VIRIAL COEFFICIENT AND HIDDEN MASS IN THE GALAXY GROUPS

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ABSTRACT. The purpose of this work is a verification of the virial mass estimations for small galaxy groups. The dynamical evolution of triple and quintuple galaxies has been studied by the numerical simulations. The dependence of the virial coefficient k(t) versus time was derived. Initially k(0) = 0. The function k(t) has some strong oscillations from 0.02 to 0.99. Generally, these oscillations are quasiperiodical ones. Such a behavior of k(t) is caused by formation in a system of close isolated temporary double subsystems. A strong correlation between the virial coefficient and the least mutual distance in the system is observed.

Such wide oscillations may add into the estimation of virial mass of the galaxy groups an uncertainty of more than one order. An additional uncertainty is introduced by the projection effect. This uncertainty for the individual estimations of the masses approach three orders. Thus any individual estimation of the virial mass is impossible for small galaxy groups.

Some possibility of statistical estimation (median or average) of the total mass, including a hidden mass, is shown for the homogeneous samples. We propose a method for these

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estimations based on a comparison of the medians of dynamical parameters (a mean size in projection and a dispersion of relative radial velocities) for the simulated and observed ensembles of the galaxy groups. This method has been applied to a sample of 46 probably physical triplets of galaxies. The probable median of the hidden mass in a volume of the triplet is about 4 M, where M is the total mass of visible matter.

On the study of galaxy groups, the problem of determination of the mass of a system on the basis of the data observed (the sizes and configurations of the systems on the sky and radial velocities of the components) arises. Usually, one uses the virial theorem in any form (Heisler et al. 1985) in order to estimate the dynamical mass of a galaxy system. The aim of this work is some verification of the virial estimates of masses for the small galaxy groups. A method of computer simulations has been used (e.g., Anosova et al. 1989).

One of the most important parameters of the dynamical state of the N-body system is the ratio of its kinetic energy T to the potential energy U: a virial coefficient $k = T/|U|$. The time averaged value for k is equal to 0.5 for an isolated stationary system of gravitating bodies, here one bears in mind a ratio of the mean kinetic energy $\overline{T}$ to the mean potential one $|\overline{U}|$.

The computer simulations (Anosova et al. 1989) have shown that the dynamical states of the multiple galaxies may strongly change during the evolution. Often the temporary isolated binaries are formed in systems which have as a rule elongated orbits. Such binaries result in some important variations of the energies T and U.

At first, let us consider a variation of the virial coefficient k versus time t, for isolated binaries in terms of the orbital eccentricity e. The functions k(t) are shown in Figure 1 for e = 0, 0.2, 0.6, 1.0. The time t is in units of the period P of the binary. The average over the period is

$$\overline{k} = \frac{1}{P} \int_0^P k(t)\, dt = 0.5 - e^2/4. \quad (1)$$

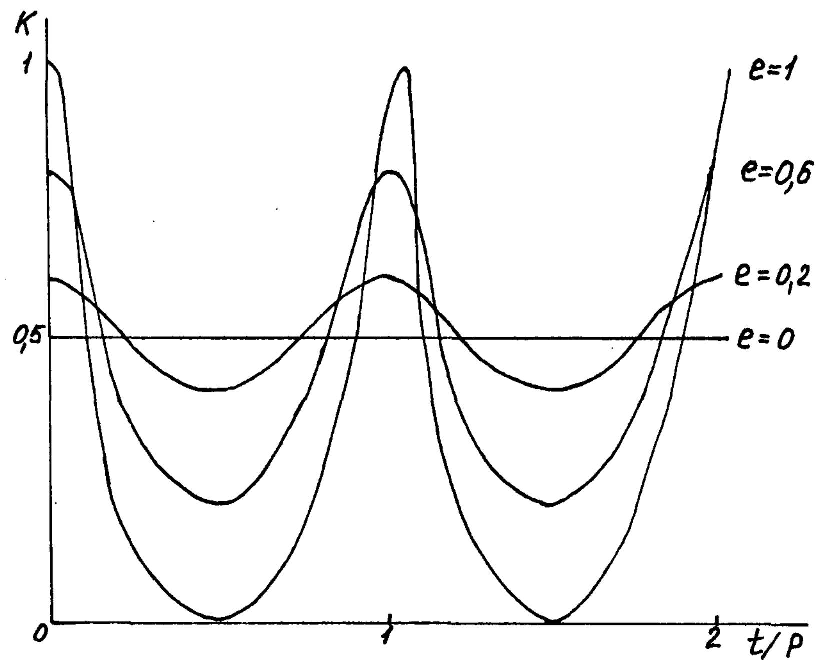

Figure 1. The dependence of the virial coefficient k(t) on time for binary systems with different eccentricities.

