*Dwarf Galaxies: From the Deep Universe to the Present*
*Proceedings IAU Symposium No. 344, 2019*
*K. B. W. McQuinn & S. Stierwalt, eds.*

doi:10.1017/S1743921318007081

# Numerical computation of gravitational field of infinitely-thin axisymmetric disc with arbitrary surface mass density profile and its application to preliminary study of rotation curve of M33

**Toshio Fukushima**

National Astronomical Observatory of Japan / SOKENDAI,
2-21-1, Ohsawa, Mitaka, Tokyo, 181-8588, Japan
email: Toshio.Fukushima@nao.ac.jp

**Abstract.** We developed a numerical method to compute the gravitational field of an infinitely-thin axisymmetric disc with an arbitrary surface mass density profile. We evaluate the gravitational potential by a split quadrature using the double exponential rule and obtain the acceleration vector by numerically differentiating the potential by Ridders' algorithm. By using the new method, we show the rotation curves of some non-trivial discs: (i) truncated power-law discs, (ii) discs with a non-negligible center hole, (iii) truncated Mestel discs with edge-softening, (iv) double power-law discs, (v) exponentially-damped power-law discs, and (vi) an exponential disc with a sinusoidal modulation of the density profile. Also, we present a couple of model fittings to the observed rotation curve of M33: (i) the standard deconvolution by assuming a spherical distributin of the dark matter and (ii) a direct fit of infinitely-thin disc mass with a double power-law distribution of the surface mass density.

**Keywords.** galaxy, gravitation

## 1. Introduction

The existence of the dark matter has been supported by the observation of anomalous rotation curves of spiral and/or disc galaxies as explained in Sofue & Rubin (2001). In many cases, the distribution model of the dark matter is limited to be spherically symmetric as illustrated in Navarro *et al.* (1996). However, is this assumption reasonable? We think that many researchers assume the spherical symmetry simply because it enables the analytical or numerical but easy computation of the associayted gravitational field.

Recently, we have developed a series of new numerical methods to compute the gravitational field of arbitrary mass distribution: (i) a infinitely-fine circular ring and an infinitely-thin uniform circular disc as Saturn's ring in Fukushima (2010), (ii) an infinitely-thin axisymmetric disc with arbitrary radial distribution of mass density as an approximation of axisymmetric disc galaxies in Fukushima (2016a), (iii) an infinitely-thin nonaxisymmetric mass distribution of arbitrary shape as an approximation of general spiral galaxies in Fukushima (2016b), (iv) an axisymmetric body with arbitrary radial/vertical distribution of mass density as thick disc and/or spheroidal galaxies in Fukushima (2016c), (v) a non-axisymmetric and infinitely-extended body as spiral, barred-spiral, or triaxial elliptic galaxies in Fukushima (2016d), (vi) a general finite body as general asteroids, small satellites, and/or comets in Fukushima (2017), and (vii) a spherical tesseroid as a building block of nearly spherical planets and large satellites in Fukushima (2018). Among these, we tried to explain the observed rotation curve of a certain spiral galaxy by using the second approach in order to answer the question about the reliability of naive assumption that the mass distribution of the dark matter associated with the galaxy is spherically symmetric.

## 2. Results

M33 is a middle-size galaxy with no apparent central mass concentration as shown in Corbeli *et al.* (2014). Approximating this galaxy as an infinitely-thin axisymmetric object, we first computed the gravitational field induced by its gas- and stars-components from their observed density distributions. The method of computation was described in Fukushima (2016a). It is a combination of (i) the split numerical quadrature introduced in Fukushima (2014), (ii) the double-exponential quadrature rule proposed in Takahashi & Mori (1973), (iii) the analytical expression of the gravitational field of a uniform circular ring expressed in terms of the complete elliptic integrals Fukushima (2010), and (iv) the fast procedures to evaluate the complete elliptic integrals developed in Fukushima (2015). In short, $\Phi(R, Z)$, the gravitational potential in the cylindrical coordinates, is expressed as a convolution integral as

