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ARTICLE

Embodied technological progress, heterogeneous multiworker firms, and unemployment

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Abstract

This paper studies the impact of technological progress on unemployment in a search-matching model with heterogeneous multiworker firms. In the model, some firms continue to reap rewards from new technologies over time and contribute to job creation, while other firms obsolesce and reduce their employment. Thus, the model captures an endogenous change in the aggregate composition of firms (the firm-composition effect). Considering this effect along with the two canonical effects—the capitalization and creative-destruction effects—I examine the importance of each through a simulation. The results show that the firm-composition effect explains almost all the variation in unemployment in the model, mainly through shrinking the number of obsolescing firms relative to surviving firms and increasing the aggregate technology adoption rate when technology progresses rapidly.

Keywords: Heterogeneous multiworker firms; technology adoption; obsolescence; unemployment

1. Introduction

This paper takes as its point of departure two facts about the US economy, illustrated in Fig. 1: labor productivity growth (henceforth "growth") and unemployment are negatively correlated, and growth is positively associated with investment in information and communication technology (ICT) at least until around 2000.[1]

Regarding these facts, two contrasting effects have been explored in the literature. First, rapid technological progress increases the value of job creation for each employer and induces it to post vacancies, leading to an increase in labor market tightness and a decrease in unemployment (the capitalization effect). Second, it also accelerates obsolescence so that technologies, embodied in jobs, become outdated and make less money, leading to a decrease in the survival duration of each job and an increase in unemployment (the creative-destruction effect).

Previous simulations quantifying these effects show that the creative-destruction effect overpowers the capitalization effect. Thus, the observed negative relationship between growth and unemployment is difficult to explain, even qualitatively. This study addresses this puzzle by introducing heterogeneous multiworker firms into a Diamond-Mortensen-Pissarides (DMP) model with embodied technology.[2] My model captures not only the capitalization effect and the creative-destruction effect but also an endogenous change in the aggregate composition of firms, which I call the firm-composition effect. Through the last channel, technology obsolescence might increase long-run employment by improving the aggregate composition of surviving firms.

In the model, while differences in prior productivity among firms are assumed to follow a continuous distribution based on Melitz (2003), each firm is broadly categorized into one of the following three types: exiting firms, firms that survive and update their technologies, and firms that survive but become technologically obsolete. Consequently, the firm-composition effect is

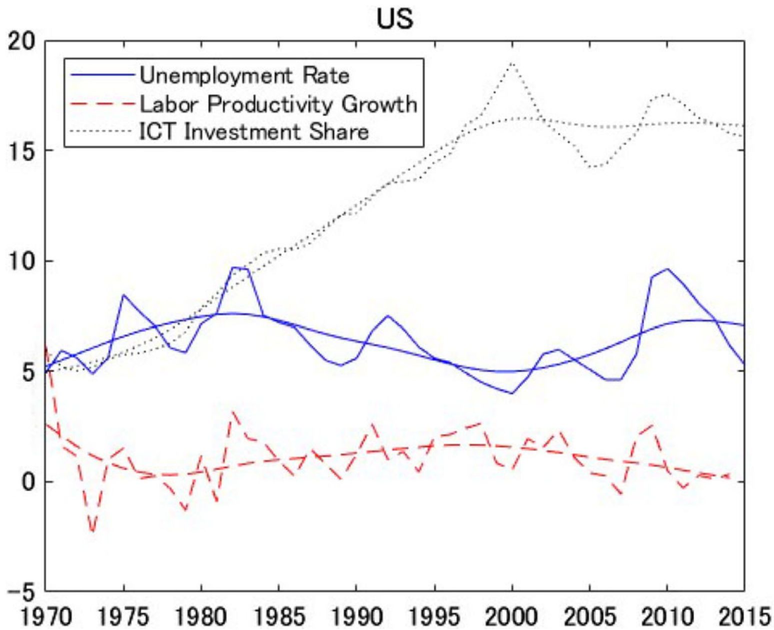

Figure 1. Unemployment, growth, and ICT investment share.
Note: The graph depicts the unemployment rate, the growth rate of labor productivity (measured as gross value added [in 2010 prices] divided by employment), and the share of ICT investment in total investment. The trended evolution is overlaid. The data sources and the trended data are the same as in Fig. 4.

evaluated based on variation in the average productivity of surviving firms by type and changes in the ratio of the number of surviving firms by type.

I show that, across all simulations presented here, the firm-composition effect explains almost all variation in unemployment in the model, with changes in the ratio of the number of surviving firms by type being the dominant factor. For example, in a benchmark simulation, the total impact of growth on unemployment is −0.259, meaning that a 1 percentage-point increase in the growth rate decreases the unemployment rate by 0.259%. This total impact includes all three effects, and it is decomposed so that $-0.259 = -0.008$ (capitalization effect) + 0.051 (creative-destruction effect) − 0.302 (firm-composition effect).

The simulated sizes of the capitalization and creative-destruction effects are similar to those found in the DMP model, as described in Appendix I. Consistent with the literature, the creative-destruction effect is stronger than the capitalization effect. However, after accounting for the firm-composition effect, the total impact of growth on unemployment is negative and compatible with the data.

To evaluate the results quantitatively, I compare the data and model prediction regarding the evolution of the unemployment rate. The model prediction is computed such that only the growth rate varies, matching the empirical evolution of growth, while the other parameter values are fixed. When the total impact of growth on unemployment is quantified with a simple regression coefficient for the data and model prediction, the results show that the model prediction accounts for 16%–36% of that implied by the data.[3]

I examine the interaction of technology and policy by asking how the total impact of growth on unemployment and its decomposition into the three effects are affected when the following parameter values change one by one: the flow unemployment value, worker bargaining power, and the entry cost of each firm. The results show that an increase in any of these parameter values monotonically increases the sizes of the total impact and all decomposed effects. Thus,

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some rigidities in the labor and product markets increase both the unemployment rate and the magnitude of the total impact while roughly preserving the relative importance of the three effects.

I also examine the interaction of the current results with finance. Drawing on Pissarides (2009) and Petrosky-Nadeau and Wasmer (2013), I simply extend the model by introducing an exogenous element in hiring cost such that the element is independent of congestion in the labor market. Similar to the business cycle context, the results show that the element magnifies the total impact of growth on unemployment and all three decomposed effects. Intuitively, heterogeneous financial conditions across firms may further amplify the firm-composition effect when obsolescing firms are financially weak and must further reduce their employment to weather episodes of illiquidity.

This paper is complementary to studies that examine the impact of growth on unemployment with a search-matching model. However, none of the seminal papers in this literature consider multiworker firms. The DMP models are based on the concept of pair matching between an employer and a worker, where each firm employs only one worker.[4] In addition, I introduce firm heterogeneity with respect to productivity.

A primary motivation of this paper stems from Pissarides and Vallanti (2007), which considers a model in which the capitalization and creative-destruction effects are jointly incorporated and identifies the puzzle that the latter effect is larger than the former effect. Miyamoto and Takahashi (2011) and Michau (2013) address this puzzle by introducing on-the-job search. Specifically, Miyamoto and Takahashi (2011) focus on the capitalization effect only and show that on-the-job search strengthens its effect; Michau (2013) focuses on the creative-destruction effect only and shows that on-the-job search attenuates its effect. I do not consider on-the-job search but attempt to reconcile data and theory by considering changes in the aggregate composition of surviving firms.

