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Gaia: Relativistic modelling and testing

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Abstract. Gaia is an ambitious space astrometry mission of ESA with a main objective to map the sky in astrometry and photometry down to a magnitude 20 by the end of the next decade. Given its extreme astrometric accuracy and the repeated observations over five years, the observation modelling is done in a fully relativistic framework and several tests of General Relativity or of its extensions can be carried out during the data processing. The paper presents an overview of the current activities in this area and of the expected performances.

Keywords. astrometry, relativity, data processing

1. Introduction

Gaia is an all-sky, high precision astrometric and photometric satellite of the European Space Agency. This is an ESA approved and fully funded mission in the C/D phase scheduled for launch in early 2012. Its design reuses the main concepts that were proved so successful with Hipparcos, namely a continuously spinning satellite with two widely separated field of views mapping the sky. Over the five years of its mission Gaia will measure the position, parallaxes and proper motions of every object brighter than visual magnitude V = 20 with an end-of-mission astrometric accuracy of 25 μas at V=15. In addition, Gaia will perform multi-band photometry for all the sources and will carry also a dedicated spectrometer to determine radial velocities for at least 100 million stars.

Beyond the sheer measurement accuracy, a major strength of Gaia follows from (i) its capability to perform an all-sky and sensitivity limited absolute astrometric survey, (ii) the unique combination into a single spacecraft of the three major electronic detectors carrying out nearly contemporaneous observations, (iii) the huge number of objects and observations which allow the accuracy on single objects to be achieved on very large samples, thus yield statistical significance. In relation with the relativistic aspect dealt with in this paper, one must also add the extreme care taken to calibrate the instrumental parameters and the availability on-board of a Rb clock regularly monitored from the ground.

2. Astrometric accuracy and relativity

2.1. *Few relevant numbers*

One can quickly recognize with a few relevant numbers the need to model astrometric observations, that is to say primarily the light path between a star and an observer, by accounting first for the finite light-velocity, and then for higher order kinematical effects or for the interaction between gravitation and the light-propagation. For observations carried out in the vicinity of the Earth, the barycentric velocity of the observer is about 30 km s^{-1}, equivalent to $v/c \sim 20$ arcsec. This is not a small angle for astronomers and the

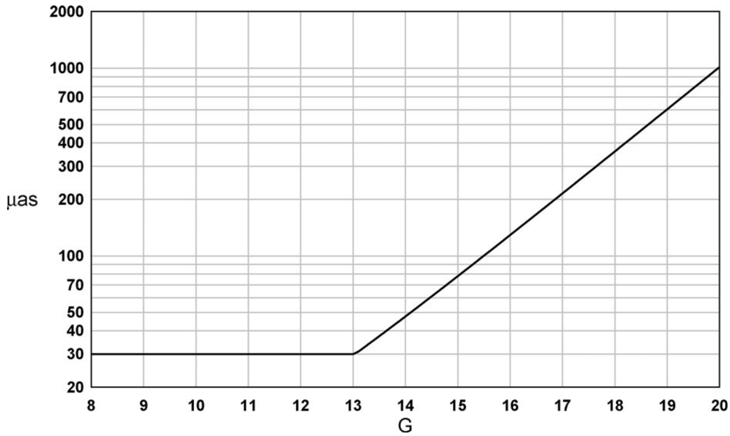

Figure 1. Astrometric accuracy expected with Gaia for a single observation over a 40s transit over the 9 astrometric CCDs. For faint stars, this is primarily determined by the photon noise, while at the bright end, the calibration and attitude determination take over.

astrometric effect was discovered in the early 18th century as the aberration of light. One can then see that the $(v/c)^2$ is of the order of one mas ($0\overset{''}{.}001$) and had to be considered systematically in the Hipparcos data processing. The full special relativity treatment is implemented for Gaia in the nominal astrometric model GREM (Gaia RElativity Model) which aims at an accuracy of $1\,\mu$as (Klioner, 2003).

