{29}	48	0	0	0	0	6	0	0	0	0	0	0
Δ_4^{30}	48	0	0	0	0	0	6	0	0	0	0	0
Δ_4^{31}	0	0	0	0	0	6	0	0	0	0	0	0
Δ_4^{32}	72	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{33}	108	0	0	0	0	0	0	0	0	0	0	0
Δ_4^{34}	72	0	0	0	1	0	4	12	6	0	4	12
Δ_4^{35}	24	0	0	1	0	4	0	6	12	4	0	10
Δ_4^{36}	156	0	0	0	2	0	6	6	0	20	0	2
Δ_4^{37}	0	0	0	2	0	6	0	0	6	0	20	6
Δ_4^{38}	96	0	0	0	0	2	8	6	0	14	0	0
Δ_4^{39}	48	0	0	0	0	8	2	0	6	0	14	0
Δ_4^{40}	42	0	0	0	0	0	0	2	2	6	6	0
Δ_4^{41}	105	0	0	0	1	0	0	0	0	0	0	5
Δ_4^{42}	125	0	0	1	0	0	0	0	0	0	0	5
Δ_4^{43}	56	0	0	0	0	0	0	1	0	0	0	0
Δ_4^{44}	70	0	0	0	0	0	0	0	1	0	0	4
Δ_4^{45}	91	0	0	0	0	0	0	2	0	1	0	0
Δ_4^{46}	83	0	0	0	0	0	0	0	2	0	1	3
Δ_5^{1}	0	15	16	0	0	0	70	0	0	0	210	0
Δ_5^{2}	0	16	15	0	0	70	0	0	0	210	0	0
Δ_5^{3}	0	0	0	0	6	0	0	0	0	0	0	0
Δ_5^{4}	0	0	0	6	0	0	0	0	0	0	0	90
Δ_5^{5}	0	0	2	0	0	3	16	12	0	54	0	0
Δ_5^{6}	0	2	0	0	0	16	3	0	12	0	54	0
Δ_5^{7}	0	0	0	0	0	4	0	19	10	0	0	0
Δ_5^{8}	96	0	0	0	0	0	4	10	19	0	0	0
Δ_5^{9}	0	0	2	0	0	18	0	0	0	3	8	32
Δ_5^{10}	48	2	0	0	0	0	18	0	0	8	3	0
Δ_5^{11}	126	0	0	0	7	0	0	0	0	28	0	1
Δ_5^{12}	0	0	0	7	0	0	0	0	0	0	28	2
Δ_5^{13}	108	0	0	0	4	0	8	24	0	0	0	0
Δ_5^{14}	0	0	0	4	0	8	0	0	24	0	0	0
Δ_5^{15}	96	0	0	0	0	0	0	2	2	0	0	0
Δ_5^{16}	54	0	0	0	0	0	12	0	0	0	0	0
Δ_5^{17}	54	0	0	0	0	12	0	0	0	0	0	0
Δ_5^{18}	27	3	0	0	0	0	3	0	0	9	0	0
Δ_5^{19}	0	0	3	0	0	3	0	0	0	0	9	0

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	Δ_5^{12}	Δ_5^{13}	Δ_5^{14}	Δ_5^{15}	Δ_5^{16}	Δ_5^{17}	Δ_5^{18}	Δ_5^{19}	Δ_5^{20}	Δ_5^{21}	Δ_5^{22}	Δ_5^{23}
Δ_4^{26}	0	16	16	8	0	0	0	0	0	0	0	0
Δ_4^{27}	24	0	0	0	0	0	0	0	0	8	8	8
Δ_4^{28}	0	0	0	0	0	24	40	24	0	16	16	72
Δ_4^{29}	0	0	0	0	24	0	24	40	0	16	16	0
Δ_4^{30}	0	0	4	0	0	8	0	32	0	24	48	60
Δ_4^{31}	0	4	0	0	8	0	32	0	0	48	24	0
Δ_4^{32}	0	12	0	27	12	0	0	0	0	0	0	0
Δ_4^{33}	0	0	12	27	0	12	0	0	0	0	0	0
Δ_4^{34}	10	1	0	6	0	12	0	0	0	0	0	27
Δ_4^{35}	12	0	1	6	12	0	0	0	0	0	0	0
Δ_4^{36}	6	0	30	12	0	0	0	4	0	0	8	0
Δ_4^{37}	2	30	0	12	0	0	4	0	0	8	0	2
Δ_4^{38}	0	0	16	7	0	8	0	8	0	0	0	0
Δ_4^{39}	0	16	0	7	8	0	8	0	0	0	0	4
Δ_4^{40}	0	7	7	0	2	2	0	0	12	0	0	2
Δ_4^{41}	5	5	0	5	0	0	0	0	0	1	1	0
Δ_4^{42}	5	0	5	5	0	0	0	0	0	1	1	5
Δ_4^{43}	4	2	2	4	2	6	2	2	0	4	2	17
Δ_4^{44}	0	2	2	4	6	2	2	2	0	2	4	4
Δ_4^{45}	3	0	3	3	2	2	0	1	8	2	1	4
Δ_4^{46}	0	3	0	3	2	2	1	0	8	1	2	2
Δ_5^{1}	0	0	0	0	0	0	560	0	0	336	0	0
Δ_5^{2}	0	0	0	0	0	0	0	560	0	0	336	0
Δ_5^{3}	90	0	60	0	0	0	0	0	0	0	0	0
Δ_5^{4}	0	60	0	0	0	0	0	0	0	0	0	225
Δ_5^{5}	0	0	32	0	0	64	0	16	0	0	0	0
Δ_5^{6}	0	32	0	0	64	0	16	0	0	0	0	0
Δ_5^{7}	0	32	0	3	0	0	0	0	0	32	32	24
Δ_5^{8}	0	0	32	3	0	0	0	0	0	32	32	0
Δ_5^{9}	0	0	0	0	0	0	16	0	0	0	0	0
Δ_5^{10}	32	0	0	0	0	0	0	16	0	0	0	96
Δ_5^{11}	2	0	0	0	0	0	0	0	0	0	14	7
Δ_5^{12}	1	0	0	0	0	0	0	0	0	14	0	14
Δ_5^{13}	0	0	2	0	4	24	0	0	0	0	0	42
Δ_5^{14}	0	2	0	0	24	4	0	0	0	0	0	0
Δ_5^{15}	0	0	0	4	0	0	0	0	0	0	0	0
Δ_5^{16}	0	3	18	0	9	30	15	0	0	0	0	0
Δ_5^{17}	0	18	3	0	30	9	0	15	0	0	0	0
Δ_5^{18}	0	0	0	0	15	0	48	66	0	0	45	81
Δ_5^{19}	0	0	0	0	0	15	66	48	0	45	0	0