The average K over the eccentricity e within the interval (0,1) is equal to

$$\langle\overline{k}\rangle = \int_0^1 \overline{k}(e)\, de = 5/12 \simeq 0.42 \quad (2)$$

in the case of a homogeneous distribution of e. Let us note that a ratio of the average of time quantities $\bar{T}$ and $|\bar{U}|$ does not depend on e, it is equal to 0.5.

Now, let us consider the behavior of the virial coefficient k versus the time t in 3- and 5-body systems. Some typical examples of k(t) are shown in Figures 2 and 3. The time t is in the units τ - the mean crossing time for the triple system (e.g., Anosova et al. 1989). One has adopted k(0) = 0 for the triple

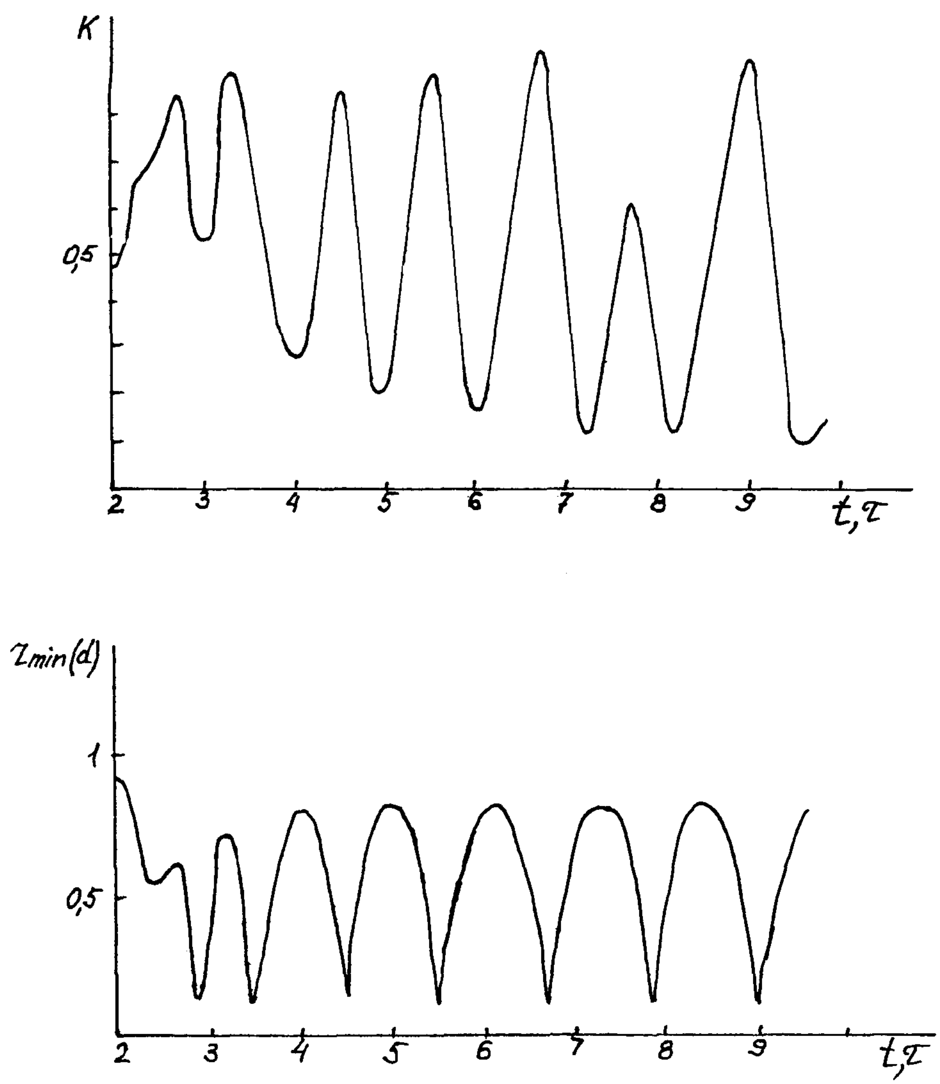

Figure 2. The dependence of the virial coefficient k(t) and the minimum separation $r_{min}(t)$ for the triple system. A strong correlation is observed.

