MITROVIĆ, D. AND ŽUBRINIĆ, D. *Fundamentals of applied functional analysis* (Pitman Monographs and Surveys in Pure and Applied Mathematics, Vol. 91, Addison Wesley Longman, Harlow, 1998), 399 pp., 0 582 24694 6, £72.

The book in question is a textbook of Applied Functional Analysis, by which, as in this case, one nowadays usually means function-analytic methods in partial differential equations. There are some books that cover similar material at a similar level. I would mention (and recommend its translation into English) H. Brezis's elegant little blue book **[1]** as well as the substantial recent book by L. C. Evans [3]; there is also some overlap with the material in Smoller's well-known book [4].

Since the topic of nonlinear partial differential equations has grown enormously in recent years, every book on it must perforce make a biased selection of topics; what is on offer in the present text seems to me a balanced and judicious one.

In general, with reservations to be made clear below, this book constitutes a valuable addition to the literature at the beginning postgraduate level. Its outstanding feature is a large number of clearly laid out examples and exercises that are (unusually at this level) provided with full solutions. On the minus side, I must say that it was not thoroughly proof-read: both references and spell-checking could do with a bit more editorial work. In most cases this causes only minor irritation, but "temepred" for "tempered" on p. 131 at the point where tempered distributions are introduced, Tietz for Tietze and Peam for Pym are more annoying.

The book starts with a three chapter discussion of Lebesgue measure, integration and the theory of L^p spaces. Much of the material is standard, but a thorough discussion of mollifiers and Nemytski operators is certainly welcome. Unfortunately, some of the presentation is pedagogically inconsistent: it is not clear why weak convergence is not included in the discussion of convergence in L^p, but is instead relegated to the Appendix. In general, the division of the material between the main text and the Appendix is somewhat arbitrary. For example, Alaoglu's theorem is a veritable cornerstone of function-analytic methods in PDEs and should have been discussed in the text. Also, in a future edition the authors should probably remove all mention of differentiability from Chapter 3 as all the necessary concepts are only introduced 300 pages later. I am sure beginning graduate students would also appreciate more guidance concerning the relevance of some of the included material, such as convergence in measure.

Chapters 4 and 5 deal with distribution theory and Sobolev spaces. The discussion of Chapter 4 is well-motivated and suitably leisurely. I expected a mention of pseudo-differential operators (or at least one reference), since all the necessary machinery for their discussion had been introduced.

Chapter 5 contains one of the best concise introductions to Sobolev spaces that I have seen. A thorough grounding in Sobolev spaces (embedding and trace theorems) is an absolute requirement in the study of PDEs and hence the hands-on approach of the authors (which includes exercises such as No. 3 on p. 197: Let Ω be an open bounded set in $\mathbb{R}^n, n \leq 6$. Let $u \in H_0^1(\Omega)$. Show that $u^2 \in H^{-1}(\Omega)$.) is very appropriate.

Chapters 6 and 7 deal with linear and nonlinear elliptic boundary value problems. Sections 1–7 of Chapter 6 give a useful overview of the linear theory and basic tools (Lax-Milgram theorem, bootstrapping arguments, methods based on the maximum principle and so on), while Section 8 inaugurates the study of solvability of nonlinear elliptic boundary value problems dealt with in Chapter 7 with the beautiful identity of Pohozhaev.

Chapter 7 is the heart of the book and presents material not usually found at this level. I found the classification scheme of Section 1 a very useful way of organizing this gargantuan topic. Both the presentation of variational methods and of the theory of monotone operators are clear and suitable for a first encounter.

Finally, Chapter 8 deals with the applications (in existence proofs and in global bifurcation theory) of the topological degree. This material is treated in a number of other sources, of which the one the authors refer the reader to, P. Rabinowitz's 1975 lecture notes, is not readily available, and another, which is missing from their bibliography, is the excellent monograph of K. Deimling **[2]**.

To summarize: this text will prove very useful to a beginning postgraduate student working in the area of nonlinear PDEs. The book is not perfect, both in the organization of the material and in the finer points of proofreading; it is to be hoped that all of these shortcomings will be corrected in subsequent printings.

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REFERENCES

1. H. BREZIS, *Analyse fonctionnelle, théorie et applications* (Masson, 1983).

2. K. DEIMLING, *Nonlinear functional analysis* (Springer-Verlag, 1985).

3. L. C. EVANS, *Partial differential equations* (Amer. Math. Soc., 1998).

4. J. SMOLLER, *Shock waves and reaction-diffusion equations* (Springer-Verlag, 1983).

PRETZEL, O. *Codes and algebraic curves* (Oxford Lecture Series in Mathematics and its Applications No. 8, Clarendon Press, 1998), xii + 192 pp., 0 19 850039 4, £35.

The algebraic-geometry codes known as geometric Goppa codes, discovered in 1981, have extraordinary error-correcting capacity. This text has the single specific aim of making them and their background comprehensible to those who, drawn by the impressive credentials, nevertheless are daunted by the formidable nature of the machinery their definition and properties involve. The author's intention is to provide a geometrically intuitive, but rigorous, approach that will harmonise with the modern algebraic one presented, for example, in the (undeniably superb) volume by H. Stichtenoth, *Algebraic function fields and codes* (Springer-Verlag, 1991), and will allow the reader to access to the relative sophistication of the latter. Arising out of an earlier text of the author, *Error correcting codes and finite fields* (Oxford, 1992), it is not in itself a general work on codes, but is, however, essentially self-contained.

The work is divided into two parts, each of which can be read largely independently of the other. Part I contains a transparent and frankly affine account of the concepts and theory of plane curves (with analogies to the theory of functions) up to the statement (only) of key theorems such as the Riemann-Roch theorem. This gives the reader a clear feel for divisors, their degree and dimensions of associated linear spaces, genus, etc. The geometric Goppa codes associated with *smooth* plane curves over a finite field F_q are then defined and their parameters and properties explained in terms of these numbers. A feature is the focus on specific examples, the (elliptic) cubic $x^3 + y^3 = 1$, the Klein quartic $x^3y + y^3 + x = 0$ and the (Hermitian) quintic $x^5 + y^5 = 1$ over F_{16}, for which a handy compact version is tabulated. There is also a full account of the error-processing algorithm of Skorobogatov-Vlădut (1990), which is fairly simple but does not allow correction up to the capability of the code, and that of Duursma (1993), which deals with this weakness but may not always be practical.

Part II contains the elements of the theory of function fields of one variable in the Chevalley-Deuring-Stichtenoth tradition but tailored to and simplified for the present context. This works well and all the theorems of Part I are duly justified. What I did miss here was a review of the definition of a geometric Goppa code in respect of a general function field and its first degree places. Though there would have been formal similarity to material in Part I, the increased scope to function fields, presented as fields of curves in higher dimensional space or as extensions of non-rational function fields, might have merited some space. This is particularly relevant to the final chapter which describes how the rates of geometric Goppa codes can approach (or even beat) the famous Gilbert (or Gilbert-Varshamov) lower bound: the codes in question cannot be those associated with plane curves. Indeed, recent work, such as that of Niederreiter and Xing, and that of Stepanov, tends to be in this direction. This having been said,