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	Δ_5^{24}	Δ_5^{25}	Δ_5^{26}	Δ_5^{27}	Δ_5^{28}	Δ_5^{29}	Δ_5^{30}	Δ_5^{31}	Δ_5^{32}	Δ_5^{33}	Δ_5^{34}	Δ_5^{35}
Δ_4^{26}	0	16	16	32	32	128	0	24	24	0	0	0
Δ_4^{27}	8	8	8	0	0	0	0	30	30	16	16	0
Δ_4^{28}	0	0	0	0	0	0	6	6	0	0	48	0
Δ_4^{29}	72	0	0	0	0	0	6	0	6	48	0	0
Δ_4^{30}	0	0	0	0	24	0	6	6	6	24	16	0
Δ_4^{31}	60	0	0	24	0	0	6	6	6	16	24	0
Δ_4^{32}	0	9	18	0	0	0	0	0	45	0	0	36
Δ_4^{33}	0	18	9	0	0	0	0	45	0	0	0	36
Δ_4^{34}	0	33	0	12	24	4	36	30	12	2	28	12
Δ_4^{35}	27	0	33	24	12	4	36	12	30	28	2	12
Δ_4^{36}	2	26	0	12	36	12	0	6	24	2	30	0
Δ_4^{37}	0	0	26	36	12	12	0	24	6	30	2	0
Δ_4^{38}	4	40	24	0	0	8	4	20	2	0	32	0
Δ_4^{39}	0	24	40	0	0	8	4	2	20	32	0	0
Δ_4^{40}	2	15	15	13	13	26	18	0	0	18	18	12
Δ_4^{41}	5	0	5	20	5	5	5	20	20	10	5	26
Δ_4^{42}	0	5	0	5	20	5	5	20	20	5	10	26
Δ_4^{43}	4	11	0	14	12	10	8	19	20	4	12	16
Δ_4^{44}	17	0	11	12	14	10	8	20	19	12	4	16
Δ_4^{45}	2	13	8	8	0	4	9	11	5	8	12	18
Δ_4^{46}	4	8	13	0	8	4	9	5	11	12	8	18
Δ_5^{1}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{2}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{3}	225	0	0	180	0	0	0	270	0	0	0	0
Δ_5^{4}	0	0	0	0	180	0	0	0	270	0	0	0
Δ_5^{5}	0	0	0	0	32	0	0	0	0	0	144	0
Δ_5^{6}	0	0	0	32	0	0	0	0	0	144	0	0
Δ_5^{7}	0	0	0	32	0	0	0	0	36	0	0	0
Δ_5^{8}	24	0	0	0	32	0	0	36	0	0	0	0
Δ_5^{9}	96	0	0	0	0	0	12	36	0	0	0	0
Δ_5^{10}	0	0	0	0	0	0	12	0	36	0	0	0
Δ_5^{11}	14	0	0	28	0	0	0	0	0	14	42	0
Δ_5^{12}	7	0	0	0	28	0	0	0	0	42	14	0
Δ_5^{13}	0	3	0	8	24	0	0	0	0	0	78	0
Δ_5^{14}	42	0	3	24	8	0	0	0	0	78	0	0
Δ_5^{15}	0	0	0	0	0	0	16	20	20	0	0	64
Δ_5^{16}	0	27	0	3	27	0	0	27	27	0	27	0
Δ_5^{17}	0	0	27	27	3	0	0	27	27	27	0	0
Δ_5^{18}	0	27	9	12	54	138	27	0	27	0	0	0
Δ_5^{19}	81	9	27	54	12	138	27	27	0	0	0	0

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	Δ_5^{36}	Δ_5^{37}	Δ_5^{38}	Δ_5^{39}	Δ_5^{40}	Δ_5^{41}	Δ_5^{42}	Δ_5^{43}	Δ_5^{44}	Δ_5^{45}	Δ_5^{46}	Δ_5^{47}
Δ_4^{26}	0	0	0	0	0	32	32	32	32	32	32	64
Δ_4^{27}	0	24	24	72	72	0	0	0	0	48	48	48
Δ_4^{28}	0	48	40	0	104	72	24	0	24	0	8	0
Δ_4^{29}	0	40	48	104	0	24	72	24	0	8	0	0
Δ_4^{30}	0	48	96	24	48	72	56	24	0	0	0	0
Δ_4^{31}	0	96	48	48	24	56	72	0	24	0	0	0
Δ_4^{32}	72	0	0	36	0	30	36	36	18	54	78	36
Δ_4^{33}	72	0	0	0	36	36	30	18	36	78	54	36
Δ_4^{34}	36	0	8	0	48	24	8	0	28	36	24	72
Δ_4^{35}	36	8	0	48	0	8	24	28	0	24	36	72
Δ_4^{36}	24	0	48	0	36	28	24	0	36	24	36	48
Δ_4^{37}	24	48	0	36	0	24	28	36	0	36	24	48
Δ_4^{38}	16	8	48	16	32	24	16	24	56	40	40	32
Δ_4^{39}	16	48	8	32	16	16	24	56	24	40	40	32
Δ_4^{40}	12	12	12	15	15	15	15	31	31	42	42	42
Δ_4^{41}	30	40	15	25	10	15	20	15	15	20	30	66
Δ_4^{42}	30	15	40	10	25	20	15	15	15	30	20	66
Δ_4^{43}	24	20	18	10	50	36	20	26	18	26	24	28
Δ_4^{44}	24	18	20	50	10	20	36	18	26	24	26	28
Δ_4^{45}	28	15	17	12	36	16	12	31	29	45	36	28
Δ_4^{46}	28	17	15	36	12	12	16	29	31	36	45	28
Δ_5^{1}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{2}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{3}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{4}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{5}	0	0	48	0	144	48	48	0	96	0	96	0
Δ_5^{6}	0	48	0	144	0	48	48	96	0	96	0	0
Δ_5^{7}	96	128	0	0	0	128	96	0	32	0	32	0
Δ_5^{8}	96	0	128	0	0	96	128	32	0	32	0	0
Δ_5^{9}	0	48	0	208	48	0	0	0	32	0	96	0
Δ_5^{10}	0	0	48	48	208	0	0	32	0	96	0	0
Δ_5^{11}	0	28	42	0	0	0	28	0	112	0	0	0
Δ_5^{12}	0	42	28	0	0	28	0	112	0	0	0	0
Δ_5^{13}	0	0	36	0	72	0	12	0	48	0	0	72
Δ_5^{14}	0	36	0	72	0	12	0	48	0	0	0	72
Δ_5^{15}	0	0	0	0	0	32	32	64	64	0	0	0
Δ_5^{16}	54	0	63	0	36	27	27	27	63	0	45	0
Δ_5^{17}	54	63	0	36	0	27	27	63	27	45	0	0
Δ_5^{18}	0	0	54	45	54	45	0	0	0	27	0	0
Δ_5^{19}	0	54	0	54	45	0	45	0	0	0	27	0

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	Δ_5^{48}	Δ_5^{49}	Δ_5^{50}	Δ_5^{51}	Δ_5^{52}	Δ_5^{53}	Δ_5^{54}	Δ_5^{55}	Δ_5^{56}	Δ_5^{57}	Δ_5^{58}	Δ_5^{59}
Δ_4^{26}	0	16	16	96	96	32	32	0	0	0	256	64
Δ_4^{27}	0	48	48	12	12	72	72	0	32	32	0	240
Δ_4^{28}	0	96	120	60	0	72	0	0	0	0	48	96
Δ_4^{29}	0	120	96	0	60	0	72	0	0	0	48	96
Δ_4^{30}	24	168	60	0	12	12	12	0	48	0	24	96
Δ_4^{31}	24	60	168	12	0	12	12	0	0	48	24	96
Δ_4^{32}	36	9	0	36	54	45	0	144	0	72	36	216
Δ_4^{33}	36	0	9	54	36	0	45	144	72	0	36	216
Δ_4^{34}	36	24	24	24	24	96	60	48	96	24	72	72
Δ_4^{35}	36	24	24	24	24	60	96	48	24	96	72	72
Δ_4^{36}	24	24	0	42	90	24	0	48	56	0	84	96
Δ_4^{37}	24	0	24	90	42	0	24	48	0	56	84	96
Δ_4^{38}	20	24	8	48	104	72	20	32	32	0	72	112
Δ_4^{39}	20	8	24	104	48	20	72	32	0	32	72	112
Δ_4^{40}	12	21	21	66	66	51	51	72	42	42	168	126
Δ_4^{41}	55	20	40	30	10	15	40	50	65	60	50	160
Δ_4^{42}	55	40	20	10	30	40	15	50	60	65	50	160
Δ_4^{43}	32	35	36	44	20	72	32	48	76	44	76	124
Δ_4^{44}	32	36	35	20	44	32	72	48	44	76	76	124
Δ_4^{45}	34	26	17	47	40	51	33	68	76	50	71	164
Δ_4^{46}	34	17	26	40	47	33	51	68	50	76	71	164
Δ_5^{1}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{2}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{3}	0	0	270	0	0	0	0	0	0	0	0	0
Δ_5^{4}	0	270	0	0	0	0	0	0	0	0	0	0
Δ_5^{5}	0	0	0	0	72	144	0	0	0	0	0	0
Δ_5^{6}	0	0	0	72	0	0	144	0	0	0	0	0
Δ_5^{7}	0	0	144	96	24	24	48	0	32	32	0	0
Δ_5^{8}	0	144	0	24	96	48	24	0	32	32	0	0
Δ_5^{9}	24	0	96	48	0	0	144	0	32	96	48	0
Δ_5^{10}	24	96	0	0	48	144	0	0	96	32	48	0
Δ_5^{11}	84	0	84	84	0	84	0	0	84	112	84	0
Δ_5^{12}	84	84	0	0	84	0	84	0	112	84	84	0
Δ_5^{13}	72	54	0	0	0	90	0	0	168	24	0	72
Δ_5^{14}	72	0	54	0	0	0	90	0	24	168	0	72
Δ_5^{15}	240	16	16	0	0	32	32	0	64	64	0	0
Δ_5^{16}	0	81	27	27	135	54	0	0	0	54	81	108
Δ_5^{17}	0	27	81	135	27	0	54	0	54	0	81	108
Δ_5^{18}	0	135	54	54	27	0	27	0	0	0	162	0
Δ_5^{19}	0	54	135	27	54	27	0	0	0	0	162	0

