

# CLAY MINERAL EFFECTS ON THE STRESS-STRAIN RESPONSE OF SOILS IN DIRECT SHEAR

*by*

ROBERT L. KONDNER and JOSÉ R. VENDRELL, Jr.\*

The Technological Institute, Northwestern University, Evanston, Illinois

## ABSTRACT

More than one hundred sets of direct shear tests were conducted on soils composed of four reference clay minerals and various combinations of the four clay minerals of the Columbia University Clay Mineral Standards Project (A.P.I. 49) to illustrate with quantitative results the influence of clay mineralogy on the stress-strain characteristics of soils and hence the practical importance of clay mineralogy in soil mechanics. The soils tested consisted of various amounts by weight of Bedford Indiana Halloysite, Mesa Alta New Mexico Kaolinite, Santa Rita New Mexico Montmorillonite and Fithian Illinois Illite prepared with distilled water. The experimental results agree quite well with hyperbolic stress-strain relations previously developed by the author. The Hvorslev strength parameters are written as exponential functions of the clay mineral content and are included in the hyperbolic stress-strain law. The ultimate shear strength of the soils are mathematically expressed in terms of the stress history, confinement, and mineralogy.

## INTRODUCTION

It has become increasingly apparent in recent years that future advances in the technology of clay-water systems will be made through the mutual cooperation of physical chemists, crystallographers, mineralogists, geologists, soil mechanicians, and rheologists. A minimizing of the importance of the work of any one group by one or more of the remaining disciplines can considerably impede progress in the basic understanding of clay-water phenomena and in the mathematical formulation of such phenomena under various past, present and future environmental conditions. Although the present paper is concerned primarily with those aspects of clay-water technology of particular interest to soil rheologists and soil mechanicians, it is hoped that other groups will keep in mind the stress-strain-time response spectra problems of the soil rheologist and make significant contributions in that area.

This paper is the first in a series of rheologic investigations to quantitatively illustrate the influence of various types and relative amounts of clay minerals on the stress-strain-time response characteristics of soils. Since certain clay minerals which may be present in very small amounts may exert a tremendous influence on the attributes of a clay type soil, it is not adequate to determine

\* Soils Engineer, Ponce, Puerto Rico.

the engineering properties of the major clay minerals themselves but the properties of various combinations of clay minerals must also be investigated.

This is necessary because the properties of a clay soil composed of various clay minerals can seldom be predicted from the properties of its clay mineral components by proportioning as a simple mixture controlled by the relative amounts of the various components. The reason for this is that the generalized principle of superposition is not valid and the various mineral components of which the soil is composed do not act independently of each other.

The present paper is concerned with the influence of four reference clay minerals and various combinations of the four clay minerals of the Columbia University, Clay Mineral Standards, Project 49, conducted under the auspices of the American Petroleum Institute, on the stress-strain characteristics of clay soils in direct shear and the mathematical formulation of this influence. The soils tested consisted of various amounts of Bedford Indiana Halloysite, Mesa Alta New Mexico Kaolinite, Santa Rita New Mexico Montmorillonite, and Fithian Illinois Illite. Although thousands of specimens of the reference minerals have been studied by numerous organizations, most of the studies have pertained to chemical or geological aspects (Kerr, 1959) with relatively little attention being given to the rheologic aspects.

It is not the purpose of this paper to present an explicit form of a general formulation for clay mineral effects on the stress-strain response of soils, but simply to demonstrate a mathematical formulation of the response of a particular group of clay minerals, which exist in nature, when they are subjected to direct shear testing. It is recognized that bulk samples of the API reference clays can be quite variable in nature and no attempt has been made to repeat the API study for the particular bulk samples used. This is no attempt to minimize the importance of the particular mineralogy, exchangeable ions, etc., on the engineering characteristics of soils, but simply to point out that the present paper is restricted to a particular group of clay minerals tested under limited conditions. The fact that the API reference clays are variable in nature makes the present results even more interesting. Thus, the present paper may shed some insight into more general formulations of the results of more extensive future studies.

The authors would like to express their appreciation to the William H. and Caroline H. Pagon Fund of Baltimore for supporting some phases of this research.

## EXPERIMENTAL TECHNIQUES

### *Direct Shear Test*

The particular direct shear apparatus used in this investigation is schematically shown in Fig. 1. The cylindrical soil specimen, diameter 2.5 in. and height 1 in., is placed in the two separate sections of the direct shear box such that half of the specimen is contained in the upper frame and half in the lower

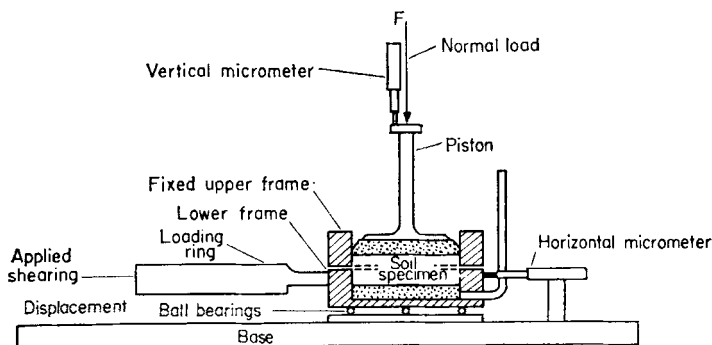


FIGURE 1.—Schematic diagram of direct shear apparatus.

frame. A vertical normal load  $F$  is applied to the soil specimen by weights acting through a hanger and piston system. The lower frame of the shear box is mounted on ball bearings in a race and is free to move horizontally relative to the upper frame which is fixed to the base frame. This horizontal displacement causes the soil specimen to shear along the horizontal plane separating the two halves of the shear box. Horizontal shear displacements are applied to the lower frame through a calibrated proving ring by a motor driven gear system. The amount of shear displacement is measured with the horizontal micrometer indicator dial and the compression or expansion of the sample in the vertical direction is obtained from the vertically mounted micrometer indicator dial.