Regarding the gravitation in the solar system, the yardstick is the light deflection by the Sun of a grazing ray, giving

$$\delta\theta = \frac{4GM_\odot}{c^2 b}\,, \tag{2.1}$$

where b is the impact parameter. For $b \sim R_\odot$ one gets about $1\overset{''}{.}75$. This was about the best astrometric accuracy with photographic plates in the early years of the 20th century and the detection was actually achieved during the famous eclipse of 1919. While the signal is larger when observing close to the solar surface, this has also several drawbacks due to the photosphere brightness in the visible and the radio wave dispersion by the solar corona. The deflection decreases as $1/b$ and is still 50 mas at 10 degrees from the Sun and 4 mas at 90 degrees. This means that deflection at any angle was within reach of Hipparcos and that for Gaia it must be modelled with great care: at 90° it is already 100 times larger than the single observation accuracy of a bright star. This is also the reason why Gaia is a good tool to test some aspects of General Relativity to an unprecedented accuracy.

There are several ways to define the astrometric performance of Gaia, using the final parallax accuracy on stars or the 1D astrometric accuracy over one transit. The latter is more interesting for the following discussion since it applies to every kind of object, whether stellar, extragalactic or solar system sources and is more relevant to discuss either global or local tests. The expected positional accuracy over one field transit, amounting to a total of 40 s integration over the nine astrometric CCDs is shown in Fig. 1 as a function of the magnitude. There are two obvious regimes, roughly for bright and fainter stars. We have a constant accuracy for sources brighter than $G = 13$ at around $30\,\mu$as; then when one goes fainter, the photon noise takes over and one sees the slow degradation in performance, with a single measurement precision of about 1 mas at $G = 20$, yielding about 0.3 mas for the parallax at mission completion. This single transit accuracy is

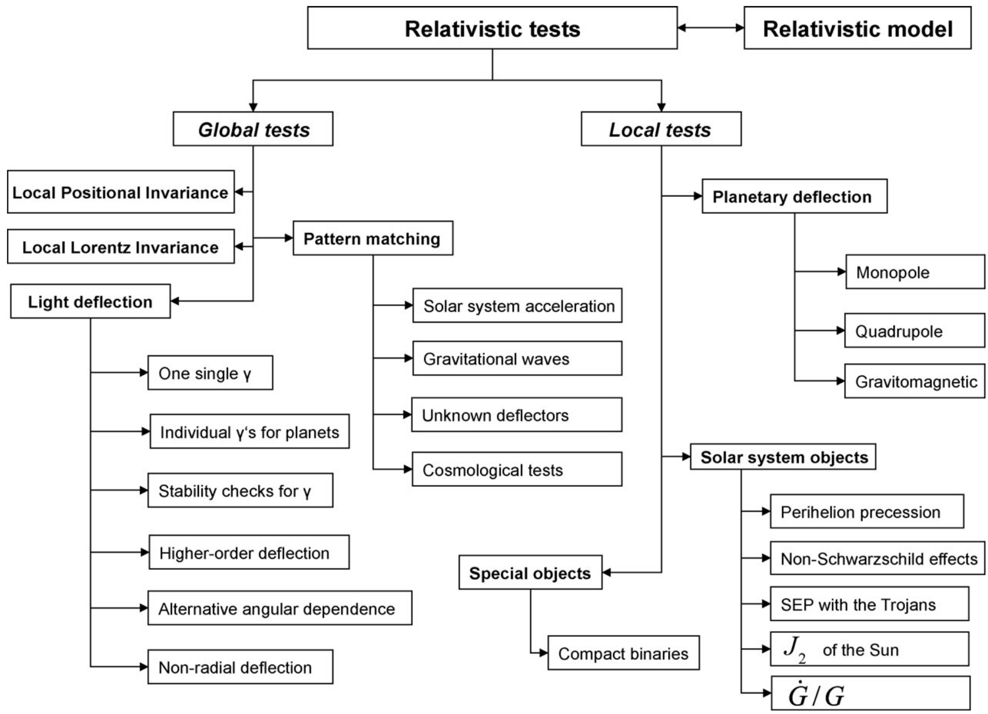

Figure 2. Summary of relativity testing that can be investigated with highly accurate space astrometry, like Gaia.

also the only meaningful one for solar system objects to fit initial conditions, masses or general physical parameters, as shown in (Hestroffer *et al.*, 2010).