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	Δ_0^1	Δ_1^1	Δ_2^1	Δ_2^2	Δ_2^3	Δ_3^1	Δ_3^2	Δ_3^3	Δ_3^4	Δ_3^5	Δ_3^6	Δ_3^7
Δ_5^{20}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{21}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{22}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{23}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{24}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{25}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{26}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{27}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{28}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{29}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{30}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{31}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{32}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{33}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{34}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{35}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{36}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{37}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{38}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{39}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{40}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{41}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{42}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{43}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{44}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{45}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{46}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{47}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{48}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{49}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{50}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{51}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{52}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{53}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{54}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{55}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{56}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{57}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{58}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{59}	0	0	0	0	0	0	0	0	0	0	0	0

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	Δ_3^8	Δ_3^9	Δ_3^{10}	Δ_4^1	Δ_4^2	Δ_4^3	Δ_4^4	Δ_4^5	Δ_4^6	Δ_4^7	Δ_4^8	Δ_4^9
Δ_5^{20}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{21}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{22}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{23}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{24}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{25}	0	0	0	0	0	0	0	2	1	2	4	0
Δ_5^{26}	0	0	0	0	0	0	0	1	2	2	0	4
Δ_5^{27}	0	0	0	0	0	0	0	0	0	0	0	2
Δ_5^{28}	0	0	0	0	0	0	0	0	0	0	2	0
Δ_5^{29}	0	0	0	0	0	1	0	0	0	0	2	2
Δ_5^{30}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{31}	0	0	0	0	0	0	0	0	1	0	0	0
Δ_5^{32}	0	0	0	0	0	0	0	1	0	0	0	0
Δ_5^{33}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{34}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{35}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{36}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{37}	0	0	0	0	0	0	0	0	0	0	0	1
Δ_5^{38}	0	0	0	0	0	0	0	0	0	0	1	0
Δ_5^{39}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{40}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{41}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{42}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{43}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{44}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{45}	0	0	0	0	0	0	0	1	0	1	0	0
Δ_5^{46}	0	0	0	0	0	0	0	0	1	1	0	0
Δ_5^{47}	0	0	0	0	0	0	1	0	0	1	0	0
Δ_5^{48}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{49}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{50}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{51}	0	0	0	0	0	0	0	0	0	0	0	1
Δ_5^{52}	0	0	0	0	0	0	0	0	0	0	1	0
Δ_5^{53}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{54}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{55}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{56}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{57}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{58}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{59}	0	0	0	0	0	0	0	0	0	0	0	0

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	Δ_4^{10}	Δ_4^{11}	Δ_4^{12}	Δ_4^{13}	Δ_4^{14}	Δ_4^{15}	Δ_4^{16}	Δ_4^{17}	Δ_4^{18}	Δ_4^{19}	Δ_4^{20}	Δ_4^{21}
Δ_5^{20}	0	0	0	0	0	0	3	0	0	0	0	0
Δ_5^{21}	0	0	0	0	0	0	0	6	0	15	0	0
Δ_5^{22}	0	0	0	0	0	0	0	6	0	0	15	0
Δ_5^{23}	1	0	0	0	8	1	0	4	0	0	0	2
Δ_5^{24}	0	1	0	0	1	8	0	4	0	0	0	2
Δ_5^{25}	1	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{26}	0	1	0	0	0	0	0	0	0	0	0	0
Δ_5^{27}	3	0	0	0	0	0	3	0	1	0	0	0
Δ_5^{28}	0	3	0	0	0	0	3	0	1	0	0	0
Δ_5^{29}	0	0	0	0	0	0	6	0	0	0	0	0
Δ_5^{30}	2	2	1	0	0	0	4	0	0	0	0	0
Δ_5^{31}	0	0	1	2	0	1	0	0	2	3	1	0
Δ_5^{32}	0	0	1	2	1	0	0	0	2	1	3	0
Δ_5^{33}	1	0	0	0	1	0	0	0	0	4	0	2
Δ_5^{34}	0	1	0	0	0	1	0	0	0	0	4	2
Δ_5^{35}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{36}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{37}	0	0	0	0	0	0	0	1	0	4	2	2
Δ_5^{38}	0	0	0	0	0	0	0	1	0	2	4	2
Δ_5^{39}	0	0	0	0	0	3	0	2	0	1	0	1
Δ_5^{40}	0	0	0	0	3	0	0	2	0	0	1	1
Δ_5^{41}	0	0	0	0	0	0	0	0	0	6	3	3
Δ_5^{42}	0	0	0	0	0	0	0	0	0	3	6	3
Δ_5^{43}	0	0	0	0	0	0	0	0	1	3	0	0
Δ_5^{44}	0	0	0	0	0	0	0	0	1	0	3	0
Δ_5^{45}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{46}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{47}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{48}	0	0	0	1	0	0	0	0	0	0	0	0
Δ_5^{49}	0	0	0	0	3	0	0	2	0	1	0	2
Δ_5^{50}	0	0	0	0	0	3	0	2	0	0	1	2
Δ_5^{51}	0	0	0	0	0	1	1	0	1	0	0	0
Δ_5^{52}	0	0	0	0	1	0	1	0	1	0	0	0
Δ_5^{53}	1	0	1	0	0	0	0	0	2	0	0	0
Δ_5^{54}	0	1	1	0	0	0	0	0	2	0	0	0
Δ_5^{55}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{56}	0	0	0	0	0	0	0	0	1	0	0	0
Δ_5^{57}	0	0	0	0	0	0	0	0	1	0	0	0
Δ_5^{58}	0	0	0	0	0	0	2	0	1	0	0	0
Δ_5^{59}	0	0	0	1	0	0	0	0	0	0	0	1