Mortensen and Pissarides (1998) clarify the mechanism connecting the capitalization and creative-destruction effects by building a model in which each employer has the choice of whether to update its own job-embodied technology. They show that their DMP model with embodied technology reduces to a simple model with disembodied technology when technology updates are implemented at no cost.[5] Drawing on Mortensen and Pissarides (1998), this paper considers embodied technology with updates to focus on both effects and their relative importance.

Hornstein et al. (2007) build a DMP model with embodied technology and examine the interaction of technology and policy. They suggest that their model can explain observed differences between the USA and Europe. However, their results rely solely on the positive impact of growth on unemployment, and they show that the size of the positive impact in Europe is larger than that in the USA. This result itself is incompatible with the evolution of unemployment in each country. This study examines this point and finds that their results are preserved but the total impact of the three effects is compatible with the data.

This study is also closely related to the strand of the literature that extends a DMP model to the product market (Blanchard and Giavazzi (2003), Ebell and Haefke (2009), Felbermayr and Prat (2011)). In this context, Felbermayr and Prat (2011) develop a search-matching model that incorporates multiworker firms and firm heterogeneity as in Melitz (2003). The model in this paper encompasses their structure and extends it by introducing embodied technology. In other words, I bridge the above strands of the literature.

This paper is organized as follows. Section 2 describes the model. Section 3 identifies the capitalization, creative-destruction, and firm-composition effects in the model. Section 4 shows the simulation results. Section 4.3 summarizes the empirical results and compares them with the model outcomes, and Section 4.4 examines the interaction of technology and policy. In Section 4.5, I discuss the interaction of the current results with finance. Section 5 concludes.

2. The economy

2.1 Setup

In my model, a firm is born as long as its expected entry value exceeds its entry cost $\hat{f}_e$. Upon entry, each firm incurs the entry cost as a sunk cost. It then realizes its productivity x, which is the source of firm heterogeneity. x is drawn from the Pareto distribution $F(x) \equiv 1 - x_{\min}^{\alpha} x^{-\alpha}$ with support $x \in [x_{\min}, \infty)$.

Because it is not optimal for firms with very low x to remain in the market, each firm then decides whether to exit. Next, each surviving firm chooses one of two firm types: *updating firm* or *obsolescing firm*. The difference between the two lies in their production function and technology cost.

The production function of an updating firm in period t is

$$a(t)xl, \tag{1}$$

where l denotes labor input and $a(t)$ represents the economy-wide technology in period t. In contrast, the production function of an obsolescing firm is

$$a(\tau)xl, \tag{2}$$

where τ denotes the firm's production start date after entry. Thus, the technology is fixed at the level at date τ. Because $a(t)$ grows at the exogenous rate of technological progress g, the gap between $a(t)$ and $a(\tau)$ increases over time.

Each updating firm incurs the technology adoption cost $\hat{I}$ (henceforth "adoption cost") in every period, while each obsolescing firm incurs it only upon entry. This study assumes that each firm chooses its own firm type only once, on entry. This assumption is harmless when the adoption cost normalized by the economy-wide technology level increases with the technology gap $t - \tau$ and when the speed of an increase in it is more than the speed of a decrease in the value of each obsolescing firm.[6]

I assume monopolistic competition in the product market. The revenue function at time t is denoted by $R(x, l, \tau, t)$ for a firm with technology vintage τ, firm-specific productivity x, and labor input l. The revenue function is as follows:[7]

$$R(x, l, \tau, t) = \left(\frac{E(t)}{n}\right)^{\frac{1}{\sigma}} (a(\tau)xl)^{\frac{\sigma-1}{\sigma}} \tag{3}$$

$E(t)$ denotes aggregate income at time t, n denotes the number of firms, and σ represents the elasticity of substitution between two differentiated goods so that $\sigma > 1$. Following Felbermayr and Prat (2011), the condition $\alpha/(\sigma - 1) > 1$ is also assumed in this paper. This ensures that the mean value of firm size for the updating firm type is bounded. The entire firm size density in equilibrium is calculated in the appendix. For updating firms, I replace τ with t such that their revenue is $R(x, l, t, t)$.

I assume workers are risk neutral, and their population is normalized to one. The workers are either employed or unemployed. Each worker earns wage $w(x, l, \tau, t)$ when employed or receives unemployment benefit $\hat{b}$ when unemployed. The inputs of the wage function—x, l, τ, and t—have the same meaning as in the revenue function. Wages are determined by bargaining as specified later.

The labor market is frictional. The matching function $m(V, u)$ represents aggregate job creation in each period; it is a function of unemployment u and aggregate job vacancies V. In line with Petrongolo and Pissarides (2001), there are constant returns to scale in this function, which increases with each argument. Labor market tightness is defined as the ratio of vacancies to unemployment, $\theta \equiv V/u$. The job-finding probability for each unemployed worker is the ratio $m(V, u)/u = m(\theta, 1)$. Similarly, the job-filling probability for each vacancy is $m(V, u)/V = m(\theta, 1)/\theta$.

At the beginning of each period, the economy-wide technology progresses. Then, each firm starts production and pays wages given its level of embodied technology. After production, separations occur such that an exogenous fraction of workers within each firm, λl, become unemployed in the next period. At the end of each period, each firm is exogenously destroyed with probability δ. Each unemployed worker applies for a vacancy and becomes employed in the next period with the job-finding probability. Each employed worker loses their job with separation probability $s = \delta + \lambda - \delta\lambda$ in the next period. The separation probability is based on a combination of firm-specific and job-specific exogenous shocks.

Time is discrete, and the common discount rate of agents (workers and firms) is r. The value functions are as follows (where the superscripts o and u distinguish the firm types):

$$J^o(x, l, \tau, t) = \max\left[\max_v \begin{bmatrix} R(x, l, \tau, t) - w(x, l, \tau, t)l - \hat{c}v \\ +\frac{1-\delta}{1+r}J^o(x, l', \tau, t+1) \end{bmatrix}, 0\right], \tag{4}$$

$$J^u(x, l, t) = \max_v \begin{bmatrix} R(x, l, t, t) - w(x, l, t, t)l - \hat{c}v \\ -\hat{I} + \frac{1-\delta}{1+r}J^u(x, l', t+1) \end{bmatrix}, \tag{5}$$

$$W^o(x, l, \tau, t) = \max\left[w(x, l, \tau, t) + \frac{1}{1+r}\left[(1-s)W^o(x, l', \tau, t+1) + sU(t+1)\right], U(t)\right], \tag{6}$$

$$W^u(x, l, t) = w(x, l, t, t) + \frac{1}{1+r}\left[(1-s)W^u(x, l', t+1) + sU(t+1)\right], \text{ and} \tag{7}$$

$$U(t) = \hat{b} + \frac{1}{1+r}\left[m(\theta, 1)\tilde{W}(t+1) + (1 - m(\theta, 1))U(t+1)\right]. \tag{8}$$

Equation (4) describes the value of an obsolescing firm at time t with productivity x, employment l, and technology vintage τ. The firm maximizes its value by choosing the number of vacancies v to post. The flow profit is the revenue minus wage payments minus the total vacancy cost. $\hat{c}$ denotes the cost of posting one vacancy. The employment of any firm evolves as $l' = (1-\lambda)l + vm(\theta, 1)/\theta$.

Equation (5) describes the value of an updating firm. I drop technology vintage from the value function because the firm uses up-to-date technology by definition. However, I introduce adoption cost. The functional form of wages is the same across firm types because the presence of the adoption cost is independent of the marginal job surplus.