2.2. *Relativistic testing with Gaia*

The whole set of relativistic experiments with Gaia can be conveniently divided into two groups:

- global tests which are related to the Gaia global astrometric solution and should use the whole set of Gaia data or at least a sizable fraction of it;
- local tests which are related to some specially designed (e.g., differential) solutions and involve a relatively small amount of selected data, for example collected during observations in the immediate vicinity of a planet.

The variety of testing that can be achieved with highly accurate astrometry is illustrated in the diagram of Fig. 2, where the main division between global and local tests is emphasized by the earlier split into two main branches.

2.2.1. *Global tests*

Global tests are performed by taking advantage of the full sky coverage and use every piece of astrometric information gathered during the mission. These tests take the form of a few additional general parameters included in the global model fitting to the observational data. The most natural and precise test of general relativity expected with Gaia is the measurement of the gravitational light deflection by the Sun with a precision of about 2×10^{-6}. Such a precision for γ is very interesting physically, since it supports or rejects some physical theories (e.g., the hypothetical "built-in" cosmological attractor mechanism of the scalar-tensor theory of gravity towards general relativity (Damour &

Nordvedt, 1993a, Damour & Nordvedt, 1993b). How this test is carried out and how the accuracy is achieved are presented in more detail in Section 3.

Another planned test is to use the relativistic aberration to test the Local Lorentz Invariance. The transformation from the unit vector of the observed direction of the incoming photon to the same vector in the barycentric frame, is basically nothing else than the Lorentz transformation with a specially chosen velocity parameter. That Lorentz transformation will be computed for each of the ≈ 80 observations of each of 10^9 sources and its effect on the observed directions is many orders of magnitude larger than the accuracy of observations. A test theory for special relativity (Robertson, 1949, Mansouri & Sexl, 1977) or any of its modern extensions can be used to add a few additional parameters into the Lorentz transformations. Those parameters can be fitted from the Gaia data with high precision. It is not expected that Gaia could compete with modern laboratory experiments (Laemmerzahl, 2006) verifying Lorentz transformation, but since Gaia is a totally different kind of experiment, this test has its own merit.

Finally, a number of experiments are related to matching certain physically interesting patterns to the measured field of proper motions or individual time-dependent positions. One of these patterns allows one to measure the acceleration of the solar system relative to the quasars. This will be achieved during the process of constructing a Gaia-based ICRF with several 10,000s QSOs as explained in (Mignard & Klioner, 2007). Another pattern allows one to constraint (Pyne *et al.*, 1996, Gwinn *et al.*, 1997) possible gravitational wave flux with a frequency $\omega < 3 \times 10^{-9}$ Hz. The accuracy that can be expected from Gaia is $\Omega_{GW} < (0.001 - 0.005)\, h^{-2}$, h being the normalized Hubble constant $h = H_0/(100\ \mathrm{km/s/Mpc})$. That limit will be several orders of magnitude better than we have from VLBI now. One can also try to fit a pattern produced by an individual gravitational wave to the individual (time-dependent) positions of the quasars. The pattern in positions produced by a plane gravitational wave is uniquely defined by 5 parameters: its amplitude (strain), its frequency, and the direction of its propagation. Although such an approach is difficult to realize (because of the huge amount of data to be processed), with this method Gaia will be sensitive to the gravitational waves with higher frequencies. The same idea of fitting a pattern to the individual positions and proper motions of sources can be used to detect a hypothetical massive body close to the solar system (e.g. a companion of the Sun).

2.2.2. *Local tests*

Three groups of local tests are currently planned for Gaia. First, special solutions for the observations close to planets of the solar system, especially close to Jupiter, as detailed hereafter in Section 3.2.

The second group of local tests is related to the solar system objects – asteroids and Near-Earth Objects (NEO). The most precise test expected here is the relativistic perihelion precession. In the famous Mercury perihelion advance test the parameter fitted from observation depends on the PPN parameters β, γ and on the solar quadrupole J_2, so that an estimate of β is only possible if J_2 is measured independently (usually, γ can be estimated separately from the same observational data from light deflection or Shapiro effect). In case of asteroids one has many bodies with different orbital parameters and this allows us to distinguish the dynamical effects of the solar quadrupole J_2 and the effects of general relativity. Thus, independent estimates of γ, β and J_2 are possible. The expected precisions are $\sim 10^{-3} - 5 \times 10^{-4}$ for β and $\sim 10^{-7} - 10^{-8}$ for solar J_2 (Hestroffer *et al.*, 2007, Hestroffer *et al.*, 2010). The same solution can be used to estimate the time variation of the Newtonian gravitational constant with an accuracy $\dot{G}/G \sim 2 \times 10^{-12}\ \mathrm{yr}^{-1}$.