The various advantages, disadvantages, and factors pertaining to the direct shear type of test such as specimen volume control, nature of boundary constraint, stress and moisture distribution, nonhomogeneity of strain, strain rate, etc., are discussed fully in the ASTM Special Technical Publication No. 131 (1952) and are not reviewed here.

#### *Soils Tested*

The soils tested consisted of the four reference clay minerals Bedford Indiana Halloysite, Mesa Alta New Mexico Kaolinite, Santa Rita New Mexico Montmorillonite and Fithian Illinois Illite and various combinations of them on a weight basis. These clay minerals are from the set of the Reference Clay Minerals, Research Project 49, sponsored by the American Petroleum Institute, for which typical characteristics such as geologic origin, chemical analyses, pH data, base exchange data, differential thermal data, diffraction data, etc., can be found in the various reports of Project 49 (Kerr *et al.*, 1949, 1950).

The clay minerals were obtained in dry form and powdered in a ball mill. Specimens of various composition were proportioned by dry weight and carefully prepared at the liquid limit using distilled water.

## RHEOLOGY CONSIDERATIONS

This paper is concerned primarily with the compositional effects of clay minerals upon the stress-strain response of clay-water systems and, as such, strain rate (time effects) as well as the explicit nature of the group of clay minerals and pore liquid have not been considered variables but have been considered constant. In examining this problem, consideration must be given to the form of deformable mechanics presently utilized in soil mechanics. Soil mechanicians have traditionally worked with a single parameter, the maximum, or failure, shear stress, for any one type of stress-strain test; that is, they worked with only one point on the stress-strain curve rather than consider the entire range of the stress-strain relation. Such a formulation is a stability or limit analysis rather than one of load deformation (stress-strain).

Limit analysis techniques for a soil are based on the failure conditions at the time of failure in the plane of failure and are expressed in terms of a maximum or limiting shear stress which is denoted as the shear strength.

The most widely used formulations of shear strength have their origins back in the eighteenth-century work of Coulomb (1776) who expressed the soil strength criterion in terms of a straight line failure envelope in a two-dimensional stress space as shown in Fig. 2. More recent developments by Terzaghi (1938) and others as reviewed by Hvorslev (1960) can be used to obtain the failure criterion in the form

$$\tau_f = \bar{c} + \sigma_c \tan \psi + \bar{\sigma}_{nf} \tan \phi_e \quad (1)$$

which, except for the inclusion of  $\bar{c}$ , is usually referred to as the Krey-Tiedemann form of the Coulomb failure criterion, where  $\bar{c}$  is a parameter representing the natural or non-strainhardened shear strength,  $\tau_f$  is the limiting shear strength,  $\sigma_c$  is the maximum stress to which the specimen has been consolidated,  $\psi$  is the cohesion angle,  $\bar{\sigma}_{nf}$  is the effective normal stress on the failure plane, and  $\phi_e$  is the effective angle of internal friction.

If the term  $\sigma_c \tan \psi$  is considered to be the increase in ultimate shear strength due to consolidation stresses and physicochemical processes and the term  $\bar{\sigma}_{nf} \tan \phi_e$  is considered to be the increase in ultimate shear strength due to the application of an effective normal stress on the failure surface which may be considered transient in nature; the form of equation (1) becomes convenient for the experimental determination of soil shear strength parameters. Consider two identical test specimens that are consolidated under a vertical stress  $\sigma_{c2}$ , of which one is sheared under a quite low effective normal stress  $(\bar{\sigma}_{nf})_1 = \sigma_{c1}$  and the other sheared at a significantly higher effective normal stress  $(\bar{\sigma}_{nf})_2 = \sigma_{c2}$ . The reduction in shear strength under the lower effective normal stress may be considered to be

$$(\Delta\tau_f)_{21} = \tau_{f22} - \tau_{f21} = (\sigma_{c2} - \sigma_{c1}) \tan \phi_e. \quad (2)$$

The parameter  $\phi_e$  can then be written as:

$$\phi_e = \tan^{-1} \left[ \frac{(\Delta\tau_f)_{21}}{(\sigma_{c2} - \sigma_{c1})} \right]. \quad (3)$$

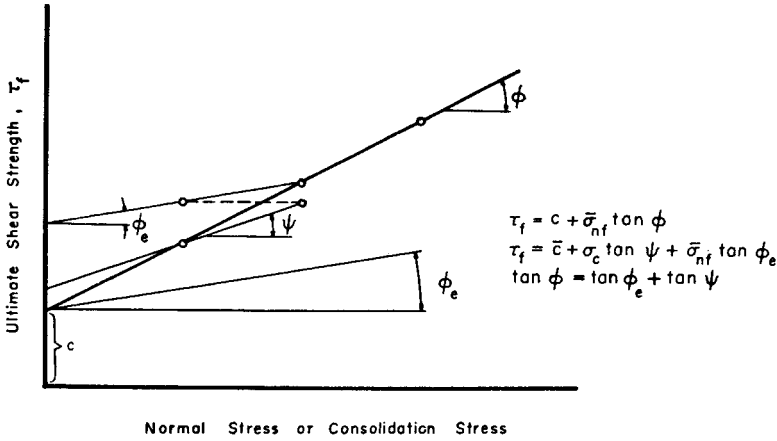


FIGURE 2.—Failure surface in two-dimensional stress space.