Equation (6) describes the value of an employed worker in an obsolescing firm, and equation (7) describes that of an employed worker in an updating firm. Equation (8) describes the value of an unemployed worker. $\tilde{W}(t+1)$ denotes the expected value of being employed in the next period. In Section 2.2, I confirm that $\tilde{W}(t+1)$ can be treated independently of specific factors in each firm. Thus, the specific form of $\tilde{W}(t+1)$ is redundant and omitted here. For a simple expression with integration, see Felbermayr and Prat (2011).

This study focuses on a balanced-growth equilibrium and assumes that some exogenous variables grow at rate g to render a nontrivial environment. The list of such exogenous variables and assumptions is $\hat{f}_e = a(t)f_e$, $\hat{I} = a(t)I$, $\hat{b} = a(t)b$, and $\hat{c} = a(t)c$. In addition, I assume $(1-\delta)(1+g)/(1+r) < 1$ to ensure that the values of firms normalized by the economy-wide technology—$J^o(x, l, \tau, t)/a(t)$ and $J^u(x, l, t)/a(t)$—are bounded. I call $(1-\delta)(1+g)/(1+r)$ the net discount factor because it emerges in those normalized values.[8] The net discount factor is assumed to be positive.

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2.2 Wage bargaining

Following Stole and Zwiebel (1996) and Smith (1999), each wage is determined by surplus sharing between a firm and a worker in the firm. Workers have no agreements among themselves or interactions in the bargaining, and each worker is treated as a marginal worker. Specifically, for an updating firm and its worker, the worker obtains the exogenous fraction β of the marginal job surplus $\partial J^u(x,l,t)/\partial l + W^u(x,l,t) - U(t)$, while the firm takes the remaining fraction $1-\beta$ of the surplus. This surplus-sharing rule is the same for an obsolescing firm and its worker. The associated expressions for the two firm types are as follows:

$$\beta\left[\frac{\partial J^o(x,l,\tau,t)}{\partial l} + W^o(x,l,\tau,t) - U(t)\right] = W^o(x,l,\tau,t) - U(t) \tag{9}$$

$$\beta\left[\frac{\partial J^u(x,l,t)}{\partial l} + W^u(x,l,t) - U(t)\right] = W^u(x,l,t) - U(t) \tag{10}$$

By using these conditions, I can characterize wages in Proposition 1.

Proposition 1. *The wage function takes the following form:*[9]

$$w(x,l,\tau,t) = \frac{\beta(\sigma-1)}{\sigma-\beta}\frac{R(x,l,\tau,t)}{l} + (1-\beta)a(t)\omega(\theta) \tag{11}$$

Here, $\omega(\theta)$ denotes the worker's reservation wage $b+\beta c\theta/[(1-\beta)(1-\delta)]$.

When $\sigma=\infty$, the wage function becomes a standard form so that revenue per worker is weighted by the worker's bargaining power β, and the reservation wage (the outside option value) is weighted by $1-\beta$.

Importantly, the recruitment cost per worker, equal to the vacancy cost $a(t)c$ divided by the job-filling probability $m(\theta,1)/\theta$, is the same for all firms. Thus, the first-order conditions for the two firm types follow such that $a(t)c\theta/m(\theta,1) = [(1-\delta)/(1+r)]\partial J^o(x,l',\tau,t+1)/\partial l = [(1-\delta)/(1+r)]\partial J^u(x,l',t+1)/\partial l$. Along with this, by using the surplus sharing conditions (9) and (10), the value of any employed worker in the next period, $W^o(x,l,\tau,t+1)$, $W^u(x,l,t+1)$, or $\tilde{W}(t+1)$, can be treated independently of firm-specific factors.

2.3 Optimal revenue and labor demand

Along with the wage function (11), the maximization problems for vacancies, from equations (4) and (5), are solved as follows.

Proposition 2. *Let $a(t)R(x,t-\tau)$ be the optimal revenue and $l(x,t-\tau)$ be the optimal labor demand in a firm with productivity x and technology gap $t-\tau$. Then, the following is true:*

$$R(x,t-\tau) = \frac{E(t)/a(t)}{n}\left[\frac{\sigma-1}{\sigma-\beta}\frac{1}{\kappa(\theta)}\right]^{\sigma-1} x^{\sigma-1}\left(\frac{1}{1+g}\right)^{(\sigma-1)(t-\tau)} \tag{12}$$

$$l(x,t-\tau) = \frac{E(t)/a(t)}{n}\left[\frac{\sigma-1}{\sigma-\beta}\frac{1}{\kappa(\theta)}\right]^{\sigma} x^{\sigma-1}\left(\frac{1}{1+g}\right)^{(\sigma-1)(t-\tau)} \tag{13}$$

Here, $\kappa(\theta)$ is defined as the employment cost such that

$$\kappa(\theta) \equiv \omega(\theta) + \frac{c\theta}{(1-\beta)m(\theta,1)}\left[\frac{1+r}{(1-\delta)(1+g)} - 1 + \lambda\right]. \tag{14}$$

Three remarks are in order. First, the optimal revenue is denoted by $a(t)R(x,t-\tau)$; thus, $R(x,t-\tau)$ is the measure normalized by $a(t)$. This notation is for convenience when considering

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the subsequent balanced-growth equilibrium. In addition, normalized aggregate income $E(t)/a(t)$ is constant in the steady state.[10]

Second, both $R(x, t-\tau)$ and $l(x, t-\tau)$ increase with $[E(t)/a(t)]/n$ and decrease with θ. The demand shifter coefficient for each firm is $[E(t)/a(t)]/n$. $\kappa(\theta)$ increases with θ because the job-filling probability $m(\theta, 1)/\theta$ decreases with θ and the reservation wage increases with it. The (flow) employment cost $\kappa(\theta)$ is defined as the sum of the reservation wage and the recruitment cost, which is adjusted to be consistent with its flow value.

Third, regarding firm-specific factors, both $R(x, t-\tau)$ and $l(x, t-\tau)$ increase with productivity x and decrease with technology gap $t-\tau$. For each updating firm, the solutions (12) and (13) become $R(x, 0)$ and $l(x, 0)$ by incorporating $\tau = t$; the technology gap term vanishes.

In deriving this proposition, the Euler equation for employment is obtained such that

$$\frac{R(x, t-\tau)}{l(x, t-\tau)} = \frac{\sigma - \beta}{\sigma - 1}\kappa(\theta). \tag{15}$$

This implies that revenue per worker in any firm equals the same aggregate value $\kappa(\theta)$ multiplied by the markup $[\sigma-\beta]/[\sigma-1]$. As a result, all wages reduce to the same value; let $w \equiv w(x, l, \tau, t)/a(t)$ be the common normalized wage.[11]

The Euler equation (15) holds not only for each updating firm but for each obsolescing firm. The endogenous shutdown period of each obsolescing firm leads to ∞ because each obsolescing firm can reduce its employment and maintain its revenue per worker. In contrast, DMP models do not separate the concepts of firm entry and job creation, and revenue (or output) per worker exogenously declines over time with technology obsolescence.