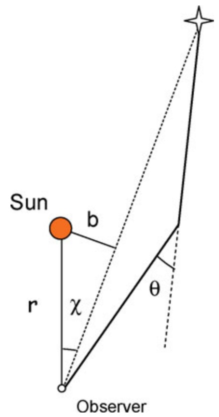

Figure 3. Geometry of the light-deflection by a solar system body at rest with respect to the observer and the source.

Perihelion precession is related to the Schwarzschild field of the Sun. It is interesting that non-Schwarzschild effects related to the post-Newtonian N-body problem will be detected with Gaia. The N-body problem in the post-Newtonian approximation is described by the Einstein-Infeld-Hoffmann (EIH) equations. The dynamical consequences of those equations are well known for major planets with their small eccentricities and inclinations, but not for asteroids. Especially, for resonant asteroids the relativistic N-body effects can be substantially enhanced. Another test is the test of the Strong Equivalence Principle (SEP) with the Trojan asteroids (asteroids orbiting the Lagrange points L_4 and L_5 of the Sun-Jupiter system) and, possibly, other resonant asteroids. Interestingly, the first example of an observable effect due to a possible violation of the SEP (Nordtvedt effect) was a shift of the position of L_4 and L_5 by about 1 arcsecond (for $\eta = 1$) as seen from the Earth. Gaia will be able to provide high-accuracy observations for all Trojan asteroids so that the expected accuracy of η is of the order of 10^{-3}.

The last group of local tests deals with specially selected, relativistic-interesting objects, the processing of which can be improved by special means. As an illustration, there is the case of compact binaries with one component being a black hole candidate. Combining usual Doppler measurements of these objects with Gaia astrometry, one can derive the mass of the invisible companion without any further assumptions (Fuchs and Bastian, 2005). For example, for the well-known system Cyg X-1 the astrometric wobble of the visible companion is expected to be 25 μas and can be measured by Gaia.

3. The light deflection

The grazing-ray expression must be corrected when observing at large angular distance from the deflecting body, which for Gaia would be always the case for the Sun. The star is located at a very large distance compared to the Sun and χ is the angular separation between the Sun and the star (Fig. 3). With the space observations to be carried out by Gaia, χ is not necessarily a small angle. In fact, it can be very small for the planets where grazing observations are feasible, but remains always larger than $\chi_{\min} = 45$ deg for the Sun. The impact parameter of the unperturbed ray is denoted by b and the

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distance between the observer (on the Earth or space-borne somewhere in the Solar System) is r. The deflector has a mass M and a radius R. To the first order in GM/c^2 and by neglecting any departure from the spherical symmetry (the so-called monopole deflection) the deflection angle is given by ,

$$\delta\theta = \frac{1+\gamma}{2}\frac{2GM}{r\,c^2}\frac{1}{\tan\frac{\chi}{2}}\,, \tag{3.1}$$

while the unit vector in the apparent direction is given by,

$$\mathbf{u} = \mathbf{u_0} + \frac{1+\gamma}{2}\frac{2GM}{c^2}\frac{[1+(\mathbf{u_0}\cdot\mathbf{r})/r]}{b^2}\,\mathbf{b}\,. \tag{3.2}$$

When the angular separation $\chi << 1$, the deflection expression reduces to the classical one with,

$$\delta\theta = \frac{1+\gamma}{2}\frac{4GM}{c^2\,b}\,. \tag{3.3}$$

Although one talks about light deflection or light bending, there is no way to measure this effect directly since the initial direction is not known. By itself the deflection is not an observable quantity and its mathematical expression is coordinate dependant. In fact one has access to the proper direction in the observer frame, and only the variation of this proper direction with time, due to varying geometry with respect to the Sun, is accessible, which eventually permits determining the deflection itself. Relevant astrometric signatures for solar system bodies are given in Table 1, both for the monopole and quadrupole deflection to be considered later. Grazing rays are only meaningful for the planets (all but Mercury which is not observable with Gaia) with a very large monopole deflection by the Gaia standard. The modeling requirement at the level of 1 μas shows that deflection by Jupiter must be included over a wide range of elongations, a non negligible computing effort, even with very optimized ephemeris access, given the number of observations.