If a third identical specimen is consolidated at  $\sigma_{c1}$  and sheared at an effective normal stress of  $\sigma_{c1}$ , its shear strength can be considered to be less than the specimen that was consolidated at  $\sigma_{c2}$  and sheared under an effective normal stress of  $\sigma_{c1}$  by an amount

$$(\Delta\tau_f)_{12} = \tau_{f21} - \tau_{f11} = (\sigma_{c2} - \sigma_{c1})\tan \psi. \tag{4}$$

The parameter  $\psi$  can then be determined as

$$\psi = \tan^{-1} \left[ \frac{(\Delta\tau_f)_{12}}{(\sigma_{c2} - \sigma_{c1})} \right]. \tag{5}$$

If the shear strength could be truly represented as the sum of the linear functions which compose equation (1), the three shear tests would be sufficient to determine  $\phi_e$  and  $\psi$  as given by equations (3) and (5). However, equations (1), (3), and (5) are only approximations to the actual soil shearing strength response and as such it is deemed advisable to repeat the process for other consolidation and effective normal stresses and average the results.

Previous research by the senior author involving uniaxial and triaxial compression of soils as well as the response of soil–solid systems indicated that the two constant hyperbolic form of mathematical relation for stress–strain has a number of very interesting and significant aspects (Kondner, 1962a, pp.19–27; Kondner and Krizek, 1962, pp.16–22; Kondner, Krizek, and Schimming, 1962, pp.20–25). It has been shown (Kondner, 1962b) that a stress–strain curve which can be represented as a rectangular hyperbola as given in Fig. 3 and written as

$$\tau = \frac{\gamma}{a + b\gamma} \tag{6}$$

can be written in the straight line form

$$\frac{\gamma}{\tau} = a + b\gamma \tag{7}$$

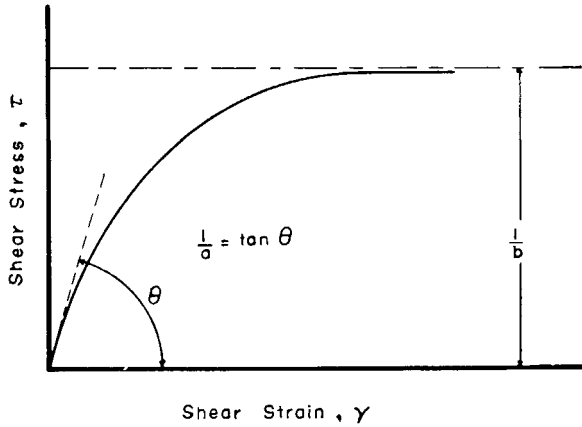


FIGURE 3.—Rectangular hyperbola representation of stress-strain.

as shown in Fig. 4, where  $\tau$  is the shear stress,  $\gamma$  is the shear strain, the intercept ( $a$ ) is inversely proportional to the initial tangent modulus of the material, and the slope ( $b$ ) is inversely proportional to the ultimate shear strength of the material as given by the mathematical limit of equation (6) as  $\gamma$  becomes very large.

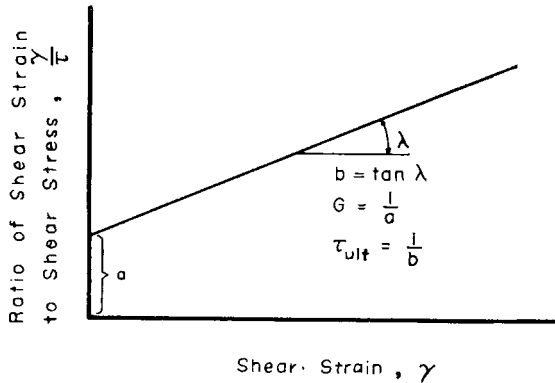


FIGURE 4.—Transformed hyperbolic representation of stress-strain.

### EXPERIMENTAL STRESS-STRAIN RESPONSE

One of the major inherent difficulties of the conventional direct shear test is that only the total shear displacement is measured. There is no positive way to obtain the shear strain because the thickness of the shear zone and the distributions of the shear strains, both across and along the shear zones, are unknown. Considering these difficulties it seems rational to assume that the

gross effective shear strains can be represented as being proportional to the total shear displacements. If the shear strain,  $\gamma$ , is replaced by the shear displacement,  $\delta$ , the form of equation (6) becomes

$$\tau = \frac{\delta}{a + b\delta} \tag{8}$$

where the coefficients  $a$  and  $b$  have the significance previously given and may be functions of the clay-water system considered, the applied strain rate, stress history, and the effective normal stress during shear. As indicated during the discussion of the hyperbolic equation, previous research indicated that the inverse of the slope  $b$  is a reasonable measure of the ultimate shear strength. The present study also gives such results.

