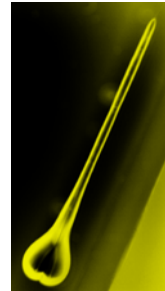# Drops sliding down an incline at large contact line velocity: What happens on the road towards rolling?

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Drops sliding down an incline exhibit fascinating shapes, which indirectly provide a great deal of information about wetting dynamics. Puthenveettil, Kumar & Hopfinger (*J. Fluid Mech.*, vol. 726, 2013, pp. 26–61) have renewed this subject by considering water and mercury drops sliding at high speed. The results raise puzzling questions: how to take into account inertia at a high-speed contact line, large contact angles, the nature of the dissipation at small scale and sliding versus rolling behaviours?

**Key words:** capillary flows, contact lines, drops

## 1. Introduction

Drops sliding down an incline offer a simple way to explore wetting dynamics, although one needs reasonably smooth and well-controlled substrates. This is perhaps why, up to the 1990s, pioneering works only addressed the yield threshold and its connections with wetting hysteresis (Dussan 1985). Later, a successful model of drop velocity just above threshold was constructed (Kim, Lee & Kang 2002), while several researchers addressed surprising shape changes when this drop velocity $U$ was progressively increased (Podgorski, Flesselles & Limat 2001; Le Grand, Daerr & Limat 2005). A remarkable instability of the drop rear was identified, in which a conical tail develops on two inclined contact lines (see figure 1), reminiscent of previous observations in the coating field (Blake & Ruschak 1979). The laws governing the cone geometry, as well as the flow structure, have been clarified in the lubrication limit. Strongly confined between two inclined contact lines, on which the flow lines must end perpendicularly, the flow adopts a self-similar structure that has been checked by particle image velocimetry (Snoeijer *et al.* 2005). This significantly constrains the cone geometry, whose typical angles are described by generalizations (Limat & Stone 2004; Snoeijer *et al.* 2005, 2007) of the Voïnov 'hydrodynamical model'. Even the curvature radius $R$ regularizing the cone at small scale was identified, and is proportional to the product of the microscopic cutoff length of the model $l_m$ (below which liquid sliding is allowed) and an exponential function of the inverse of

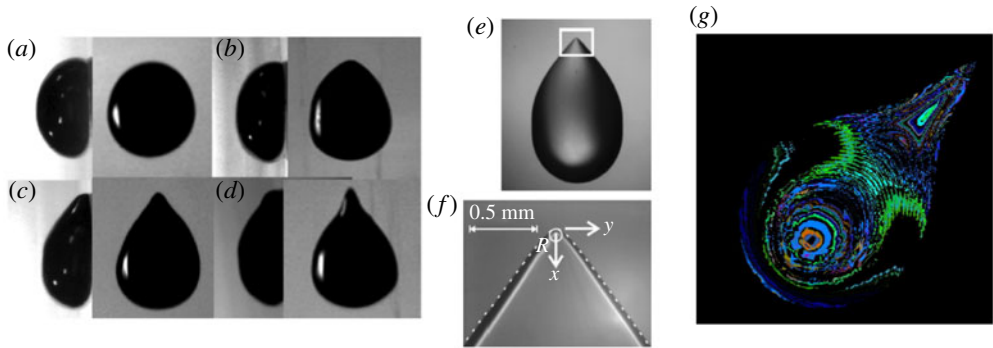† Email address for correspondence: limat@pmmh.espci.fr

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FIGURE 1. From left to right: mercury drops observed by Puthenveettil *et al.* (2013); a silicon oil drop observed by Peters *et al.* (2009), with the definition of the tip radius *R*; and a recent numerical simulation using a VOF method, by Maglio & Legendre (2013).

the capillary number (Peters *et al.* 2009), i.e. $R \simeq l_m \exp(\theta_S^3/[9Ca])$, in which $\theta_S$ is the static receding contact angle and $Ca = \mu U/\gamma$, where $\mu$ is the dynamic viscosity and $\gamma$ is the surface tension. This exponential function is due to the logarithmic thickness profile found in the hydrodynamical model. However, the success of this model is qualified at best, as experiments with water have revealed unexpectedly low values of $l_m$, sometimes even subatomic (Podgorski *et al.* 2001; Winkels *et al.* 2011). This fact was suspected to be linked to the effect of liquid inertia, heretofore neglected.

Indeed, the behaviour of drops reaching a high Reynolds number remained largely unknown. Another limitation was the low values of the equilibrium contact angles explored, and also the very limited number of contact angles considered. Clearly, these limitations prevent one from understanding how things change when one moves from sliding drops in partial wetting conditions (Podgorski *et al.* 2001; Kim *et al.* 2002) to drops rolling on hydrophobic substrates (Mahadevan & Pomeau 1999; Richard & Quéré 1999), in which both velocities and contact angles are increased dramatically. This question is far from trivial, as the divergence of viscous stresses near contact lines disappears when drops 'roll' (Mahadevan & Pomeau 1999), although the transition scenario is still unclear (Thampi, Adhikari & Govindarajan 2013).