Table 1. Relativistic light deflection in the solar system. R_A is the angular radius of body A. The columns with $\delta\theta = 1\mu$as give the planetary or solar elongation where the deflection reaches that value.

Body	Monopole		Quadrupole	
	grazing mas	χ $\delta\theta = 1\,\mu$as	grazing μas	χ $\delta\theta = 1\,\mu$as
Sun	17,000	180°		
Mercury	0.083	0.15°		
Venus	0.49	4.5°		
Mars	0.12	0.4°		
Jupiter	16.3	90°	240	8 R_J
Saturn	5.8	17°	95	4 R_S
Uranus	2.1	1.2°	8	2 R_U
Neptune	2.5	0.9°	10	2 R_N

3.1. *The determination of space curvature*

The deflection expression in Eq. (3.1) has been derived in the framework of the PPN extension of the GR and includes γ as free parameter. As said earlier a single observation

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of a bright star at 90° from the Sun allows potentially to detect deviation from $\gamma = 1$ to 0.01. Given the number of stars (about 10 million stars brighter than $V = 13$) and the repeated observations, one sees immediately that Gaia has a huge potential to evaluate γ to a never achieved accuracy. Consider first the ideal and best case, where the instrument modelling and calibration are fully under control and that the remaining noise is purely random. A straight comparison with Hipparcos provides already a good order of magnitude of the performance achievable with Gaia under these assumptions. This is shown in Table 2 where the different sources of improvement are scaled to Hipparcos. Altogether one can expect with the bright stars something 2000 better than the Hipparcos results, or a determination of γ with the solar deflection as good as 2×10^{-6}.

The estimate seems rather conservative, neglecting the contribution of stars fainter than $V = 13$, but despite that, this is a very challenging goal. It rests upon the statistical improvement with the square root of the number of observations and disregards any adverse unmodelled systematic effect minutely correlated with the deflection astrometric signature as small as $\sim 2 \times 10^{-6} \times 4$mas, or 0.008 μas. The difficulty would not be to solve for γ, but to assess that one has not left any other global parameter, not orthogonal to the subspace determined by the condition equations of γ. Otherwise, the true meaning of the parameter so determined will be questionable. The real challenge to do better than 10^{-5} lies precisely here and extreme care will be exercised to look for correlations with small instrumental parameters, with the zero-point parallax, or the aberration correction, to quote a few of these effects.

Table 2. Determination of the PPN parameter γ with space astrometry scaled on Hipparcos and using only the brightest stars.

Hipparcos	**Gaia**		σ_H / σ_G
10^5 stars $V < 10$	8×10^6 stars $V < 13$		
2.5×10^6 abscissas	6.0×10^8 abscissas	$\Longrightarrow$	15
$\sigma \sim 3$ to 8 mas	$\sigma \sim 40$ μas	$\Longrightarrow$	125
$\chi > 47°$	$\chi > 45°$		
$\Downarrow$	$\Downarrow$		$\Downarrow$
$\sigma_\gamma \sim 3 \times 10^{-3}$	$\sigma_\gamma \sim 2 \times 10^{-6}$	$\Longleftarrow$	2 000

Compared to other experiments or tests related to the measurement of γ, Gaia deals directly with light deflection and not with gravitational signal retardation, as in Viking-like, time dependence of Shapiro delay, as in Cassini-like ones (Bertotti *et al.*, 2003), or differential Shapiro delay (i.e. the difference between the Shapiro delays for signal from the same source received by two spatially separate observing sites), as in VLBI. Before Gaia only direct measurements of stellar positions during total solar eclipses, or the special case of Hipparcos, provided us with measurements of true light deflection. Gaia is going to improve the current best result available from Hipparcos by more than 3 orders of magnitude.