Figure 5 is a typical family of shear stress versus shear displacement curves

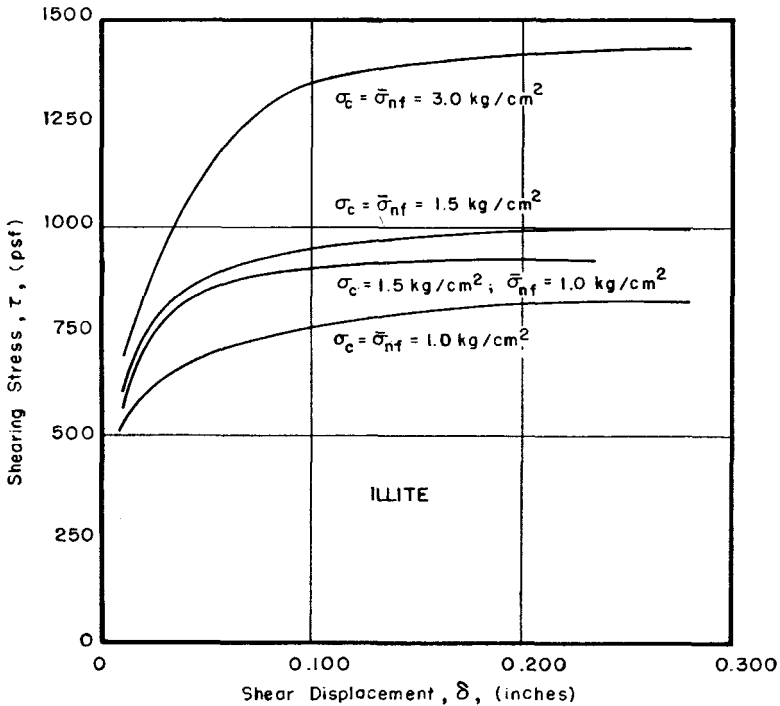


FIGURE 5.—Stress-strain curves for various consolidation and normal pressures

for various values of the effective normal stress on the shear plane. All four specimens were prepared at the liquid limit, consolidated under various values of normal stress,  $\sigma_c$ , and then sheared under various values of normal stress  $\bar{\sigma}_{nf}$ . As previously discussed, by consolidating a specimen with a normal

stress  $\sigma_c$  and then shearing the specimen with a normal stress less than  $\sigma_c$ , it is possible to represent the ultimate shear strength in the form of equation (1).

It should be emphasized that the durations of the various individual tests were approximately 12 min with a shear deformation rate of approximately 0.025 in. per min. For most tests, with the possible exception of some on the 100 per cent montmorillonite, the primary consolidation as indicated by consolidation tests was substantially complete during shear.

Figure 6 is a hyperbolic plot in the form of  $\delta/\tau$  versus  $\delta$  for the data given

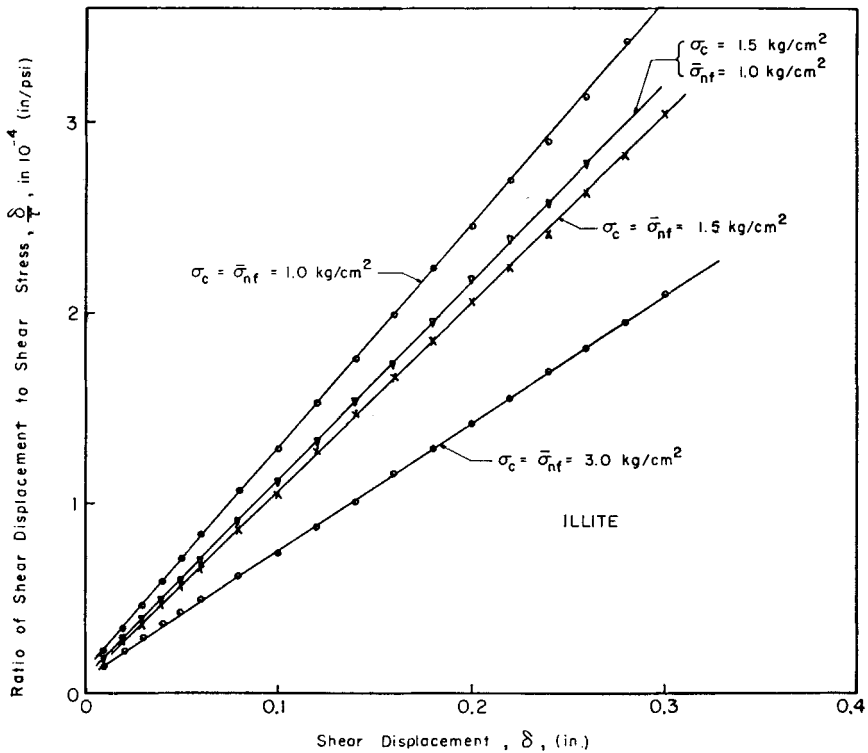


FIGURE 6.—Stress-strain response in transformed hyperbolic form.

in Fig. 5. The applicability of the two-constant hyperbolic form as a representation of the stress-strain response is indicated by the agreement between the straight lines and the experimental data. Note that the slopes and intercepts,  $b$  and  $a$  respectively, are functions of  $\sigma_c$  and  $\bar{\sigma}_{nf}$ . Since  $1/b$  is a measure of the ultimate value of the shear strength, a plot of  $1/b$  versus  $\bar{\sigma}_{nf}$  should be similar to the failure envelope obtained from the straight line approximation to the failure surface in the two-dimensional stress space as given by the conventional modified Coulomb condition. Figure 7 is such a plot of  $1/b$  versus  $\bar{\sigma}_{nf}$  and