$$\Phi(R, Z) \equiv \int_0^\infty \Sigma(R')\Psi(R'; R, Z)dR', \tag{2.1}$$

where $\Psi(R'; R, Z)$ is the Green function expressed in terms of $K(m)$, the complete elliptic integral of the first kind as

$$\Psi(R'; R, Z) \equiv \frac{-4GR'}{\sqrt{(R'+R)^2 + Z^2}} K\left(\frac{4RR'}{(R'+R)^2 + Z^2}\right), \tag{2.2}$$

where $G$ is Newton's constant of universal attraction. Before the actual numerical quadrature, the convolution integral is split at the singular point of the Green function, $R' = R$, so as to the double exponential rule is effectively applicated:

$$\Phi(R, Z) = \int_0^R \Sigma(R')\Psi(R'; R, Z)dR' + \int_R^\infty \Sigma(R')\Psi(R'; R, Z)dR'. \tag{2.3}$$

This integration interval splitting must be also executed at possible other points associated with the discontinuity and the non-analyticity of the surface density function, $\Sigma(R')$. At any rate, we added the effect of a spherically symemtric mass distribution of the dark matter modelled in Navarro *et al.* (1996) and determined some parameters of the double power-law model function so as to fit to the observation. The rotation curve of the best fit model is illustrated in Fig. 1, which exhibits a hump in the region when $R$, the distance from the symmetry axis, takes an intermediate value as $R \approx 8$ kpc.

Next, assuming that the whole mass distribution of M33 including both the ordinary gas and stars and the dark matter is infinitely-thin, axisymmetric, and following a double-power-law density distribution with respect to $R$ as

$$\Sigma(R) \equiv \Sigma_S \left(R/R_S\right)^{-c} \left[1 + \left(R/R_S\right)^{1/a}\right]^{(c-b)a}, \tag{2.4}$$

where $a$, $b$, $c$, $R_S$, and $\Sigma_S$ are the model parameters, we determined the parameters such that the resulting rotation curve fits to the observational data. The result of fitting is shown in Fig. 2. Obviously, this simple model explains the observed rotation curve better than the standard treatment illustrated in Fig. 1. At any rate, the amount of the determined mass density distribution is much larger than that of observed density distributions of the stars and the gas as clearly seen in Fig. 3.

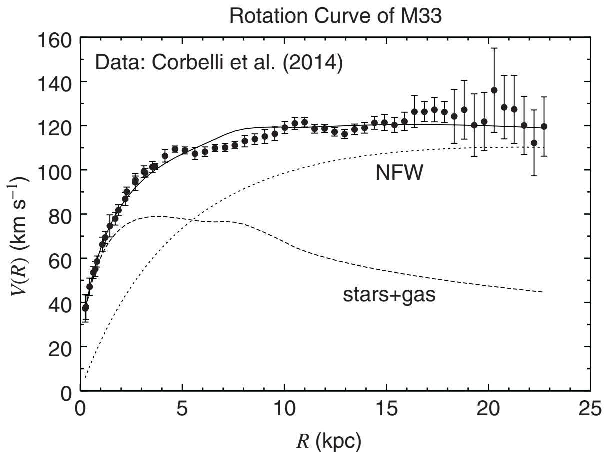


**Figure 1.** Rotation curve of M33 fitted by the standard model. The contribution of the infinitely thin disc model of ordinary matter, i.e. the gas and the stars, was computed by the new method explained in Fukushima (2016a). Meanwhile, that of the dark matter was derived by assuming the spherically symmetric model of dark matter described in Navarro *et al.* (1996). The observation data of the circular velocity of M33 are quoted from Corbeli *et al.* (2014)