But, as mentioned earlier, Gaia is sensitive to gravitational light deflection in a wide range of angular distances between the observed source and the Sun. This allows one not only to measure γ, but also to map the angular dependence of the light deflection by fitting a number of additional parameters describing alternative light deflection models. The most general way to do this is to fit an expansion of the deflection signal in terms of vector spherical harmonics. The fits of relativistic parameters in Gaia will be done simultaneously with the fits of all other parameters. This is also an advantage compared to many other experiments, when the relativistic fit is a special post-processing on the residuals of the "main solution" that assumes general relativity to be valid. In the latter case the correlations between the relativistic parameters (e.g. γ) and other parameters fitted in the main solution are completely neglected. This leads to a substantial uncertainty in the real accuracy of estimates of the relativistic parameters. One more advantage of Gaia is that full-scale simulations of the observations and data processing are being performed to test the suggested data processing algorithms. This allows to reliably check if the observational data and the selected data processing algorithms are really sensitive to a given signal (see for example Hobbs *et al.*, 2010).

3.2. *Planetary light deflection*

During the course of the Gaia observing program, planets will also enter the fields of view, typically 60 times during the mission (it's below the average at low ecliptic latitude). The planets themselves won't be observable as being both too big in angular size and too bright for the detector. However, faint satellites of Jupiter and Saturn, as well as stars that would happen to be around, will be normally detected and subsequently observed. Observing outside the atmosphere with CCDs will allow to see stars fairly close to the planetary disk, probably as close as 0.1 radius from the surface, at least on the leading side (blooming due to the transit of the bright planet may hinder similar observations on the trailing side). The light bending by the giant planets given in Table 1 is very significant for Gaia. The monopole light deflection by Jupiter will be detected at several degrees from Jupiter or Saturn. Although the deflection is smaller at large distance, there are more stars contributing to the signal. The deflection decreases as the inverse of the impact parameter, while the number of sources grows in the same proportion, meaning that each circular annulus contributes evenly to the final determination of γ. Altogether these observations alone will lead to an independent determination of γ with a precision $\sim 10^{-3}$, that is, with about the same accuracy as Hipparcos for the Solar deflection (Froeschlé *et al.*, 1997). What matters here, is not the accuracy of γ, which is low compared to the main determination with the Sun, but the fact it can be achieved with a different body with no relation to the Sun.

Jupiter is not at rest with respect to the barycenter of the solar system and clearly the deflection pattern of a moving body is not identical to that of a static one and this can be evidenced with Gaia observations. Simulations have shown that the relevant parameter for this translational motion could be determined to an accuracy of 2×10^{-3}, i.e. two orders of magnitude better than current best results (Anglada, Klioner & Torra, 2007).

Given the sensitivity of Gaia astrometry one can hope also to detect the departure of the deflection pattern from a pure isotropic monopole deflection and see the non-radial signature brought about by the quadrupole moment. For Jupiter the magnitude is 240 μas (Table 1) for a grazing ray and falls off rapidly as the cube of the impact parameter. Therefore, it will be really observable on the few instances where a bright star is seen very close to the Jupiter disk, preferentially on the leading side (defined relative to the scan motion). The main properties of the quadrupole deflection and its detection with Gaia have been investigated in (Crosta & Mignard, 2006) and (Anglada, Klioner & Torra, 2007). Simulations have shown that the detection will be very challenging, but could be achieved to better than a 3-sigma level. Since the evidence is based on a few favorable transits of bright stars, this is not a statistical effect and it must be looked at on a case by case basis with realistic simulations of the observations. For a given orbit of Gaia, it is shown that there exist good initial conditions for the scanning law (there are two free parameters) that yield favorable close approaches between Jupiter and bright stars, which at the end increase the detection of the quadrupole deflection up to an 8-sigma level as shown in (de Bruijne, 2010).

However, the selection of these initial conditions may conflict with other mission requirements and the choice will be, at the end, a compromise and no risk will be taken that could impact the mission core science in exchange of an uncertain detection of the quadrupole deflection. Moreover, the deflection is also sensitive to the actual orbit of Gaia, since the apparent direction of Jupiter must be known to better than 0.05 of its radius, or better than 3500 km. In practice, the orbit of Gaia to that accuracy won't be known before the final injection that would take place a few days after the launch.

Acknowledgement

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