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	Δ_4^{22}	Δ_4^{23}	Δ_4^{24}	Δ_4^{25}	Δ_4^{26}	Δ_4^{27}	Δ_4^{28}	Δ_4^{29}	Δ_4^{30}	Δ_4^{31}	Δ_4^{32}	Δ_4^{33}
Δ_5^{20}	0	0	21	0	0	0	0	0	0	0	0	0
Δ_5^{21}	0	0	0	0	0	5	10	10	15	30	0	0
Δ_5^{22}	0	0	0	0	0	5	10	10	30	15	0	0
Δ_5^{23}	0	0	0	0	0	4	36	0	30	0	0	0
Δ_5^{24}	0	0	0	0	0	4	0	36	0	30	0	0
Δ_5^{25}	0	12	6	0	6	4	0	0	0	0	6	12
Δ_5^{26}	0	12	6	0	6	4	0	0	0	0	12	6
Δ_5^{27}	15	0	0	0	9	0	0	0	0	9	0	0
Δ_5^{28}	15	0	0	0	9	0	0	0	9	0	0	0
Δ_5^{29}	21	9	6	0	36	0	0	0	0	0	0	0
Δ_5^{30}	6	8	16	0	0	0	2	2	2	2	0	0
Δ_5^{31}	2	0	0	7	6	10	2	0	2	2	0	20
Δ_5^{32}	2	0	0	7	6	10	0	2	2	2	20	0
Δ_5^{33}	4	4	0	0	0	4	0	12	6	4	0	0
Δ_5^{34}	4	4	0	0	0	4	12	0	4	6	0	0
Δ_5^{35}	0	0	5	20	0	0	0	0	0	0	10	10
Δ_5^{36}	0	0	3	12	0	0	0	0	0	0	12	12
Δ_5^{37}	0	0	0	0	0	3	6	5	6	12	0	0
Δ_5^{38}	0	0	0	0	0	3	5	6	12	6	0	0
Δ_5^{39}	0	3	3	0	0	9	0	13	3	6	6	0
Δ_5^{40}	0	3	3	0	0	9	13	0	6	3	0	6
Δ_5^{41}	0	0	0	6	3	0	9	3	9	7	5	6
Δ_5^{42}	0	0	0	6	3	0	3	9	7	9	6	5
Δ_5^{43}	0	6	1	9	3	0	0	3	3	0	6	3
Δ_5^{44}	0	6	1	9	3	0	3	0	0	3	3	6
Δ_5^{45}	3	5	4	3	3	6	0	1	0	0	9	13
Δ_5^{46}	3	5	4	3	3	6	1	0	0	0	13	9
Δ_5^{47}	10	0	0	0	5	5	0	0	0	0	5	5
Δ_5^{48}	1	0	0	7	0	0	0	0	2	2	4	4
Δ_5^{49}	0	0	0	0	1	4	8	10	14	5	1	0
Δ_5^{50}	0	0	0	0	1	4	10	8	5	14	0	1
Δ_5^{51}	0	0	1	2	6	1	5	0	0	1	4	6
Δ_5^{52}	0	0	1	2	6	1	0	5	1	0	6	4
Δ_5^{53}	0	6	5	1	2	6	6	0	1	1	5	0
Δ_5^{54}	0	6	5	1	2	6	0	6	1	1	0	5
Δ_5^{55}	0	0	0	3	0	0	0	0	0	0	12	12
Δ_5^{56}	3	0	2	3	0	2	0	0	3	0	0	6
Δ_5^{57}	3	0	2	3	0	2	0	0	0	3	6	0
Δ_5^{58}	5	5	4	0	8	0	2	2	1	1	2	2
Δ_5^{59}	0	0	0	0	1	5	2	2	2	2	6	6

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	Δ_4^{34}	Δ_4^{35}	Δ_4^{36}	Δ_4^{37}	Δ_4^{38}	Δ_4^{39}	Δ_4^{40}	Δ_4^{41}	Δ_4^{42}	Δ_4^{43}	Δ_4^{44}	Δ_4^{45}
Δ_5^{20}	0	0	0	0	0	0	63	0	0	0	0	126
Δ_5^{21}	0	0	0	10	0	0	0	6	6	30	15	30
Δ_5^{22}	0	0	10	0	0	0	0	6	6	15	30	15
Δ_5^{23}	27	0	0	2	0	6	8	0	24	102	24	48
Δ_5^{24}	0	27	2	0	6	0	8	24	0	24	102	24
Δ_5^{25}	33	0	26	0	60	36	60	0	24	66	0	156
Δ_5^{26}	0	33	0	26	36	60	60	24	0	0	66	96
Δ_5^{27}	9	18	9	27	0	0	39	72	18	63	54	72
Δ_5^{28}	18	9	27	9	0	0	39	18	72	54	63	0
Δ_5^{29}	3	3	9	9	9	9	78	18	18	45	45	36
Δ_5^{30}	24	24	0	0	4	4	48	16	16	32	32	72
Δ_5^{31}	20	8	4	16	20	2	0	64	64	76	80	88
Δ_5^{32}	8	20	16	4	2	20	0	64	64	80	76	40
Δ_5^{33}	1	14	1	15	0	24	36	24	12	12	36	48
Δ_5^{34}	14	1	15	1	24	0	36	12	24	36	12	72
Δ_5^{35}	5	5	0	0	0	0	20	52	52	40	40	90
Δ_5^{36}	9	9	6	6	6	6	12	36	36	36	36	84
Δ_5^{37}	0	2	0	12	3	18	12	48	18	30	27	45
Δ_5^{38}	2	0	12	0	18	3	12	18	48	27	30	51
Δ_5^{39}	0	12	0	9	6	12	15	30	12	15	75	36
Δ_5^{40}	12	0	9	0	12	6	15	12	30	75	15	108
Δ_5^{41}	6	2	7	6	9	6	15	18	24	54	30	48
Δ_5^{42}	2	6	6	7	6	9	15	24	18	30	54	36
Δ_5^{43}	0	7	0	9	9	21	31	18	18	39	27	93
Δ_5^{44}	7	0	9	0	21	9	31	18	18	27	39	87
Δ_5^{45}	9	6	6	9	15	15	42	24	36	39	36	135
Δ_5^{46}	6	9	9	6	15	15	42	36	24	36	39	108
Δ_5^{47}	15	15	10	10	10	10	35	66	66	35	35	70
Δ_5^{48}	6	6	4	4	5	5	8	44	44	32	32	68
Δ_5^{49}	4	4	4	0	6	2	14	16	32	35	36	52
Δ_5^{50}	4	4	0	4	2	6	14	32	16	36	35	34
Δ_5^{51}	4	4	7	15	12	26	44	24	8	44	20	94
Δ_5^{52}	4	4	15	7	26	12	44	8	24	20	44	80
Δ_5^{53}	16	10	4	0	18	5	34	12	32	72	32	102
Δ_5^{54}	10	16	0	4	5	18	34	32	12	32	72	66
Δ_5^{55}	6	6	6	6	6	6	36	30	30	36	36	102
Δ_5^{56}	12	3	7	0	6	0	21	39	36	57	33	114
Δ_5^{57}	3	12	0	7	0	6	21	36	39	33	57	75
Δ_5^{58}	6	6	7	7	9	9	56	20	20	38	38	71
Δ_5^{59}	3	3	4	4	7	7	21	32	32	31	31	82