## 2. Overview

Puthenveettil, Kumar & Hopfinger (2013) first reconsider the drop velocity selection by introducing a boundary layer at the solid/liquid interface, in which the shear is localized. This leads to a new mobility law that works nicely for both water and mercury. They then adopt a strategy similar to that of Le Grand *et al.* (2005), extracting advancing and receding angles from side views, and comparing the results to available models of wetting dynamics. First, the classical hydrodynamic description with a slip length can again fit the data well, but the slip length required is much too small and even unphysical, as in Winkels *et al.* (2011). The inertia terms in a generalized model seem negligible, which is not so surprising, as the effective Reynolds number defined in terms of the local thickness vanishes asymptotically near the contact line. Molecular kinetic theory also gives reasonable fits, but the price to be paid appears to be a rather strong asymmetry between wetting and dewetting. The authors also consider the so-called 'interface creation' model, which has caused

controversy recently in terms of its conception of surface tension, and they explain that a careful choice of parameters could lead to consistency between their data and those of Le Grand *et al.* (2005) while restoring a certain symmetry. In the author's opinion, in the presence of wetting hysteresis, this principle of symmetry is questionable, and the use of an asymmetrical description of activated jumps at small scales, possibly combined at large scale with viscous dissipation, remains a simpler alternative.

Finally, the authors prove that even in their situation, there is cone formation at the drop rear. Modelling of the observed structure remains puzzling, as the inertial terms (of order $\rho U^2/x$, where $x$ is the distance to the cone tip) do not scale as the capillary pressure gradient ($\gamma/x^2$), which is balanced at all scales by viscous dissipation ($\mu U/x^2$) in the lubrication limit (Limat & Stone 2004).

## 3. Future

Perhaps the most puzzling issue is as follows: a hydrodynamical model describes the spatial self-organization (logarithmic profiles, correlations between angles, the self-similar structure of the flow etc.) reasonably well, but the price to be paid (at least for water and now mercury) seems to be the acceptance of an unphysical cutoff length scale $l_m$. Here, it is not necessarily a good thing to set models against each other as incompatible approaches. One is perhaps faced with a situation in which several sources of dissipation are involved, each one being relevant at a different scale. This can be illustrated by the 'combined model' (Petrov & Petrov 1992), whose simpler form in the small contact angle limit states

$$\theta^3 = \theta_m^3 \pm 9Ca \log(\xi/l_m), \quad \theta_m^2 = \theta_S^2 \pm 2\alpha Ca, \quad (3.1)$$

in which $\theta$ is the slope of the interface at a distance $\xi$ from the contact line, the logarithmic dependence accounting for viscous dissipation ('bending' of the interface), while the static equilibrium angle $\theta_S$ is replaced by a velocity-dependent effective angle $\theta_m$ defined at very small scale. Here, $\alpha$ is some microscopic sliding constant, which could be related to molecular interactions with thermal activation mechanisms (again, see, e.g., Petrov & Petrov 1992), or to the effect of surface defects on the pinning of the contact line (Rolley & Guthmann 2007), or indeed to any other source of dissipation at small scale. In the limit $\theta_S \ll \alpha \ll \theta_S^2/(2Ca)$, one can reorganize (3.1) to obtain an 'à la Voïnov' description that reads $\theta^3 = \theta_S^3 \pm 9Ca \log(\xi/l'_m)$, where $l'_m$ is an effective cutoff given now by the relationship $l'_m = l_m \exp(-\alpha/[3\theta_S])$, which can indeed be much smaller than $l_m$. A trivial extension to the three-dimensional description of the corner tip in Peters *et al.* (2009) is possible and would indeed lead to the same microscopic apparent scale in the formula describing the contact line curvature $R$. Again, although the hydrodynamic description of the spatial interface structure remains correct, its extrapolation to small scale seems to lead to an unphysically small cutoff, but this is only reflecting the existence of a very large dissipation occurring at microscopic scales, of different physical origin.

Here, the key obstacle for further progress could be our imperfect knowledge of dissipation at small scales. One would need very well-controlled substrates having well-defined interaction rules with the contact line. Such substrates can now be devised by combining the present expertise in surface chemistry with nanotechnology facilities in clean rooms. An obstacle is that it seems easier to fabricate perfectly hydrophobic materials than surfaces of controlled wettability. In addition, it will be difficult to vary the values assumed by the reference static contact angle. Here, powerful assistance could come from numerical simulations of contact lines which have made huge

progress by adapting interface codes (VOF and level set methods) to contact line dynamics (see, e.g., Maglio & Legendre 2013), even allowing the inclusion of inertia effects as well as finite slopes of the interface. Complex situations could then be explored, even in parameter regions that are not accessible to analytical models or experiments.

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