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Abstracts of Australasian PhD theses

Finite evaluation of the class number of quadratic number fields

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This thesis explores a number of relationships between the ideal class number, h of a quadratic number field, $Q(\sqrt{d})$, and its square free determinant, d . From these several new results have been discovered as well as a number of improvements being made to existing techniques.

Chapter One is devoted to a redevelopment by elementary methods, of the theory of genera over quadratic number fields, with an emphasis on ease of computation of characters. For fields of negative determinant we include -1 as a prime divisor of d , which results in a unified theory for real and complex quadratic fields.

In Chapter Two, we develop algorithms to count precisely the class number h , by determining one representative ideal of each ideal class. For complex fields this algorithm is a modification of existing methods, but for real fields we make greater use of continued fraction expansions to derive a new and efficient algorithm.

In Chapter Three we generalise the techniques for recent work by H. Hasse, H. Reichardt and others on the divisibility of the class number h by 2^3 , for some specific types of fields. We broaden the coverage of fields, and introduce further criteria. We also expose the underlying techniques, which leads to more general classification of $h \pmod{2^n}$ for some small values of n , for any quadratic number field.

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In Chapter Four we have a generalisation of Euler's polynomials, which are associated with the complex quadratic fields of class number one. This generalisation is devoted mainly to complex quadratic fields of class number two, although some generalisations to all such fields with one ideal class per genus is considered.

By applying Lagrange's continued fraction algorithm in Chapter Five we generate real quadratic fields which have a tendency to have a relatively large number of ideal classes per genus. Some of the results of these methods include the precise formulae for fundamental units for many fields, and a generalisation of the interesting sequence $d_n = (2^n+3)^2 - 8$ considered by Shanks [1].

An appendix containing class number evaluation programs and a sample of tables produced concludes the thesis.

Reference

[1] Daniel Shanks, "On Gauss's class number problems", *Math. Comp.* 23 (1969), 151-163.