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	Δ_4^{46}	Δ_5^1	Δ_5^2	Δ_5^3	Δ_5^4	Δ_5^5	Δ_5^6	Δ_5^7	Δ_5^8	Δ_5^9	Δ_5^{10}	Δ_5^{11}
Δ_5^{20}	126	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{21}	15	1	0	0	0	0	0	10	10	0	0	0
Δ_5^{22}	30	0	1	0	0	0	0	10	10	0	0	5
Δ_5^{23}	24	0	0	0	5	0	0	6	0	0	24	2
Δ_5^{24}	48	0	0	5	0	0	0	0	6	24	0	4
Δ_5^{25}	96	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{26}	156	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{27}	0	0	0	3	0	0	2	6	0	0	0	6
Δ_5^{28}	72	0	0	0	3	2	0	0	6	0	0	0
Δ_5^{29}	36	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{30}	72	0	0	0	0	0	0	0	0	2	2	0
Δ_5^{31}	40	0	0	4	0	0	0	0	6	6	0	0
Δ_5^{32}	88	0	0	0	4	0	0	6	0	0	6	0
Δ_5^{33}	72	0	0	0	0	0	6	0	0	0	0	2
Δ_5^{34}	48	0	0	0	0	6	0	0	0	0	0	6
Δ_5^{35}	90	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{36}	84	0	0	0	0	0	0	6	6	0	0	0
Δ_5^{37}	51	0	0	0	0	0	1	8	0	3	0	2
Δ_5^{38}	45	0	0	0	0	1	0	0	8	0	3	3
Δ_5^{39}	108	0	0	0	0	0	3	0	0	13	3	0
Δ_5^{40}	36	0	0	0	0	3	0	0	0	3	13	0
Δ_5^{41}	36	0	0	0	0	1	1	8	6	0	0	0
Δ_5^{42}	48	0	0	0	0	1	1	6	8	0	0	2
Δ_5^{43}	87	0	0	0	0	0	2	0	2	0	2	0
Δ_5^{44}	93	0	0	0	0	2	0	2	0	2	0	8
Δ_5^{45}	108	0	0	0	0	0	2	0	2	0	6	0
Δ_5^{46}	135	0	0	0	0	2	0	2	0	6	0	0
Δ_5^{47}	70	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{48}	68	0	0	0	0	0	0	0	0	1	1	4
Δ_5^{49}	34	0	0	0	1	0	0	0	6	0	4	0
Δ_5^{50}	52	0	0	1	0	0	0	6	0	4	0	4
Δ_5^{51}	80	0	0	0	0	0	1	4	1	2	0	4
Δ_5^{52}	94	0	0	0	0	1	0	1	4	0	2	0
Δ_5^{53}	66	0	0	0	0	2	0	1	2	0	6	4
Δ_5^{54}	102	0	0	0	0	0	2	2	1	6	0	0
Δ_5^{55}	102	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{56}	75	0	0	0	0	0	0	1	1	1	3	3
Δ_5^{57}	114	0	0	0	0	0	0	1	1	3	1	4
Δ_5^{58}	71	0	0	0	0	0	0	0	0	1	1	2
Δ_5^{59}	82	0	0	0	0	0	0	0	0	0	0	0

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	Δ_5^{12}	Δ_5^{13}	Δ_5^{14}	Δ_5^{15}	Δ_5^{16}	Δ_5^{17}	Δ_5^{18}	Δ_5^{19}	Δ_5^{20}	Δ_5^{21}	Δ_5^{22}	Δ_5^{23}
Δ_5^{20}	0	0	0	0	0	0	0	0	24	42	42	0
Δ_5^{21}	5	0	0	0	0	0	0	25	40	46	36	30
Δ_5^{22}	0	0	0	0	0	0	25	0	40	36	46	0
Δ_5^{23}	4	14	0	0	0	0	36	0	0	24	0	32
Δ_5^{24}	2	0	14	0	0	0	0	36	0	0	24	26
Δ_5^{25}	0	1	0	0	12	0	12	4	0	0	8	4
Δ_5^{26}	0	0	1	0	0	12	4	12	0	8	0	0
Δ_5^{27}	0	2	6	0	1	9	4	18	0	18	6	45
Δ_5^{28}	6	6	2	0	9	1	18	4	0	6	18	12
Δ_5^{29}	0	0	0	0	0	0	46	46	0	39	39	60
Δ_5^{30}	0	0	0	4	0	0	8	8	0	32	32	16
Δ_5^{31}	0	0	0	5	8	8	0	8	0	0	0	4
Δ_5^{32}	0	0	0	5	8	8	8	0	0	0	0	0
Δ_5^{33}	6	0	13	0	0	6	0	0	8	12	0	12
Δ_5^{34}	2	13	0	0	6	0	0	0	8	0	12	6
Δ_5^{35}	0	0	0	10	0	0	0	0	0	2	2	0
Δ_5^{36}	0	0	0	0	6	6	0	0	0	0	0	0
Δ_5^{37}	3	0	3	0	0	7	0	6	16	14	6	15
Δ_5^{38}	2	3	0	0	7	0	6	0	16	6	14	0
Δ_5^{39}	0	0	6	0	0	4	5	6	4	2	14	3
Δ_5^{40}	0	6	0	0	4	0	6	5	4	14	2	0
Δ_5^{41}	2	0	1	3	3	3	5	0	12	12	8	6
Δ_5^{42}	0	1	0	3	3	3	0	5	12	8	12	6
Δ_5^{43}	8	0	4	6	3	7	0	0	0	0	0	0
Δ_5^{44}	0	4	0	6	7	3	0	0	0	0	0	6
Δ_5^{45}	0	0	0	0	0	5	3	0	0	9	0	0
Δ_5^{46}	0	0	0	0	5	0	0	3	0	0	9	0
Δ_5^{47}	0	5	5	0	0	0	0	0	0	1	1	10
Δ_5^{48}	4	4	4	15	0	0	0	0	0	0	0	0
Δ_5^{49}	4	3	0	1	6	2	10	4	8	12	6	9
Δ_5^{50}	0	0	3	1	2	6	4	10	8	6	12	24
Δ_5^{51}	0	0	0	0	2	10	4	2	8	6	2	12
Δ_5^{52}	4	0	0	0	10	2	2	4	8	2	6	0
Δ_5^{53}	0	5	0	2	4	0	0	2	0	10	4	1
Δ_5^{54}	4	0	5	2	0	4	2	0	0	4	10	4
Δ_5^{55}	0	0	0	0	0	0	0	0	0	0	0	0
Δ_5^{56}	4	7	1	3	0	3	0	0	6	6	0	3
Δ_5^{57}	3	1	7	3	3	0	0	0	6	0	6	3
Δ_5^{58}	2	0	0	0	3	3	6	6	4	10	10	15
Δ_5^{59}	0	1	1	0	2	2	0	0	8	2	2	2

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	Δ_5^{24}	Δ_5^{25}	Δ_5^{26}	Δ_5^{27}	Δ_5^{28}	Δ_5^{29}	Δ_5^{30}	Δ_5^{31}	Δ_5^{32}	Δ_5^{33}	Δ_5^{34}	Δ_5^{35}
Δ_5^{20}	0	0	0	0	0	0	0	0	0	21	21	0
Δ_5^{21}	0	0	10	30	10	65	60	0	0	30	0	6
Δ_5^{22}	30	10	0	10	30	65	60	0	0	0	30	6
Δ_5^{23}	26	4	0	60	16	80	24	6	0	24	12	0
Δ_5^{24}	32	0	4	16	60	80	24	0	6	12	24	0
Δ_5^{25}	0	2	2	0	0	0	0	0	54	28	2	24
Δ_5^{26}	4	2	2	0	0	0	0	54	0	2	28	24
Δ_5^{27}	12	0	0	21	25	8	9	18	18	0	0	18
Δ_5^{28}	45	0	0	25	21	8	9	18	18	0	0	18
Δ_5^{29}	60	0	0	8	8	34	0	9	9	0	0	0
Δ_5^{30}	16	0	0	8	8	0	31	18	18	16	16	16
Δ_5^{31}	0	0	36	16	16	8	18	11	20	0	16	16
Δ_5^{32}	4	36	0	16	16	8	18	20	11	16	0	16
Δ_5^{33}	6	14	1	0	0	0	12	0	12	15	22	0
Δ_5^{34}	12	1	14	0	0	0	12	12	0	22	15	0
Δ_5^{35}	0	10	10	10	10	0	10	10	10	0	0	30
Δ_5^{36}	0	0	0	6	6	0	0	0	0	18	18	33
Δ_5^{37}	0	12	7	4	6	11	33	0	9	6	24	6
Δ_5^{38}	15	7	12	6	4	11	33	9	0	24	6	6
Δ_5^{39}	0	21	3	0	27	18	15	6	3	0	33	6
Δ_5^{40}	3	3	21	27	0	18	15	3	6	33	0	6
Δ_5^{41}	6	6	6	23	12	12	27	6	3	12	15	12
Δ_5^{42}	6	6	6	12	23	12	27	3	6	15	12	12
Δ_5^{43}	6	9	2	3	6	3	3	9	0	15	21	24
Δ_5^{44}	0	2	9	6	3	3	3	0	9	21	15	24
Δ_5^{45}	0	0	0	6	4	0	0	18	0	16	12	18
Δ_5^{46}	0	0	0	4	6	0	0	0	18	12	16	18
Δ_5^{47}	10	0	0	0	0	0	0	15	15	0	0	20
Δ_5^{48}	0	8	8	4	4	0	3	9	9	12	12	24
Δ_5^{49}	24	2	6	32	18	30	16	20	5	16	2	0
Δ_5^{50}	9	6	2	18	32	30	16	5	20	2	16	0
Δ_5^{51}	0	4	12	6	2	6	8	6	11	10	26	8
Δ_5^{52}	12	12	4	2	6	6	8	11	6	26	10	8
Δ_5^{53}	4	4	8	8	2	8	6	5	12	30	8	16
Δ_5^{54}	1	8	4	2	8	8	6	12	5	8	30	16
Δ_5^{55}	0	0	0	0	0	0	3	6	6	12	12	33
Δ_5^{56}	3	1	12	16	2	3	6	9	9	18	6	18
Δ_5^{57}	3	12	1	2	16	3	6	9	9	6	18	18
Δ_5^{58}	15	2	2	3	3	7	4	10	10	13	13	8
Δ_5^{59}	2	4	4	5	5	0	6	6	6	13	13	20