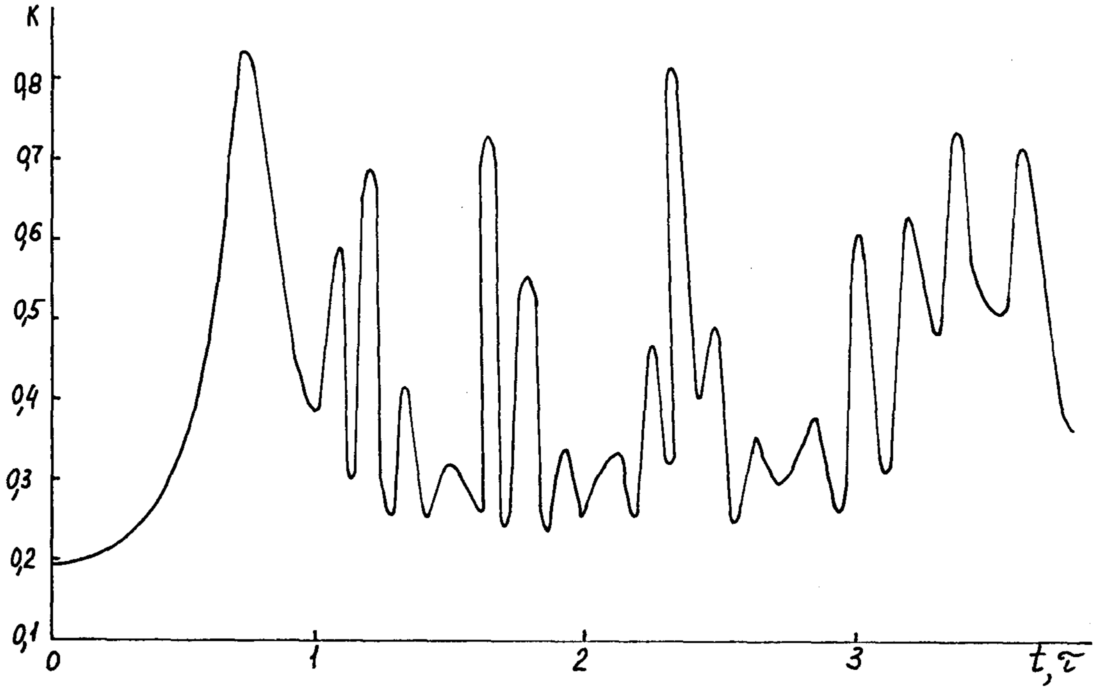

Figure 3. The dependence k(t) for the quintruple system.

systems; the dependences k(t) have been obtained, beginning from $t = 2\tau$ as the triple system "forgets" its intial state after this time. The functions k(t) have strong oscillations within the interval (0.02, 0.099). Often these oscillations for the triple systems have a quasiperiodical character; such a behavior is caused by the formation of temporary double subsystems. This conclusion is supported by the strong correlation between k and the minimum mutual separation r_{min} in a triplet (see Figure 2). The coefficients of correlation between k and r_{min} are shown in Table 1 for the models with equal-mass as well as different mass compoents. Here N is the number of realizations considered, and Δt is the time interval between two neighoring realizations. This Table shows strong correlation between k and r_{min} for all ratios of the masses considered. The coefficient of correlation weakly depends on the ratio of the masses.

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Table 1

Mass ratio	1:1:1	1:1:3	1:3:3	1:2:6	1:3:9	1:4:12	1:5:15
Correlat. Coefficient	-0.88	-0.90	-0.83	-0.89	-0.81	-0.72	-0.79
N	525	525	525	2626	525	2625	2625
Δt	0.5	0.5	0.5	0.1	0.5	0.1	0.1

The distribution of f(k) is shown in Figure 4. It has been constructed from 4400 realizations fixed in $\Delta t = 0.2\tau$ within the interval $(2, 10)\tau$. The moments of this distribution are equal to $k = 0.41$, $\sigma_k = 0.21$.