As growth-related variables, I define the net discount factor G_1 and the downsizing operator G_2 as follows:[12]

$$G_1 \equiv \frac{(1-\delta)(1+g)}{1+r} \text{ and} \tag{16}$$

$$G_2 \equiv \frac{l(x, t+1-\tau)}{l(x, t-\tau)} = \left(\frac{1}{1+g}\right)^{\sigma-1} \tag{17}$$

2.4 Technology choice and free entry

There are two expressions for the entry value of each firm: $J^u(x, 0, t)$ and $J^o(x, 0, t, t) - a(t)I$. The former is the value of an updating firm with $l=0$; in other words, it is the value when a new entrant chooses to be an updating firm. The latter is the value when a new entrant chooses to be an obsolescing firm. $a(t)I$ is subtracted because the initial adoption cost is required for any firm type and, as in equation (4), $J^o(x, 0, t, t)$ does not include it. In addition, initial adopted technology is up-to-date—that is, $\tau = t$. The following proposition describes the entry value by firm type.

Proposition 3. *Let $J^u_e(x, t) \equiv J^u(x, 0, t)$ be the entry value for an updating firm and $J^o_e(x, t) \equiv J^o(x, 0, t, t) - a(t)I$ be the entry value for an obsolescing firm. Then, these values are obtained as follows:*

$$J^u_e(x, t) = a(t)\left[\frac{G_1}{1-G_1}\left[\frac{1-\beta}{\sigma-\beta}R(x, 0) - I\right] - I\right] \textit{ and} \tag{18}$$

$$J^o_e(x, t) = a(t)\left[\frac{G_1}{1-G_1G_2}\frac{1-\beta}{\sigma-\beta}R(x, 0) - I\right] \tag{19}$$

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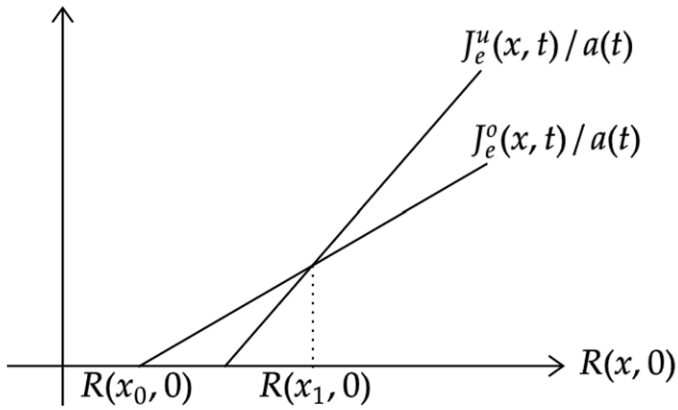

Figure 2. Entry value by firm type.

Fig. 2 depicts the two possible entry values, (18) and (19), in the same graph. The vertical axis is the entry value normalized by $a(t)$, and the horizontal axis is $R(x,0)$. Because $R(x,0)$ monotonically increases with x, this graph reflects the relationship between the normalized entry value and firm-specific productivity.

In brief, a firm with high (low) x optimally chooses the updating (obsolescing) firm type because the slope of $J_e^u(x,t)/a(t)$ is higher than that of $J_e^o(x,t)/a(t)$ and the height of $J_e^u(x,t)/a(t)$ is lower than that of $J_e^o(x,t)/a(t)$ at $R(x,0)=0$.

Specifically, there are two productivity cutoffs, x_0 and x_1, that satisfy

$$J_e^o(x_0,t)=0 \text{ and} \tag{20}$$

$$J_e^o(x_1,t)=J_e^u(x_1,t), \tag{21}$$

where I term x_0 the exit cutoff and x_1 the technology cutoff. In steady state, a firm with x strictly higher than x_1 is an updating firm; a firm with x such that $x_0 \le x \le x_1$ holds is an obsolescing firm; and a firm with x strictly lower than x_0 exits the market.

These cutoffs are simply characterized by combining the free entry condition as follows:

$$a(t)f_e=\int_{x_{\min}}^{\infty} \max\left[J_e^u(x,t), J_e^o(x,t), 0\right] dF(x) \tag{22}$$

Here, the right-hand side is the expected entry profit before paying the entry cost $a(t)f_e$ and realizing x. For convenience, the cutoff ratio is defined as follows:

$$\phi \equiv \frac{x_0}{x_1} \tag{23}$$

ϕ^α equals the ratio of the number of updating firms to that of surviving firms, denoted as $[1-F(x_1)]/[1-F(x_0)]$, which is referred to as the adoption rate for technology in this economy. The proposition below uncovers x_0 and ϕ (and x_1 following equation [23]).

Proposition 4. *From equations (12) and (18)–(23), I derive the following equations:*

$$\phi^{\sigma-1}=1-G_2 \text{ and} \tag{24}$$

$$f_e=\left(\frac{x_{\min}}{x_0}\right)^\alpha \frac{\sigma-1}{\alpha-\sigma+1} I\left[\frac{G_1}{1-G_1}\phi^\alpha+1\right] \tag{25}$$

These equations solve ϕ and x_0.

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Two remarks are in order. First, the job-cut rate in each obsolescing firm, $1 - G_2$, positively affects the adoption rate ϕ^{α}. Because an increase in the growth rate g accelerates technology obsolescence and increases $1 - G_2$, it also increases ϕ^{α}. So even if rapid growth harms some firms, it improves the composition of surviving firms in the aggregate.

Second, the right-hand side of equation (25) depends on the growth-related term $[G_1/[1 - G_1]]\phi^{\alpha}$. This implies that the entry decision of each potential entrant is dominantly associated with the chance of being an updating firm.[13] An increase in g improves the success rate among surviving firms and profitability upon success, thereby promoting firm entry and increasing the exit cutoff.

Notably, a decrease in the entry cost does not affect the adoption rate. It promotes firm entry and increases the exit cutoff but also increases the technology cutoff so that the cutoff ratio is unchanged. As demonstrated later, updating firms has a substantial effect on job creation. Thus, the effectiveness of horizontal product market deregulation may be limited.

2.5 Closing the model

To close the model, I first specify the aggregate resource constraint. It is the equality between aggregate income and the total revenue of surviving firms. Because there are two firm types, the aggregate resource constraint takes the following somewhat complicated form:

$$E(t) = n\left(1-\phi^{\alpha}\right)\int_{x_0}^{x_1}\sum_{t-\tau=0}^{\infty} a(t)R(x, t-\tau)\frac{(1-\delta)^{t-\tau}\delta dF(x)}{F(x_1)-F(x_0)} \tag{26}$$

$$+ n\phi^{\alpha}\int_{x_1}^{\infty} a(t)R(x,0)\frac{dF(x)}{1-F(x_1)}$$

Here, the first (second) term on the right-hand side is the sum of revenues of obsolescing (updating) firms. The number of obsolescing firms is $n(1-\phi^{\alpha})$, and that of updating firms is $n\phi^{\alpha}$.