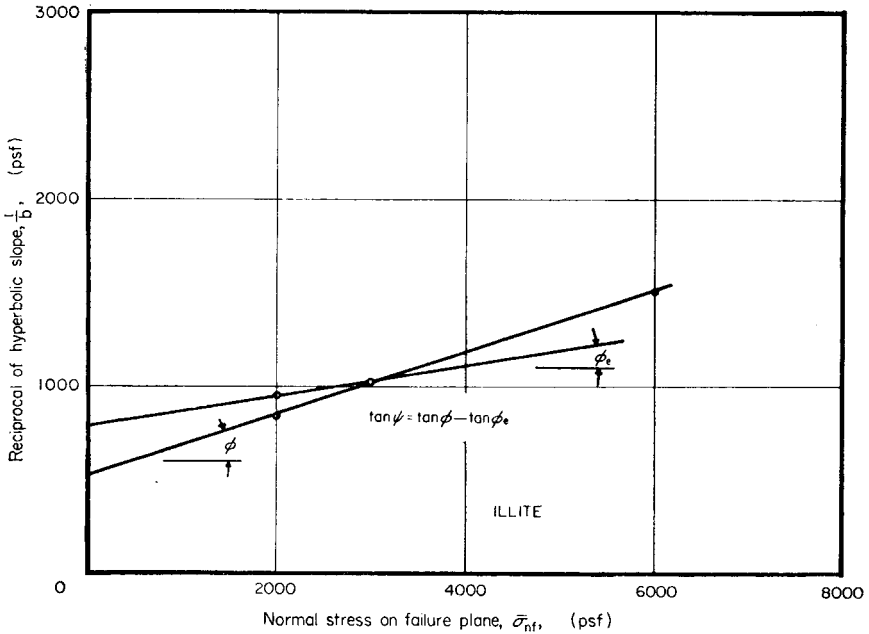


FIGURE 7.—Experimental failure surface in two-dimensional stress space.

gives a measure of the conventional angle of internal friction,  $\phi$ , and so-called cohesion parameter,  $c$ . By using the results of the specimen consolidated to a higher stress and then sheared at a lower stress, one is able to obtain the modified Krey-Tiedemann form of equation (1). The various parameters included in equation (1) are indicated on Fig. 7. Thus, the parameter  $1/b$  can be written as

$$\frac{1}{b} = \bar{k}[\bar{c} + \sigma_c \tan \psi + \bar{\sigma}_{nf} \tan \phi_e] \tag{9}$$

where  $\bar{k}$  is the correlation coefficient between  $1/b$  and the actual measured ultimate shear strength. Figure 8 is a plot of the ultimate shear stress predicted using  $1/b$  and the actual measured value, for the entire test program. The dashed line indicates a perfect correlation factor of one while the solid line is the actual correlation factor of 1.14. Thus, the ultimate strength is approximately 88 per cent of  $1/b$ .

The results given in Figs. 5, 6, and 7 are typical of those obtained throughout the test program for the various clay minerals and combinations of them. Table 1 summarizes the results obtained for the four reference clay minerals studied. The effects of combinations of the clay minerals are given in the next section.

The correlation between the intercept  $a$  and both the consolidation stress,

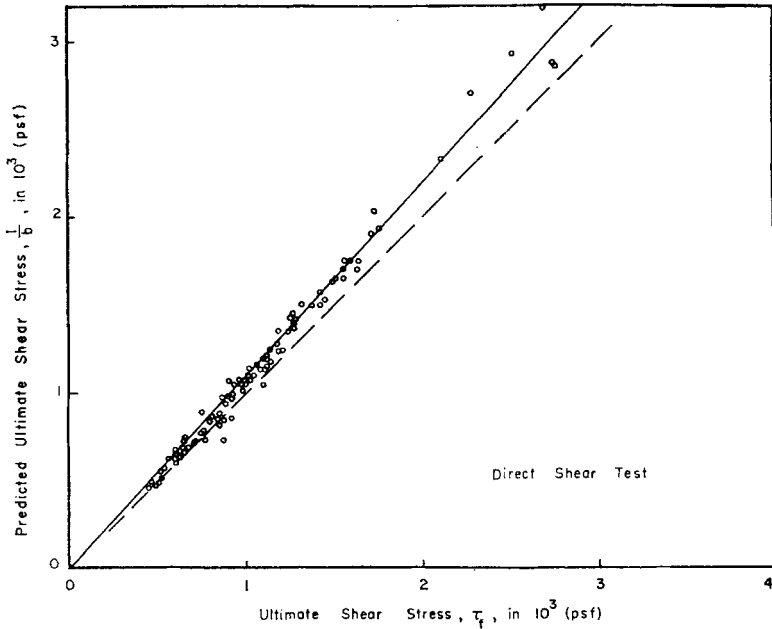


FIGURE 8.—Correlation of hyperbolic strength and measured strength.