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	Δ_5^{36}	Δ_5^{37}	Δ_5^{38}	Δ_5^{39}	Δ_5^{40}	Δ_5^{41}	Δ_5^{42}	Δ_5^{43}	Δ_5^{44}	Δ_5^{45}	Δ_5^{46}	Δ_5^{47}
Δ_5^{20}	0	84	84	21	21	63	63	0	0	0	0	0
Δ_5^{21}	0	70	30	10	70	60	40	0	0	45	0	6
Δ_5^{22}	0	30	70	70	10	40	60	0	0	0	45	6
Δ_5^{23}	0	60	0	12	0	24	24	0	24	0	0	48
Δ_5^{24}	0	0	60	0	12	24	24	24	0	0	0	48
Δ_5^{25}	0	48	28	84	12	24	24	36	8	0	0	0
Δ_5^{26}	0	28	48	12	84	24	24	8	36	0	0	0
Δ_5^{27}	18	12	18	0	81	69	36	9	18	18	12	0
Δ_5^{28}	18	18	12	81	0	36	69	18	9	12	18	0
Δ_5^{29}	0	33	33	54	54	36	36	9	9	0	0	0
Δ_5^{30}	0	88	88	40	40	72	72	8	8	0	0	0
Δ_5^{31}	0	0	24	16	8	16	8	24	0	48	0	48
Δ_5^{32}	0	24	0	8	16	8	16	0	24	0	48	48
Δ_5^{33}	36	12	48	0	66	24	30	30	42	32	24	0
Δ_5^{34}	36	48	12	66	0	30	24	42	30	24	32	0
Δ_5^{35}	55	10	10	10	10	20	20	40	40	30	30	40
Δ_5^{36}	36	12	12	12	12	6	6	36	36	18	18	72
Δ_5^{37}	12	47	54	14	61	42	31	24	36	27	15	18
Δ_5^{38}	12	54	47	61	14	31	42	36	24	15	27	18
Δ_5^{39}	12	14	61	24	50	12	15	16	33	18	21	24
Δ_5^{40}	12	61	14	50	24	15	12	33	16	21	18	24
Δ_5^{41}	6	42	31	12	15	55	57	18	27	18	21	36
Δ_5^{42}	6	31	42	15	12	57	55	27	18	21	18	36
Δ_5^{43}	36	24	36	16	33	18	27	39	25	26	30	42
Δ_5^{44}	36	36	24	33	16	27	18	25	39	30	26	42
Δ_5^{45}	18	27	15	18	21	18	21	26	30	25	25	30
Δ_5^{46}	18	15	27	21	18	21	18	30	26	25	25	30
Δ_5^{47}	60	15	15	20	20	30	30	35	35	25	25	2
Δ_5^{48}	68	8	8	8	8	20	20	28	28	36	36	40
Δ_5^{49}	8	30	18	42	28	26	34	16	8	16	18	20
Δ_5^{50}	8	18	30	28	42	34	26	8	16	18	16	20
Δ_5^{51}	12	30	30	32	66	22	12	18	20	28	22	48
Δ_5^{52}	12	30	30	66	32	12	22	20	18	22	28	48
Δ_5^{53}	24	54	28	28	10	44	42	30	20	12	20	32
Δ_5^{54}	24	28	54	10	28	42	44	20	30	20	12	32
Δ_5^{55}	51	12	12	12	12	12	12	36	36	24	24	48
Δ_5^{56}	42	33	3	16	10	21	21	29	15	29	35	36
Δ_5^{57}	42	3	33	10	16	21	21	15	29	35	29	36
Δ_5^{58}	12	36	36	42	42	40	40	18	18	16	16	20
Δ_5^{59}	32	21	21	20	20	20	20	35	35	27	27	19

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	Δ_5^{48}	Δ_5^{49}	Δ_5^{50}	Δ_5^{51}	Δ_5^{52}	Δ_5^{53}	Δ_5^{54}	Δ_5^{55}	Δ_5^{56}	Δ_5^{57}	Δ_5^{58}	Δ_5^{59}
Δ_5^{20}	0	63	63	63	63	0	0	0	63	63	63	252
Δ_5^{21}	0	90	45	45	15	75	30	0	60	0	150	60
Δ_5^{22}	0	45	90	15	45	30	75	0	0	60	150	60
Δ_5^{23}	0	54	144	72	0	6	24	0	24	24	180	48
Δ_5^{24}	0	144	54	0	72	24	6	0	24	24	180	48
Δ_5^{25}	48	12	36	24	72	24	48	0	8	96	24	96
Δ_5^{26}	48	36	12	72	24	48	24	0	96	8	24	96
Δ_5^{27}	18	144	81	27	9	36	9	0	96	12	27	90
Δ_5^{28}	18	81	144	9	27	9	36	0	12	96	27	90
Δ_5^{29}	0	135	135	27	27	36	36	0	18	18	63	0
Δ_5^{30}	12	64	64	32	32	24	24	16	32	32	32	96
Δ_5^{31}	36	80	20	24	44	20	48	32	48	48	80	96
Δ_5^{32}	36	20	80	44	24	48	20	32	48	48	80	96
Δ_5^{33}	36	48	6	30	78	90	24	48	72	24	78	156
Δ_5^{34}	36	6	48	78	30	24	90	48	24	72	78	156
Δ_5^{35}	60	0	0	20	20	40	40	110	60	60	40	200
Δ_5^{36}	102	12	12	18	18	36	36	102	84	84	36	192
Δ_5^{37}	12	45	27	45	45	81	42	24	66	6	108	126
Δ_5^{38}	12	27	45	45	45	42	81	24	6	66	108	126
Δ_5^{39}	12	63	42	48	99	42	15	24	32	20	126	120
Δ_5^{40}	12	42	63	99	48	15	42	24	20	32	126	120
Δ_5^{41}	30	39	51	33	18	66	63	24	42	42	120	120
Δ_5^{42}	30	51	39	18	33	63	66	24	42	42	120	120
Δ_5^{43}	42	24	12	27	30	45	30	72	58	30	54	210
Δ_5^{44}	42	12	24	30	27	30	45	72	30	58	54	210
Δ_5^{45}	54	24	27	42	33	18	30	48	58	70	48	162
Δ_5^{46}	54	27	24	33	42	30	18	48	70	58	48	162
Δ_5^{47}	50	25	25	60	60	40	40	80	60	60	50	95
Δ_5^{48}	65	8	8	28	28	34	34	132	68	68	48	216
Δ_5^{49}	8	76	84	32	42	41	44	24	40	44	162	120
Δ_5^{50}	8	84	76	42	32	44	41	24	44	40	162	120
Δ_5^{51}	28	32	42	55	42	40	24	32	60	40	54	160
Δ_5^{52}	28	42	32	42	55	24	40	32	40	60	54	160
Δ_5^{53}	34	41	44	40	24	23	44	44	28	44	62	152
Δ_5^{54}	34	44	41	24	40	44	23	44	44	28	62	152
Δ_5^{55}	99	18	18	24	24	33	33	87	60	60	36	228
Δ_5^{56}	51	30	33	45	30	21	33	60	52	68	51	186
Δ_5^{57}	51	33	30	30	45	33	21	60	68	52	51	186
Δ_5^{58}	24	81	81	27	27	31	31	24	34	34	78	164
Δ_5^{59}	54	30	30	40	40	38	38	76	62	62	82	198

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Appendix

This appendix relates the work in [33] and [34] to the data about $\mathcal{G}$ in this paper. First we give the correspondence between the G_a-orbits in [34] and here.