To be consistent with Melitz (2003) and Felbermayr and Prat (2011), this paper defines the average of surviving firms with respect to x, denoted by $\tilde{x}$, so that the relationship below holds:

$$\frac{E(t)}{n} = a(t)R(\tilde{x}, 0) \tag{27}$$

Because $E(t)/n$ is the average revenue of surviving firms, the above equation states that the average productivity $\tilde{x}$ is defined such that the revenue of a firm with $x = \tilde{x}$ and $t - \tau = 0$ equals the average revenue. Specifically, average productivity is given as

$$\tilde{x} \equiv \left[(1-\phi^{\alpha})\frac{\delta}{1-G_2(1-\delta)}\int_{x_0}^{x_1} x^{\sigma-1}\frac{dF(x)}{F(x_1)-F(x_0)} + \phi^{\alpha}\int_{x_1}^{\infty} x^{\sigma-1}\frac{dF(x)}{1-F(x_1)}\right]^{\frac{1}{\sigma-1}}, \tag{28}$$

where this expression accounts for the two firm types. The term $\delta/[1 - G_2(1-\delta)] = \delta\sum_{t-\tau=0}^{\infty} G_2^{t-\tau}(1-\delta)^{t-\tau} < 1$ corresponds to the weighted downsizing operator for new and old obsolescing firms. If there is no obsolescence, $G_2 = 1$ and $\delta/[1 - G_2(1-\delta)] = 1$.[14]

Importantly, from equations (12), (15), and (27), it is clear that average productivity equals revenue per worker.[15] In other words, the Euler equation (29) for employment pins down the relationship between labor market tightness and average productivity:

$$\frac{\sigma-\beta}{\sigma-1}\kappa(\theta) = \tilde{x} \tag{29}$$

This equation implies that when there are more-productive firms, more vacancies are posted. Thus, an increase in $\tilde{x}$ increases θ. The unique existence of θ requires $\tilde{x}[(\sigma-1)/(\sigma-\beta)] - b > 0$. Because productivity cutoffs and $\tilde{x}$ are solved in advance, equation (29) solves θ.

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For a given θ, unemployment u is immediately solved by the equality condition between job creation and destruction:

$$um(\theta, 1) = (1 - u)s \Leftrightarrow u = \frac{s}{m(\theta, 1) + s} \tag{30}$$

Thus, an increase in θ increases $m(\theta, 1)$ and decreases u.

Finally, the number of surviving firms n is solved by the equality condition of aggregate labor demand and the number of employed workers:

$$L^u + L^o = 1 - u \tag{31}$$

Here, L^u denotes the total labor demand of updating firms and L^o denotes that of obsolescing firms. The total labor demand by firm type is given as

$$L^u = n\phi^\alpha \int_{x_1}^{\infty} l(x, 0) \frac{dF(x)}{1 - F(x_1)} \text{ and} \tag{32}$$

$$L^o = n\left(1 - \phi^\alpha\right) \int_{x_0}^{x_1} \sum_{t-\tau=0}^{\infty} l(x, t-\tau) \frac{(1-\delta)^{t-\tau}\delta dF(x)}{F(x_1) - F(x_0)}. \tag{33}$$

In summary, the equilibrium is defined as follows.

Definition 5. *A balanced-growth equilibrium is defined as a list of unemployment u, number of surviving firms n, labor market tightness θ, exit cutoff x_0, adoption rate ϕ^α, aggregate income $E(t)/a(t)$, values $\{J^o(x, l, \tau, t)/a(t), J^u(x, l, t)/a(t), W^o(x, l, \tau, t)/a(t), W^u(x, l, t)/a(t)$, and $U(t)/a(t)\}$, revenue $R(x, t-\tau)$, labor demand $l(x, t-\tau)$, and wage w, where the following is true:*

- *$J^o(x, l, \tau, t)$, $J^u(x, l, t)$, $W^o(x, l, \tau, t)$, $W^u(x, l, t)$, and $U(t)$ satisfy the Bellman equations (4)–(8) and the bargaining equations (9) and (10)*
- *Vacancy posting is optimal such that $R(x, t-\tau)$ and $l(x, t-\tau)$ satisfy equations (12) and (13)*
- *Firm-type choice is optimal such that x_0 and ϕ^α satisfy equations (20) and (21)*
- *There is free entry such that equation (22) holds*
- *The aggregate resource constraint is given as equation (26)*
- *u and n are obtained to satisfy equations (30) and (31)*

3. The impact of growth on unemployment

How does unemployment u change when the growth rate g increases? There are three channels at work.

First, there is an increase in the net discount factor in the Euler equation (29) for employment when $\tilde{x}$ is fixed—the capitalization effect. This effect reflects rapid improvement in productivity and an increase in the job creation value of each firm. Because the effect increases labor market tightness θ, unemployment decreases. Specifically, from equation (29), the capitalization effect is computed as follows:

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$$\frac{d\theta}{dg} = \left[\frac{d\omega(\theta)}{d\theta} + \frac{c}{1-\beta}\frac{d[\theta/m(\theta,1)]}{d\theta}\left(G_1^{-1} - 1 + \lambda\right)\right]^{-1} \tag{34}$$

$$\times \left[\underbrace{-\frac{c\theta}{(1-\beta)m(\theta,1)}\frac{dG_1^{-1}}{dg}}_{\substack{(+)\\ \text{Capitalization Effect}}} + \frac{\sigma-1}{\sigma-\beta}\frac{d\tilde{x}}{dg}\right]$$

Because $d\omega(\theta)/d\theta$ and $d[\theta/m(\theta,1)]/d\theta$ are positive, the second row determines the sign of $d\theta/dg$. With the help of equation (30), we obtain du/dg.

Second, there is a decrease in the weighted downsizing operator $\delta/[1-G_2(1-\delta)]$ in the definition of $\tilde{x}$ (equation [28])—the current-form creative-destruction effect. In this economy featuring multiworker firms, the structure of the creative-destruction effect is different from that in DMP models.[16,17] Intuitively, a decrease in the downsizing operator G_2 (equivalently, an increase in the job-cut rate $1-G_2$) in each obsolescing firm increases unemployment. No other channel is associated with obsolescence and related job cuts when aggregate firm composition (represented by the two productivity cutoffs) is fixed.

A change in aggregate firm composition is the third effect—the firm-composition effect. This effect can be described by changes in the average productivity of updating firms $\tilde{x}^u$, that of obsolescing firms $\tilde{x}^o$, and the adoption rate ϕ^α, where the average productivity by firm type is set such that $\tilde{x}^u \equiv [\int_{x_1}^{\infty} x^{\sigma-1}dF(x)/[1-F(x_1)]]^{1/(\sigma-1)}$ and $\tilde{x}^o \equiv [\int_{x_0}^{x_1} x^{\sigma-1}dF(x)/[F(x_1)-F(x_0)]]^{1/(\sigma-1)}$.

In particular, the content of $d\tilde{x}/dg$ in equation (34) is given as follows:

$$\frac{d\tilde{x}}{dg}\frac{\sigma-1}{\tilde{x}^{2-\sigma}} = \underbrace{(1-\phi^\alpha)(\tilde{x}^o)^{\sigma-1}\frac{d}{dg}\left[\frac{\delta}{1-G_2(1-\delta)}\right]}_{\substack{(-)\\ \text{Current-Form Creative-Destruction Effect}}} \tag{35}$$

$$+\,\phi^\alpha\frac{d\,(\tilde{x}^u)^{\sigma-1}}{dg}$$

$$+\,(1-\phi^\alpha)\frac{\delta}{1-G_2(1-\delta)}\frac{d\,(\tilde{x}^o)^{\sigma-1}}{dg}$$

$$+\underbrace{\left[(\tilde{x}^u)^{\sigma-1} - \frac{\delta}{1-G_2(1-\delta)}(\tilde{x}^o)^{\sigma-1}\right]\frac{d\phi^\alpha}{dg}}_{(+)}$$

Here, the first row computes the creative-destruction effect and the remaining rows jointly compute the firm-composition effect. The fourth row in equation (35) is unambiguously positive when $g>0$ because $d\phi^\alpha/dg>0$ and $\tilde{x}^u > \tilde{x}^o$ hold. As demonstrated in the next section, the term in the fourth row is dominant in reducing unemployment.

The total impact of growth on unemployment is ambiguous given the three competing channels.

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4. Simulation

In this section, I quantify the impact of growth on unemployment, du/dg, and the importance of each channel. Then, I compare du/dg in the model with the data. In addition, I show the extent to which my model's results improve on those of a standard DMP model.