TABLE I.—EFFECTIVE COHESION AND FRICTION ANGLES OF THE CLAY MINERALS STUDIED

Mineral	Effective Cohesive Angle, $\psi$ , in degrees	Effective Friction Angle, $\phi_c$ , in degrees
Halloysite	25.0	11.0
Montmorillonite	10.6	0.0
Illite	5.7	4.2
Kaolinite	4.2	5.2

$\sigma_c$ , as well as the effective normal stress,  $\bar{\sigma}_n$ , is not nearly as clearly defined as it is for  $b$ . In general, as  $\sigma_c$  increases the intercept  $a$  decreases, that is, as the consolidation pressure increases the initial tangent shear modulus,  $1/a$ , given by the hyperbolic fit increases. There are a considerable number of cases, for the maximum normal and consolidation stresses, in which the first several points of the hyperbolic form of the stress-strain plot lie below the straight line fit, indicating a shear modulus greater than that given by  $1/a$ . In addition, some of these intercepts are larger than those obtained from the corresponding intermediate values of  $\sigma_c$ . The average shear moduli, in terms of shear displacements, obtained from the hyperbolic relation range from

326 to 630 lb/in<sup>3</sup>. To obtain the shear modulus in conventional units of stress the shear strains instead of shear displacements would be needed. It was observed that the value of  $\bar{\sigma}_n$  did not seem to affect the value of the intercept, but such a generalization is not justified because the normal stresses were always less than or equal to the consolidation pressures. It is felt that much of the uncertainty regarding the intercept is due to the experimental difficulties of controlling the initial conditions of the test, particularly seating effects.

## COMPOSITIONAL VARIATIONS

### *Theoretical Development*

The parameter  $\phi_e$  of equations (1) and (3) and indicated in Fig. 7 for a particular mineral content, is actually a function of the composition of the various minerals involved in the solid phase as well as many additional variables which were held constant in this study. The following development is presented for the case of a two component solid phase, that is, a combination of various amounts of two particular clay minerals.

Preliminary tests as well as consideration of the plasticity characteristics of such mixtures, indicated that the rate of change of  $\phi_e$  with respect to a change in composition was proportional to a constant plus the particular value of  $\phi_e$ . Expressed mathematically this gives the following linear differential equation

$$\frac{d\phi_e}{dP} + B\phi_e = k \quad (10)$$

where  $P$  is the composition variable defined as

$$P = \frac{100 - P_1}{P_1} = \frac{P_2}{P_1}, \quad (11)$$

with  $P_1$  and  $P_2$  the percentages of the two clay minerals considered.

Use of an integration factor to form an exact differential gives a solution of the form

$$\phi_e = k + D e^{-BP} \quad (12)$$

where  $D$  is a constant of integration.

The constants  $D$  and  $k$  can be evaluated by considering the conditions at the ends of the compositional interval considered, namely,

$$\phi_e = \phi_{e_1} \quad \text{at } P = 0, \text{ that is, } P_1 = 100\% \text{ and } P_2 = 0\% \quad (13)$$

$$\phi_e = \phi_{e_2} \quad \text{at } P = \infty, \text{ that is, } P_2 = 100\% \text{ and } P_1 = 0\% \quad (14)$$

Use of the condition of equation (13) for  $P_1 = 100\%$  gives

$$\phi_e = \phi_{e_1} = k + D \quad (15)$$

or

$$D = \phi_{e_1} - k. \quad (16)$$

Substitution into equation (12) gives

$$\phi_e = \phi_{e1} e^{-BP} + k(1 - e^{-BP}). \quad (17)$$

Use of the condition of equation (14) for  $P_2 = 100$  per cent gives

$$\phi_e = \phi_{e2} = k \quad (18)$$

Therefore, the parameter  $\phi_e$  can be written as

$$\phi_e = \phi_{e1} e^{-BP} + \phi_{e2}(1 - e^{-BP}). \quad (19)$$

Equation (19) can be rearranged in the following form

$$\phi_e - \phi_{e2} = (\phi_{e1} - \phi_{e2}) e^{-BP}. \quad (20)$$

Hence,

$$\frac{\phi_e - \phi_{e2}}{\phi_{e1} - \phi_{e2}} = e^{-BP} \quad (21)$$

and

$$BP = \log_{10} \left[ \left( \frac{\phi_{e1} - \phi_{e2}}{\phi_e - \phi_{e2}} \right)^{2.3} \right] \quad (22)$$

Thus, equation (22) is a straight line relationship between the compositional ratio,  $P$ , and the term

$$\log_{10} \left[ \left( \frac{\phi_{e1} - \phi_{e2}}{\phi_e - \phi_{e2}} \right)^{2.3} \right]$$

where  $B$  is the slope of the straight line. By plotting the experimental data in the form of equation (22) it is possible to check the applicability of equation (19) to represent the response of the material due to compositional change and to obtain the parameter  $B$ .

### Experimental Results

Figure 9 is a typical plot of equation (22). This particular figure is for the halloysite-kaolinite series. Thus, knowing the friction parameter for halloysite,  $\phi_{e1}$ , and that for kaolinite,  $\phi_{e2}$ , the parameter  $B$  can be obtained from Fig. 9 and substituted into equation (19) to give for the halloysite-kaolinite series the relation

$$\phi_{eH-K} = \phi_{eH} e^{-BP} + \phi_{eK}(1 - e^{-BP}) \quad (23)$$

where

$$P = \frac{P_K}{P_H} \quad (24)$$

Figure 10 is a conventional phase diagram plot of  $\phi_{eH-K}$  versus composition in which the left value of the abscissa is 100 per cent halloysite and 0 per cent kaolinite and the right side is 100 per cent kaolinite and 0 per cent halloysite with linear variations of both between the end points. The curve represents equation (23) and the points represent the experimental data. It is interesting to note that equation (23) and hence Fig. 10 is the superposition of two terms.