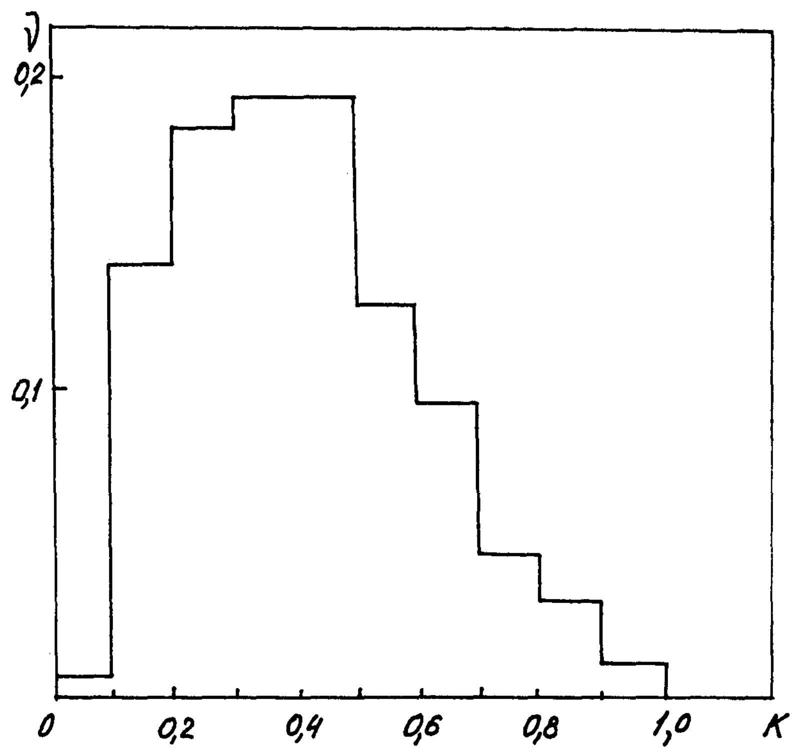

Figure 4. Distribution f(k) for the 4400 realizations of the simulated triplets.

Such strong oscillations of k(t) may make an uncertainty in the estimation of the virial mass of the galaxy groups of more than one order for an individual group. Some additional uncertainty will be made by the effect of projection - using projected distances and radial velocities instead of the three-dimensional sizes and velocities. The projection uncertainty of the individual virial mass of the galaxy group may be as large as three orders. In order to estimate the characteristic maximum uncertainty in the virial mass by both above effects, we have considered a sample from the simulated catalogues of triples with equal-mass components during the evolution. We have determined the medians of the minimum and maximum virial mass of the triplets $(M_{min})_{1/2} = 10^{-2.63}$ and $(M_{max})_{1/2} = 10^{0.25}$ in units of the actual mass of the triplet. Thus the characteristic scattering between the maximum and minimum virial estimations of the mass reaches to 3 orders. Therefore one may make a conclusion that any reliable individual estimation of the virial mass is impossible for galaxy triplets.

By the numerical simulations, we have shown some possibility for reliable statistical estimates (median or average) of the total mass of the galaxy triplets taking into account dark matter. This estimation is certain for homogenous samples of the objects. A statistical method has been proposed in order to make such estimates. This method is based on a comparison of the medians of the dynamical parameters (an average projected size r of the system and a dispersion s of the relative radial velocities of the galaxies) for the simulated and observed samples of galaxy groups. It has been applied to a sample of 46 probably physical systems from a list by Karachentseva et al. (1988), the selection of the probably physical triplets has been carried out using the criterion of Anosova (1987).

The simulated catalogues are compiled for equal-mass components and the different hidden mass M_o, as well as for different-mass components. The medians $r_{1/2}$ and $s_{1/2}$ are compared for the simulated catalogues and the physical galaxy triplets after a corresponding renormalization. It has been assumed that the simulated and observed ensembles are in agreement if the medians $r_{1/2}$ and $s_{1/2}$ do not differ more than 10% simultaneously. The results are shown in Table 2 and Table 3.

Table 2

Hidden mass	0	50	75	100	125
Number of the "coincidences" within 10%	0	4	7	5	4
Ave. dark mass within a volume of triplet	0	10.6	13.9	14.5	16.4

Table 3

Ratio of masses	1:1:1	1:1:3	1:3:3	1:2:6	1:3:9	1:4:12	1:5:15
Number of the "coincidences" within 10%	0	0	0	2	2	4	8

In the first line of Table 2, the hidden mass, distributed isothermally inside a sphere with radius equal to 10 d (d is the mean size of the triplets), is indicated in units of the mass of one component of the triplet. At the second line, a number of the compatible (within 10%) ensembles (26 out of 100) triplets are represented. At the last line, the mean hidden mass inside the volume of a triplet is shown (a dark mass inside a sphere with radius equal to a distance from a centre of its distribution to the most distant body from this centre). The probable median of the dark mass in the volume of the triplet is equal to about 4 M, where M is a total mass of the triplet. A noticeable number of "coincidences" are observed in a coase of the significant dispersion of the masses as 1:5:15 in the models without any hidden mass. However, the masses of the components of the galaxy triplets are similar (Karachentsev 1987), therefore a preference may be given to the dark matter hypothesis. May be, some superposition of two effects takes place that demands a further verification.

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