4.1 Benchmark calibration

The model is calibrated to the US economy. The period is annual. The matching function is specified such that $m(V, u) = m_0 u^\eta V^{1-\eta}$. The parameter values are set as follows.

The discount rate r and the base growth rate g are 0.04 and 0.02, respectively. In line with Elsby et al. (2013), the monthly job-separation probability $s/12$ is 0.036. The firm-destruction probability δ is 0.092, computed as the mean value over the 1978–2014 period using data from Business Dynamics Statistics. The data are available since 1978. The end year is set to be consistent with the period that is adopted in the subsequent simulation.

The elasticity of matching function η is 0.5, based on Petrongolo and Pissarides (2001). Following Felbermayr and Prat (2011), the bargaining power of each worker β is 0.5 so that $\beta = \eta$ holds. As mentioned in Felbermayr and Prat (2011), when allowing multiworker firms, the Hosios condition $\beta = \eta$ is not sufficient to ensure an efficient allocation. In the social planner's problem here, the derivatives of total revenue to u and θ are not simple. In contrast, in the DMP setting, the typical total revenue is exogenous output per worker multiplied by $1 - u$ (thus, its derivative with respect to u is simply output per worker multiplied by -1). Because of the absence of well-established estimates, I set the bargaining power $\beta = \eta$ as a standard choice. Additionally, an independent change in β generates a monotonic effect on du/dg; the question of whether the value of β is near its optimal value seems uninteresting.

According to Ebell and Haefke (2009), the entry cost in the USA in 1997 equaled 0.6 months of per capita income, and the entry cost in 1978 amounted to 5.2 months of per capita income. I simply use the mean value of these estimates such that $f_e = [(0.6 + 5.2)/2] \times 1/12 = 0.24$, although the main results are robust to this decision. I show the sensitivity of the three effects later. I use the value of per capita income equal to one, based on the normalization $\tilde{x} = 1$, which is mentioned later. Although the average productivity $\tilde{x}$ and per capita income $E(t)/a(t)$ are not the same, these values are numerically very similar so that $\tilde{x} = 1$ (normalization) and $E(t)/a(t) = 0.9401$ (solution) hold.

The remaining parameter values cannot be set directly. Thus, these values are obtained such that important targeted moments in the model equal those in the data.[18]

The scale of matching function m_0, the flow value of unemployment b, and the vacancy cost c are 7.99, 0.27, and 0.54, respectively. These values are obtained to replicate the data moments: labor market tightness equals 0.72, from Pissarides (2009); the monthly job-finding probability equals 0.565, from Elsby et al. (2013); the replacement rate (the ratio of the unemployment flow value to the wage b/w) equals 0.37, from the OECD Statistics database.[19]

The elasticity of substitution between two differentiated goods σ is 2.56 such that the markup equals 1.32, from Christopoulou and Vermeulen (2012).[20] The shape of the productivity distribution α is 1.66 such that the size distribution of firms is approximately Zipf and its tail index $\alpha/(\sigma - 1)$ equals 1.06, from Axtell (2001) and Luttmer (2007).

The minimum level of firm-specific productivity $x_{\min}$ is 0.008 such that the average productivity $\tilde{x}$ equals 1 as a normalization. The adoption cost I is 1.83 such that the average firm size equals 19.51 as the mean value over the 1978–2014 period, from Business Dynamics Statistics.

The parameter values are summarized in Table 1. Under these parameter values, the equilibrium solution is $u = 0.0599$, $n = 0.0482$, $w = 0.7289$, $E(t)/a(t) = 0.9401$, $x_0 = 0.1638$, and $x_1 = 1.5296$. Of course, $\theta = 0.72$ and $\tilde{x} = 1$ hold because these are the targeted moment and the normalization.

Table 1. Parameter values

Parameter	Description	Value
r	Discount rate	0.04
g	Growth rate	0.02
$s/12$	Job-separation probability	0.036
δ	Firm-destruction probability	0.092
η	Matching-function elasticity	0.5
β	Worker bargaining power	0.5
f_e	Entry cost	0.24
m_0	Matching-function scale	7.99
b	Flow unemployment value	0.27
c	Vacancy cost	0.54
σ	Elasticity of substitution	2.56
α	Productivity distribution shape	1.66
x_{min}	Minimum productivity	0.008
I	Adoption cost	1.83

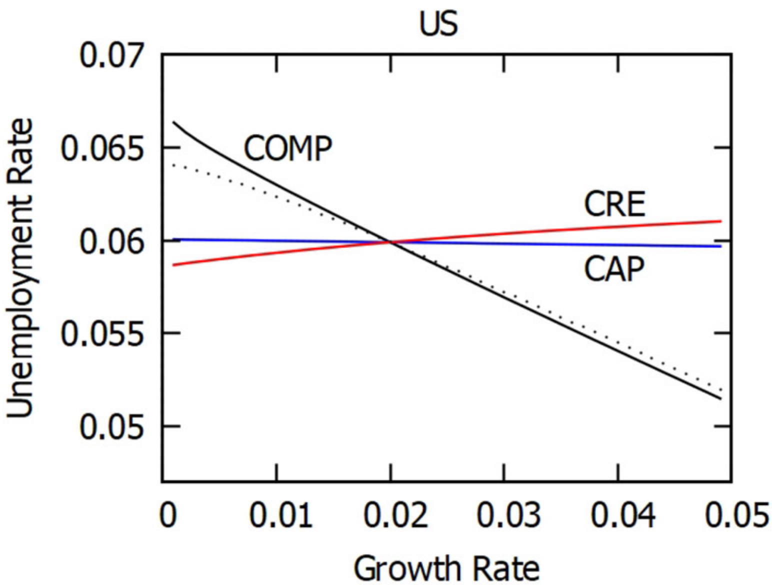

Figure 3. The impact of growth on unemployment.
Note: The total impact of growth on unemployment is decomposed into three effects so that CAP, CRE, and COMP represent the capitalization effect, the creative-destruction effect, and the firm-composition effect, respectively. The dotted line corresponds to the total impact of the three effects.

4.2 Results

Fig. 3 graphically shows the total impact of growth on unemployment du/dg (as the dotted line) and its decomposition into the three effects. When the growth rate increases, the capitalization effect (labeled "CAP") slightly reduces unemployment, while the creative-destruction effect

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Table 2. Decomposition and calibration sensitivity

Description	Benchmark	$b/w = 0.71$	Markup = 1.6
du/dg	−0.2586	−0.5412	−0.2548
CAP	−0.0078	−0.0078	−0.0078
CRE	+ 0.0508	+ 0.1082	+ 0.0739
COMP	−0.3016	−0.6413	−0.3207
COMP1	+ 0.9855	+ 2.103	+ 1.9608
COMP2	+ 0.0074	+ 0.0158	+ 0.0477
COMP3	−1.2916	−2.7386	−2.3181

Note: The total impact of growth on unemployment du/dg is decomposed into three effects: CAP, CRE, and COMP. COMP is further decomposed into COMP1, COMP2, and COMP3, which are associated with the second to fourth rows in equation (35), respectively. In addition to Benchmark (the result under the benchmark calibration), I check alternative calibration cases such that the value of the targeted moment is $b/w = 0.71$ or Markup = 1.6, and the entry cost f_e is given as 0.6/12 or 5.2/12. Each case for f_e is omitted because its result is identical to the Benchmark column.

(labeled "CRE") increases it. Consistent with the literature, the latter effect is stronger than the former effect.