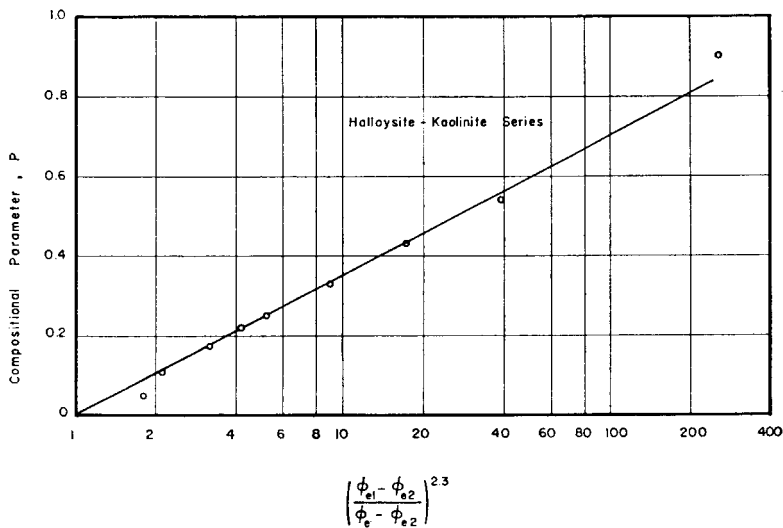


FIGURE 9.—Test plot for compositional variation equation.

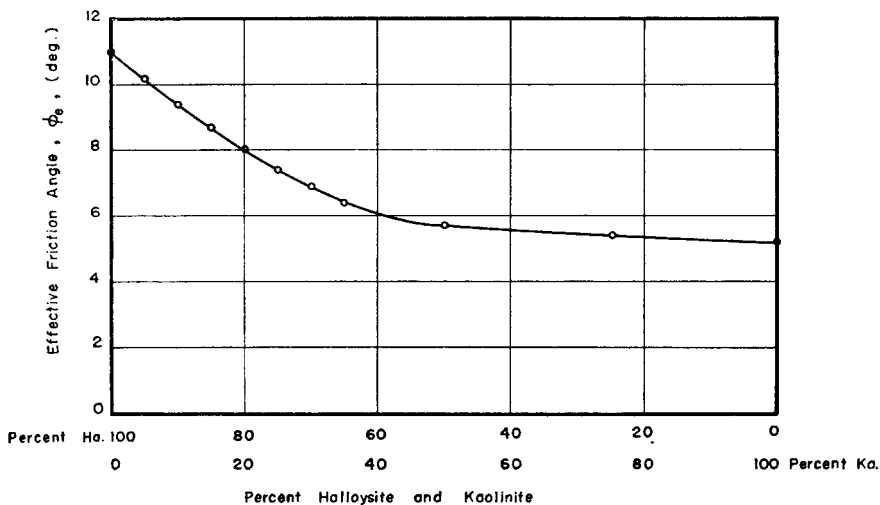


FIGURE 10.—Halloysite-kaolinite phase diagram.

These two terms are plotted individually in Fig. 11. It can be seen that the addition of the two curves in Fig. 11 give the curve of Fig. 10.

Similar relations have been obtained for the remaining permutations obtained by combining any two of the four clay minerals studied.

Although the above development of compositional effects has been presented

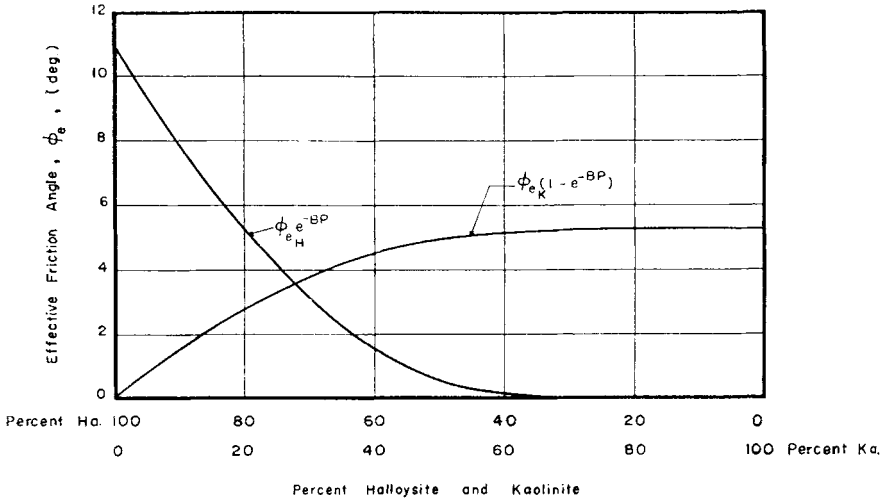


FIGURE 11.—Components of compositional variation equation, halloysite-kaolinite series.

in terms of the so-called effective friction angle,  $\phi_e$ , similar developments can be obtained for the cohesion angle,  $\psi$ , and hence, for the parameter  $\phi$ , which is the angle of inclination of the straight line failure envelope, since

$$\tan \phi = \tan \psi + \tan \phi_e \tag{25}$$

Table 1 gives the various values of  $\phi_e$  and  $\psi$  for the reference clay minerals studied. Table 2 gives the various values of  $B$  for various combinations of the clay minerals.