Name in [34]	Name here	Name in [34]	Name here	Name in [34]	Name here
$\Delta_1(a)$	$\Delta_1^1(a)$	$\Delta_3^4(a)$	$\Delta_3^1(a)$	$\Delta_4^1(a)$	$\Delta_4^{16}(a)$
$\Delta_2^1(a)$	$\Delta_2^2(a)$	$\Delta_3^5(a)$	$\Delta_3^9(a)$	$\Delta_4^2(a)$	$\Delta_4^3(a)$
$\Delta_2^2(a)$	$\Delta_2^3(a)$	$\Delta_3^6(a)$	$\Delta_3^8(a)$	$\Delta_4^3(a)$	$\Delta_4^{18}(a)$
$\Delta_2^3(a)$	$\Delta_2^1(a)$	$\Delta_3^7(a)$	$\Delta_3^6(a)$	$\Delta_4^4(a)$	$\Delta_4^{17}(a)$
$\Delta_3^1(a)$	$\Delta_3^3(a)$	$\Delta_3^8(a)$	$\Delta_3^2(a)$	$\Delta_4^5(a)$	$\Delta_4^7(a)$
$\Delta_3^2(a)$	$\Delta_3^4(a)$	$\Delta_3^9(a)$	$\Delta_3^{10}(a)$	$\Delta_4^6(a)$	$\Delta_4^{21}(a)$
$\Delta_3^3(a)$	$\Delta_3^5(a)$	$\Delta_3^{10}(a)$	$\Delta_3^7(a)$		

The remainder of the appendix describes, for each of the G_a-orbits in the first three discs of a, the octad orbits (equivalently, the line orbits in a residue) in terms of the permutation representation for G given in § 4. We emphasize that below we are using the names in [34] for these G_a-orbits; the notation for L-orbits may be found in [33].

$\Delta_1(a)$, $L = Stab_G\{\Lambda_1\}$, where

$\Lambda_1 = \{6032, 6158, 6734, 22\,973, 22\,975, 22\,977, 38\,858, 83\,012\}$.

L-orbit	Size	Octad number	L-orbit	Size	Octad number
α_8	1	1	α_2	448	62
α_0	30	248	α_4	280	2

$\Delta_2^1(a)$, $L = Stab_G\{\Lambda_1\}$, where

$\Lambda_1 = \{22\,973, 22\,977, 38\,858, 83\,012\}$ and Λ_2 is the sextet given by
$\{\{4, 20, 77, 349\}, \{6393, 21\,350, 49\,646, 61\,991\},$
$\{2951, 3008, 3320, 12\,882\}, \{948, 970, 1080, 17\,319\},$
$\{17\,400, 21\,982, 22\,598, 62\,004\}, \{22\,973, 22\,977, 38\,858, 83\,012\}\}$.

L-orbit	Size	Octad number	L-orbit	Size	Octad number
$\alpha_{4,4^2}$	5	1	$\alpha_{2,2^4}$	240	3
$\alpha_{0,4^2}$	10	101	$\alpha_{0,2^4}$	120	344
$\alpha_{1,31^5}$	320	59	$\alpha_{3,31^5}$	64	5

$\Delta_2^2(a)$, $L = Stab_G\{\Lambda_1\}$, where

$\Lambda_1 = \{2, 43, 948, 16\,365, 17\,319, 22\,977, 29\,733, 83\,012\}$ and
$\Lambda_2 = \{22\,977, 83\,012\}$.

L-orbit	Size	Octad number	L-orbit	Size	Octad number
$\alpha_{8,2}$	1	1	$\alpha_{2,1}$	192	62
$\alpha_{2,2}$	16	111	$\alpha_{4,0}$	60	55
$\alpha_{4,2}$	60	2	$\alpha_{2,0}$	240	176
$\alpha_{4,1}$	160	6	$\alpha_{0,0}$	30	248

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$\Delta_2^3(a)$, $L = Stab_G\{\Lambda_1\}$, where

Λ_1 is the trio given by

$\{\{540, 573, 583, 586, 590, 1177, 1192, 1200\},$
$\{306\,821, 306\,823, 306\,922, 306\,923, 306\,925, 306\,927, 306\,935, 306\,936\},$
$\{2, 43, 183, 792, 948, 970, 1080, 17\,319\}\}.$

L-orbit	Size	Octad number	L-orbit	Size	Octad number
α_{80^2}	3	1	α_{42^2}	672	100
α_{4^2}	84	2			

$\Delta_3^1(a)$, $L = Stab_G\{\Lambda_1\}$, where

$\Lambda_1 = \{22\,973, 22\,977, 83\,012\}.$

L-orbit	Size	Octad number	L-orbit	Size	Octad number
α_3	21	1	α_1	360	6
α_2	168	3	α_0	210	101

$\Delta_3^2(a)$, $L = Stab_G\{\Lambda_1, \Lambda_2\}$, where

$\Lambda_1 = \{37\,797, 38\,920, 60\,738, 61\,698, 62\,101, 62\,131, 62\,135, 62\,140\}$ and
$\Lambda_2 = \{22\,977, 83\,012\}.$

L-orbit	Size	Octad number	L-orbit	Size	Octad number
$\alpha_{8,0}$	1	759	$\alpha_{2,2}$	56	8
$\alpha_{0,2}$	7	1	$\alpha_{4,1}$	112	146
$\alpha_{0,0}$	7	26	$\alpha_{4,0}^{(2)}$	112	744
$\alpha_{4,2}$	14	136	$\alpha_{2,0}$	168	49
$\alpha_{0,1}$	16	3	$\alpha_{2,1}$	224	5
$\alpha_{4,0}^{(1)}$	42	745			

$\Delta_3^3(a)$, $L = Stab_G\{\Lambda_1\}$, where

$\Lambda_1 = \{479, 1125, 1151, 2252, 6955, 16\,379, 22\,977, 83\,012\}$ and
$\Lambda_2 = \{22\,977, 83\,012\}.$

L-orbit	Size	Octad number	L-orbit	Size	Octad number
$\alpha_{8,2}$	1	1	$\alpha_{2,1}$	192	62
$\alpha_{2,2}$	16	100	$\alpha_{4,0}$	60	13
$\alpha_{4,2}$	60	2	$\alpha_{2,0}$	240	87
$\alpha_{4,1}$	160	3	$\alpha_{0,0}$	30	248

$\Delta_3^4(a)$, $L = Stab_G\{\Lambda_1\}$, where

$\Lambda_1 = \{22\,977, 83\,012\}.$

L-orbit	Size	Octad number	L-orbit	Size	Octad number
α_2	77	5	α_0	330	55
α_1	352	6			

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$\Delta_3^5(a)$, $L = Stab_G\{\Lambda_1, \Lambda_2, \Lambda_3\}$, where

$\Lambda_1 = \{445, 452, 1059, 1125, 16\,105, 17\,319, 28\,307, 83\,012\}$,
$\Lambda_1 = \{17\,319\}$,
$\Lambda_1 = \{83\,012\}$.