However, these canonical effects are small relative to the firm-composition effect (labeled "COMP"). Rapid technology obsolescence is not always undesirable because it not only increases the job-cut rate in each obsolescing firm but reduces the number of such firms and induces firms to choose the updating firm type. As a result, the composition of surviving firms and the associated worker reallocation improve in the aggregate.[21]

Table 2 numerically shows the total impact du/dg and its decomposition. The column "Benchmark" reports the result under the benchmark calibration.[22] COMP is approximately 40 times larger than CAP. The size of CAP is similar to that obtained in standard DMP models (under fully disembodied technology) because equation (34) is essentially unchanged across models when $d\tilde{x}/dg = 0$. In Appendix I, I describe the simple DMP model with the capitalization and creative-destruction effects and simulate du/dg.

In addition, in Table 2, COMP is further decomposed into COMP1, COMP2, and COMP3. While the second to fourth rows in equation (35) jointly compute COMP, COMP1 is computed by taking into account only the second row in equation (35); it reflects a change in the average productivity of updating firms. COMP1 is positive because some less productive firms become updating firms and the average productivity of the updating firm type decreases. This reduces labor market tightness and increases unemployment. Similarly, COMP2 is computed by using only the third row in equation (35); it reflects a change in the average productivity of obsolescing firms. COMP2 is relatively insignificant because this channel involves competing effects: a decrease in the technology cutoff x_1 and an increase in the exit cutoff x_0. Finally, COMP3 is computed by activating only the fourth row in equation (35), and it reflects an increase in the adoption rate. The results imply that COMP3 is the dominant source of COMP.

In Table 2, the results of alternative calibration cases are also shown. First, the targeted data moment for b/w is changed to 0.71, as suggested by Hall and Milgrom (2008). As discussed in the literature, the opportunity cost of employment may be too low, when it does not include the value of leisure or home production.[23] In this alternative case, the sizes of CRE and COMP increase while CAP remains unchanged. In total, the size of du/dg increases.

In the second case, the targeted markup is changed to 1.6. De Loecker et al. (2020) document the evolution of market power in the USA, where the average markup steadily increased from 1.2 in 1980 to 1.6 in 2016.[24] Although the size of du/dg slightly decreases, the result is still robust.

In the third case, f_e is changed in the calibration process. Modifying it does not change the benchmark result, which is reported in Table 2. This is because such a change in f_e is fully absorbed

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in $x_{\min}$, while the other parameter values are unaltered; as in equation (25), f_e itself does not matter as long as $f_e/[x_{\min}^{\alpha}]$ is constant.

4.3 Data versus model

Fig. 4 demonstrates the evolution of the unemployment rate from the data and model predictions. The model predictions are computed such that only the growth rate varies as in its empirical evolution (see the bottom graph), while the other parameter values are fixed.

To directly observe model performance, Table 3 shows the regression estimates for which each evolution in Fig. 4 is used. Each regression coefficient in Table 3 corresponds to the estimated value of du/dg. Regarding its value, the results imply that the model accounts for 16%–36% of the variation in the data.

Additionally, in this exercise, the relative importance of the three effects for du/dg holds similarly to that in Table 2 (for each case in each column, respectively) because these effects are computed in an almost linear manner across different growth rates as visualized in Fig. 3. Thus, this model explains an important new part of the data in contrast to standard DMP models, which explain at most the size of CAP.

Regarding the empirical literature, the implied du/dg is -0.47 in Blanchard and Wolfers (2000, Table 4), -1.49 in Pissarides and Vallanti (2007), and -1.15 in Miyamoto and Takahashi (2011). The first paper's estimate is for 20 OECD countries, while the other estimates are for the USA.

Regarding the standard DMP model in Appendix I, the implied du/dg is -0.0072 when the ratio of disembodied technology equals 0.99 as the exogenous adoption rate $\phi^{\alpha}=0.99$ (equivalently when the remaining ratio 0.01 is embodied technology). The sign reversal of du/dg occurs between $\phi^{\alpha}=0.9$ and $\phi^{\alpha}=0.8$. When $\phi^{\alpha}=0.1$, the implied du/dg is $+0.0388$. For low ϕ^{α}, the standard DMP model fails to explain the data even qualitatively.

The size of the implied du/dg from the data increases over time. In Table 4, the regression results by period are reported; the total period 1970–2014 is truncated at the peak and trough of the growth rate in Fig. 4. Because the regression coefficients for the model, reported in Table 3, are almost unchanged regarding period separation, model performance deteriorates as time passes. If this tendency is true, the value of investigating the sources of du/dg increases.

4.4 Interaction with policy

Table 5 shows how the decomposition of du/dg is modified when the following policy-related parameter values are changed independently: the flow unemployment value b, worker bargaining power β, or the entry cost f_e. When $b=0.27$, $\beta=0.5$, and $f_e=0.24$, the decomposition result is the same as in the second column (labeled "Benchmark") in Table 2.

Two remarks are in order. First, an increase in each parameter value monotonically increases the sizes of du/dg and all decomposed outcomes. This is common to the three parameters. In other words, some rigidities in the labor and product markets increase both the unemployment rate and the magnitude of du/dg. This result helps us understand the observed differences between the USA and Europe as suggested in Hornstein et al. (2007).[25]

Second, a change in b appears to generate a stronger effect than the other parameters. In Table 5, the range of β is not narrow and that of f_e is associated with the US entry costs between 1978 and 1997 as suggested in Ebell and Haefke (2009). In addition, $b=0.5$ is near the value when the targeted data moment of b/w in the calibration equals 0.71 (although this alternative calibration case also changes the value of c). A change in b from 0.27 (as the benchmark value) to 0.5 increases the size of du/dg by 0.41. As found in Aguiar and Hurst (2008) and Aguiar et al. (2021), leisure time and contents change over time; b does not reflect mere unemployment insurance and seems an important factor in the evolution of du/dg.

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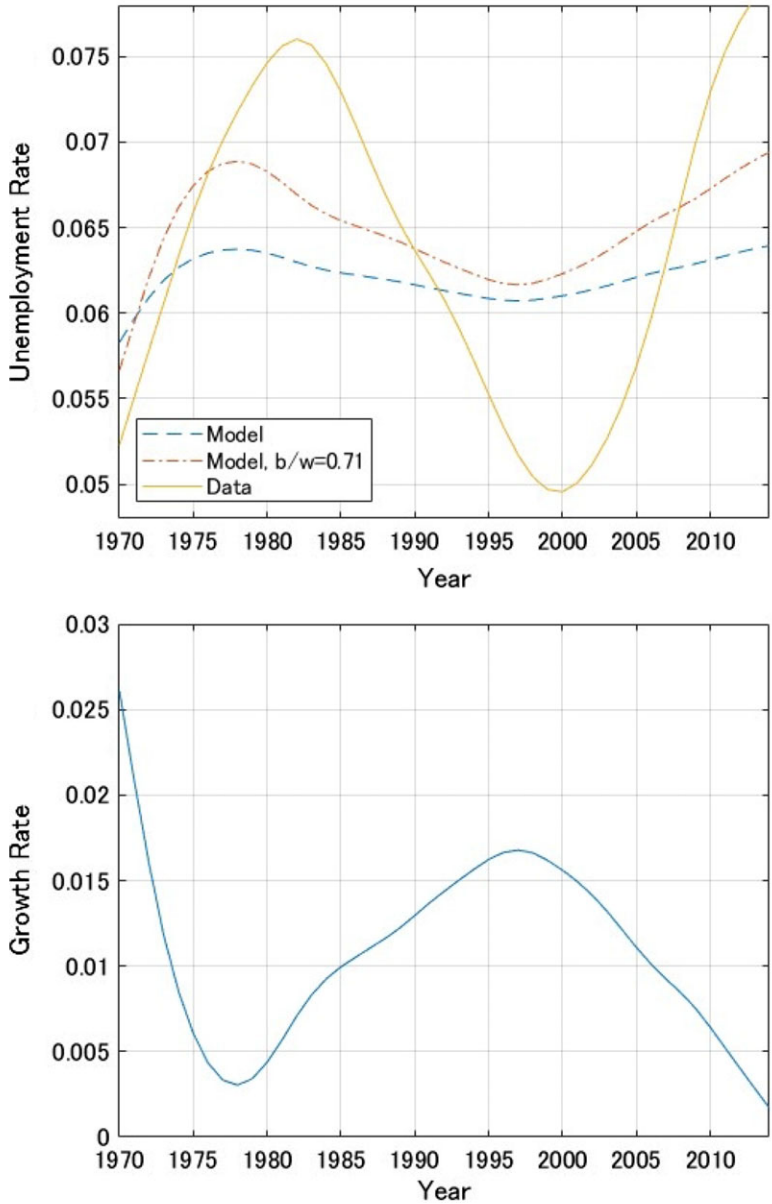