TABLE 2.—VALUE OF THE DIMENSIONLESS COMPOSITION RATE PARAMETER B

Mineral Group	Composition Rate Parameter B
Illite-montmorillonite	10.30
Kaolinite-montmorillonite	7.69
Halloysite-montmorillonite	5.44
Halloysite-illite	3.33
Halloysite-kaolinite	2.94
Kaolinite-illite	2.18

Although the above development is for a two-phase clay mineral system, it could probably be extended to a more general multi-phase system in the form of the solution of a family of differential equations.

## CONCLUSIONS

In summary, the general form of the hyperbolic stress-strain relations in direct shear for the particular group of clay-water systems composed of various percentages of the reference clay minerals considered in this study is represented as

$$\tau = \frac{\gamma}{a + (\bar{k}[\bar{c} + \sigma_c \tan \psi + \bar{\sigma}_{nf} \tan \phi_e])^{-1} \gamma} \quad (26)$$

where

$$\psi = \psi_1 e^{-BP} + \psi_2(1 - e^{-BP}), \quad (27)$$

$$\phi_e = \phi_{e1} e^{-BP} + \phi_{e2}(1 - e^{-BP}), \quad (19)$$

$$P = \frac{P_2}{P_1}, \quad (11)$$

$P_1$  and  $P_2$  are the percentages by weight of the two clay minerals of which the clay-water system is composed,  $\phi_e$  is the effective friction angle,  $\psi$  is the effective cohesion angle, the subscripts one and two refer to the two clay minerals,  $\sigma_c$  is the consolidation pressure to which the sample has been subjected,  $\sigma_{nf}$  is the effective normal stress on the failure surface,  $B$  is an experimental coefficient depending upon the two clay minerals considered,  $\bar{c}$  is an experimental coefficient depending upon the clay minerals considered and stress history of the material,  $\bar{k}$  is a correlation coefficient of 1.14,  $a$  is a function of  $\bar{\sigma}_n$ ,  $\gamma$  is a measure of the shear strain, and  $\tau$  is the shear stress. It must be emphasized that equations (19), (26), and (27) have been developed for direct shear tests in which the shear displacement rate was constant and the water phase of the soil tested was distilled water. In addition, the range of consolidation and normal pressures was from 1 to 3 tons/ft<sup>2</sup>.

Because of the variability of clay minerals in general and the variability of the properties of those types tested in particular, it should be emphasized that this paper does not present an explicit form of a general formulation for clay mineral effects but is a demonstration of a mathematical formulation of the response of a particular group of clay minerals subjected to direct shear testing. It is not the particular numerical values of the various coefficients that are significant, but the general functional form of the response that is important. The various mathematical forms may shed insight into the formulation of the results of future studies of clay mineral effects on the rheologic response of clay soils.

## REFERENCES

- A.S.T.M. (1952), Symposium on direct shear testing of soils, S.T.P. No. 131, 87 pp.  
 Bates, T. F., Hildebrand, F. A., and Swineford, A. (1950) Morphology and structure of endellite and halloysite, *Amer. Min.*, v.35, pp.463-484.  
 Coulomb, C. A. (1776) Essai sur une application des règles de maximis et minimis à quelques problemes de statique, relatifs à l'architecture: *Mémoires de l'Académie des Sciences (Savants Etrangers)* v.7, pp.343-382.

- Grim, R. E. (1953) *Clay Mineralogy*, McGraw-Hill, New York, 384 pp.
- Hvorslev, M. J. (1960) Physical components of the shear strength of saturated clays, *A.S.C.E. Research Conference on Shear Strength of Cohesive Soils*, pp.169-273.
- Kerr, P. F., and Kulp, J. L. (1949) Reference clay localities—United States: *Amer. Petroleum Inst.*, Project 49, Report No. 2, Columbia University, 103 pp.
- Kerr, P. F., and Hamilton, P. K. (1949) Glossary of Clay mineral names: *Amer. Petroleum Inst.*, Project 49, Report No. 1, Columbia University, 68 pp.
- Kerr, P. F., *et al.* (1950) Analytical data on reference clay minerals: *Amer. Petroleum Inst.*, Project 49, Report No. 7, Columbia University, 160 pp.
- Kerr, P. F. (1959) Discussion, Physico-chemical properties of soils: clay minerals, *J. Soil Mechanics and Foundations Division*, A.S.C.E., v.85, pp.73-78.
- Kondner, R. L. (1962a) Friction pile groups in cohesive soil, *J. Soil Mechanics and Foundations Division*, A.S.C.E., v.88, pp.117-149.
- Kondner, R. L. (1962b) Hyperbolic stress-strain relation in direct shear, to be submitted for publication.
- Kondner, R. L., and Krizek, R. J. (1962) Correlation of load bearing tests on soils, *Proc. H.R.B.*, v.41, pp.557-590.
- Kondner, R. L., Krizek, R. J., and Schimming, B. B. (1962) Lateral stability of rigid poles subjected to an applied couple, to be submitted for publication, 36 pp.
- Terzaghi, K., (1938) Die coulombsche gleichung für scherwiderstand bindiger böden, *Die Bautechnik*, v.16, pp.343-346.
- Terzaghi, K., (1943) *Theoretical Soil Mechanics*, John Wiley, New York, 510 pp.