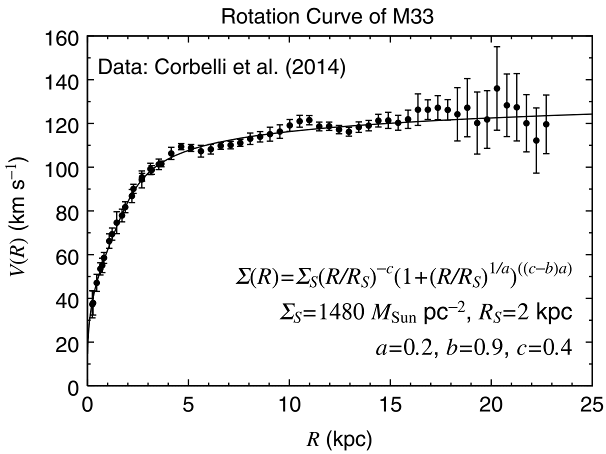


**Figure 2.** Rotation curve of M33 fitted by an infinitely thin double power disc model. Same as Fig. 1 but fitted for the infinitely thin density distribution described by a double power law disc model and obtained by using the new method of gravity computation where $a$, $b$, $c$, $R_S$, and $\Sigma_S$ are the model parameters to be tuned.

## 3. Conclusion

As illustrated in Figs 1 and 2, the spherically symmetric dark matter model is not suitable to the observed rotation curve in this specific case of M33. Of course, this is just one example. We need more evidences or anti-evidences of the applicability of infinitely-thin axisymmetric disc model.

Also, the inifinitely-thin axisymmetric mass distribution is an extreme assumption. The reality must be between them, namely a thick axisymmetric disc, if we ignore the effect of nonaxisymmetry such as caused by spiral arms. In order to go beyond this limitation, we must investigate the gravitational field of a more realistic density distribution and conduct their confrontation with the kinematic observational data.

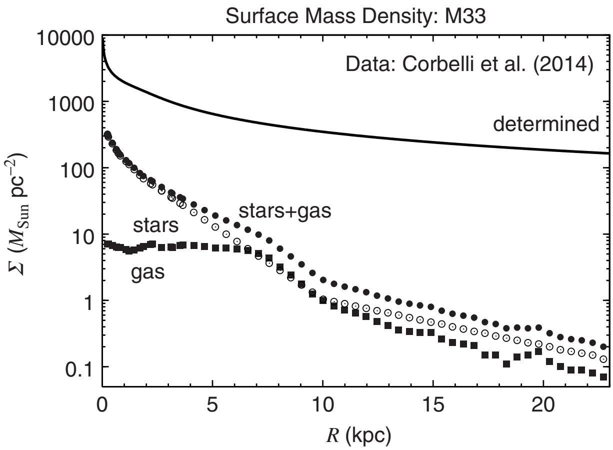


**Figure 3.** Determined surface density distribution of M33.

## References


Corbelli E., Thilker D., Zibetti S., Giovanardi C., & Salucci P. 2014, *A&Ap*, 572, A23
Fukushima, T. 2010, *CMDA*, 108, 339
Fukushima, T. 2014, *Appl. Math. Comput.*, 238, 485
Fukushima, T. 2015, *J. Comput. Appl. Math.*, 282, 71
Fukushima, T. 2016a, *MNRAS*, 456, 3702
Fukushima, T. 2016b, *MNRAS*, 459, 3825
Fukushima, T. 2016c, *MNRAS*, 462, 2138
Fukushima, T. 2016d, *MNRAS*, 463, 1500
Fukushima, T. 2017, *AJ*, 154, 145
Fukushima, T. 2018, *J. Geodesy*, https://doi.org/10.1007/s00190-018-1126-2
Navarro J. F., Frenk C. S., & White S. D. M. 1996, *ApJ*, 462, 563
Sofue Y., & Rubin V. 2001, *ARAA*, 39, 137
Takahashi H., & Mori M. 1973, *Numer. Math.*, 21, 206