L-orbit	Size	Octad number	L-orbit	Size	Octad number
$\alpha_{8,1,1}$	1	1	$\alpha^{(2)}_{4,0,1}$	40	23
$\alpha^{(1)}_{0,0,0}$	10	617	$\alpha_{4,1,1}$	60	2
$\alpha_{2,1,1}$	16	111	$\alpha_{4,0,0}$	60	55
$\alpha^{(2)}_{0,0,0}$	20	248	$\alpha_{2,1,0}$	96	100
$\alpha^{(1)}_{4,1,0}$	40	11	$\alpha_{2,0,1}$	96	300
$\alpha^{(2)}_{4,1,0}$	40	81	$\alpha_{2,0,0}$	240	176
$\alpha^{(1)}_{4,0,1}$	40	13			

$\Delta_3^6(a)$, $L = Stab_G\{\Lambda_1, \Lambda_2, \Lambda_3\}$, where

$\Lambda_1 = \{4, 970, 1080, 12\,882, 17\,319, 21\,350, 22\,598, 83\,012\}$,
$\Lambda_1 = \{970, 1080, 17\,319, 83\,012\}$,
$\Lambda_1 = \{83\,012\}$.

L-orbit	Size	Octad number	L-orbit	Size	Octad number
$\alpha_{8,4,1}$	1	1	$\alpha_{4,2,1}$	72	2
$\alpha^{(1)}_{0,0,0}$	6	248	$\alpha_{2,0,0}$	96	491
$\alpha^{(2)}_{0,0,0}$	24	504	$\alpha_{2,1,0}$	192	195
$\alpha_{4,4,1}$	4	15	$\alpha_{2,2,0}$	48	226
$\alpha_{4,1,1}$	16	21	$\alpha_{4,0,0}$	4	102
$\alpha_{2,2,1}$	48	213	$\alpha_{4,1,0}$	48	10
$\alpha_{4,3,1}$	48	17	$\alpha_{4,2,0}$	72	6
$\alpha_{2,1,1}$	64	150	$\alpha_{4,3,0}$	16	65

$\Delta_3^7(a)$, $L = Stab_G\{\Lambda_1, \Lambda_2, \Lambda_3\}$, where

$\Lambda_1 = \{4, 349, 970, 3320, 12\,882, 17\,319, 49\,646, 61\,991\}$,
$\Lambda_2 = \{11\,170, 12\,411, 12\,416, 12\,422, 20\,545, 20\,551, 20\,560, 22\,613\}$,
Λ_3 is the partition of Λ_1 given by $\{4, 349\}$, $\{970, 17\,319\}$, $\{3320, 12\,882\}$, $\{49\,646, 61\,991\}$.

L-orbit	Size	Octad number	L-orbit	Size	Octad number
$\alpha_{8,0,2^4}$	1	1	$\alpha_{4,0,2^2}$	12	400
$\alpha_{0,8,0^4}$	1	595	$\alpha_{4,2,1^4}$	32	44
$\alpha_{0,0,0^4}$	1	635	$\alpha_{4,2,21^2}$	192	2
$\alpha^{(1)}_{0,4,0^4}$	12	730	$\alpha_{2,2,2}$	32	261
$\alpha^{(2)}_{0,4,0^4}$	16	504	$\alpha_{2,4,2}$	32	510
$\alpha_{4,4,1^4}$	16	24	$\alpha_{2,2,1^2}$	192	408
$\alpha_{4,0,1^4}$	16	56	$\alpha_{2,4,1^4}$	192	406
$\alpha_{4,4,2^2}$	12	113			

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$\Delta_3^8(a)$, $L = Stab_G\{\Lambda_1, \Lambda_2, \Lambda_3\}$, where

Λ_1 is the sextet whose tetrads are $\{540, 573, 583, 590\}, \{300\,337, 301\,248, 301\,594, 305\,089\}$, $\{300\,364, 300\,688, 301\,606, 305\,099\}, \{948, 970, 1080, 17\,319\}, \{1749, 1850, 1883, 1896\}$, $\{2951, 3008, 3320, 12\,882\}$,
$\Lambda_2 = \{540, 573, 583, 590, 300\,337, 300\,364, 300\,688, 301\,248, 301\,594, 301\,606, 305\,089, 305\,099\}$
$\Lambda_3 = \{948, 970, 1080, 1749, 1850, 1883, 1896, 2951, 3008, 3320, 12\,882, 17\,319\}$.

L-orbit	Size	Octad number	L-orbit	Size	Octad number
$\alpha_{4^2,8,0}$	3	751	$\alpha_{2^4,2,6}$	72	3
$\alpha_{4^2,0,8}$	3	1	$\alpha_{2^4,4,4}$	216	100
$\alpha_{4^2,4,4}$	9	723	$\alpha_{31^5,5,3}$	192	114
$\alpha_{2^4,6,2}$	72	214	$\alpha_{31^5,3,5}$	192	5

$\Delta_3^9(a)$, $L = Stab_G\{\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4\}$, where

$\Lambda_1 = \{2, 445, 452, 948, 1059, 1151, 16\,105, 16\,379\}$,
$\Lambda_2 = \{30\,887, 34\,121, 52\,240, 57\,768, 102\,195, 142\,053, 273\,221, 297\,652\}$,
$\Lambda_3 = \{34\,642, 51\,319, 56\,950, 79\,889, 102\,237, 142\,051, 302\,809, 302\,904\}$,
$\Lambda_4 = \{2, 948\}$.

L-orbit	Size	Octad number	L-orbit	Size	Octad number
$\alpha_{8,0,0,2}$	1	1	$\alpha_{4,4,0,0}$	6	105
$\alpha_{0,8,0,0}$	1	741	$\alpha_{4,0,4,0}$	6	81
$\alpha_{0,0,8,0}$	1	594	$\alpha_{4,2,2,0}$	48	11
$\alpha^{(1)}_{0,4,4,0}$	12	368	$\alpha_{2,2,4,2}$	8	116
$\alpha^{(2)}_{0,4,4,0}$	16	248	$\alpha_{2,4,2,2}$	8	188
$\alpha_{4,4,0,2}$	6	94	$\alpha_{2,2,4,1}$	96	62
$\alpha_{4,0,4,2}$	6	23	$\alpha_{2,4,2,1}$	96	108
$\alpha_{4,2,2,2}$	48	3	$\alpha^{(1)}_{2,4,2,0}$	24	235
$\alpha_{4,4,0,1}$	16	18	$\alpha^{(2)}_{2,4,2,0}$	96	100
$\alpha_{4,0,4,1}$	16	38	$\alpha^{(1)}_{2,2,4,0}$	24	253
$\alpha^{(1)}_{4,2,2,1}$	96	2	$\alpha^{(2)}_{2,2,4,0}$	96	150
$\alpha^{(2)}_{4,2,2,1}$	32	5			

$\Delta_3^{10}(a)$, $L = Stab_G\{\Lambda_1, \Lambda_2, \Lambda_3\}$, where

$\Lambda_1 = \{43, 948, 17\,319, 29\,733\}$,
$\Lambda_2 = \{158\,373, 169\,472\}$,
$\Lambda_3 = \{182\,449, 194\,482\}$.

L-orbit	Size	Octad number	L-orbit	Size	Octad number
$\alpha_{1,1,0}$	128	653	$\alpha_{3,0,1}$	32	5
$\alpha_{1,0,1}$	128	657	$\alpha_{4,2,2}$	1	136
$\alpha_{1,1,2}$	32	649	$\alpha_{4,0,0}$	4	662
$\alpha_{1,2,1}$	32	292	$\alpha^{(1)}_{0,0,0}$	6	101
$\alpha_{2,2,0}$	24	77	$\alpha^{(2)}_{0,0,0}$	24	607
$\alpha_{2,0,2}$	24	14	$\alpha_{0,2,2}$	4	1
$\alpha_{2,1,1}$	96	24	$\alpha_{0,2,0}$	16	519
$\alpha_{2,0,0}$	96	3	$\alpha_{0,0,2}$	16	511
$\alpha_{3,1,0}$	32	10	$\alpha_{0,1,1}$	64	386

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