Figure 4. Unemployment evolution, data versus model.
Note: The evolution of the unemployment rate is depicted for data and model predictions (which correspond to the calibration setups labeled "Benchmark" and "$b/w = 0.71$" in Table 2) as a response to the empirical variation of the growth rate. The annual data are constructed from the 2017 release of the EUKLEMS database and the OECD Statistics database. The growth rate is calculated as gross value added (in 2010 prices) divided by employment. The data are trended through Hodrick and Prescott filtering with the smoothing parameter value 100.

4.5 Interaction with finance

Empirically, the labor market appears to have a strong link with the stock market.[26] Complementary to this view, financial imperfections may amplify the impact of growth on unemployment in the current simulation. Petrosky-Nadeau and Wasmer (2013) consider a two-stage

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Table 3. Regression results: data versus model

Description	Data	Model	Model, $b/w = 0.71$
Intercept	0.0798	0.0646	0.0706
	(0.0030)	(0.0001)	(0.0000)
Coefficient	−1.4654	−0.2274	−0.5308
	(0.3073)	(0.0059)	(0.0039)
Adjusted R^2	0.694	0.995	0.999

Note: The independent variable is the growth rate g, and the dependent variable is the unemployment rate u, using the evolution in Fig. 4. Newey-West HAC standard errors are shown in parentheses.

Table 4. Regression results by period

Description	Data, Period1	Data, Period2	Data, Period3
	1970–1978	1979–1997	1998–2014
Intercept	0.0717	0.0869	0.0853
	(0.0011)	(0.0000)	(0.0011)
Coefficient	−0.8058	−1.8280	−2.3215
	(0.0679)	(0.0042)	(0.1163)
Adjusted R^2	0.948	0.839	0.968

Note: The regression result for the data in Table 3 is further examined. Specifically, I separate the total period 1970–2014 into three periods and get the regression results for each period.

Table 5. Decomposition and policy

Description	$b = 0.1$	$b = 0.5$	$\beta = 0.2$	$\beta = 0.8$	$f_e = 0.6/12$	$f_e = 5.2/12$
du/dg	−0.1663	−0.6658	−0.1746	−0.3957	−0.1034	−0.4626
CAP	−0.0058	−0.0144	−0.0065	−0.0141	−0.0023	−0.0142
CRE	+0.0325	+0.1321	+0.0341	+0.0773	+0.0205	+0.0909
COMP	−0.1929	−0.7832	−0.2021	−0.4588	−0.1215	−0.5390
COMP1	+0.6299	+2.5664	+0.6607	+1.4986	+0.3968	+1.7631
COMP2	+0.0047	+0.0193	+0.0050	+0.0113	+0.003	+0.0133
COMP3	−0.8268	−3.3468	−0.8654	−1.9653	−0.5208	−2.3068
$u\vert_{g=0.02}$	0.0516	0.0824	0.0352	0.1024	0.0323	0.0819

Note: The decomposition results are shown under different parameter values for the flow unemployment value b, worker bargaining power β, and the entry cost f_e. The remaining parameter values, which are not specified in the first row, equal those of the benchmark calibration. Thus, the current exercise does not examine alternative calibration cases but looks for a change in each parameter value one by one. The first column is the same as in Table 2 except for the additional information $u|_{g=0.02}$, which reports the unemployment rate at $g = 0.02$.

matching environment initially through the credit market and then through the labor market. Their model implies that the financial sector introduces a new element to hiring costs such that element is independent of congestion in the labor market. As a result, their model shows that the elasticity of labor market tightness to productivity shocks is amplified in the business cycle context.[27]

A simple extension regarding this point is examined here. Specifically, I replace the hiring cost c by the modified hiring cost $c + [m(\theta, 1)/\theta]H$. All setups except for this are unchanged. H denotes the new exogenous element in hiring costs. In other words, the total cost of employing one worker is $c\theta/m(\theta, 1) + H$.[28] Thus, H is a fixed and independent term.

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Table 6. Decomposition and additional exogenous term H in hiring costs

Description	$H = 0.01$	$H = 0.02$	$H = 0.01, b/w = 0.71$	$H = 0.02, b/w = 0.71$
du/dg	-0.2849	-0.3170	-0.6731	-0.8900
CAP	-0.0086	-0.0095	-0.0097	-0.0128
CRE	$+0.0560$	$+0.0623$	$+0.1346$	$+0.1780$
COMP	-0.3322	-0.3697	-0.7974	-1.0544
COMP1	$+1.0857$	$+1.2085$	$+2.6180$	$+3.4656$
COMP2	$+0.0082$	$+0.0091$	$+0.0196$	-0.0260
COMP3	-1.4224	-1.5827	-3.4029	-4.4950
$c + \frac{m(\theta,1)}{\theta} H\|_{g=0.02}$	0.5429	0.5429	0.2551	0.2551
c	0.4488	0.3546	0.161	0.0668
$\frac{m(\theta,1)}{\theta} H\|_{g=0.02}$	0.0942	0.1883	0.0942	0.1883

Note: Alternative calibrations and their decomposition results are shown, where the value of the additional exogenous term H is initially given and then I implement the same procedure as in the benchmark calibration except that the total hiring cost is modified to $c + m(\theta, 1)H/\theta$. In addition to the benchmark calibration (with the targeted data moment $b/w = 0.37$), the case for the targeted data moment $b/w = 0.71$ is also shown for different values of H.

Table 6 shows the results under different H. The last two rows in Table 6 report the implied size of each element in hiring costs. Checking these relative sizes, I select $H = 0.01$ and 0.02 to demonstrate the outcomes. Similar to the business cycle context, at least, the presence of the fixed term H magnifies the current results.

In addition, heterogeneous financial conditions across firms may further amplify the results. Eckstein et al. (2019) use firm-level data and document that lower credit ratings are associated with more volatile employment and higher interest rate volatility. They show that in the 2008 financial crisis, firms with lower credit ratings experienced greater declines in employment.[29] In my model, a change in the adoption rate is positively linked to the job-cut rate, as suggested in Proposition 4. Thus, if obsolescing (less productive) firms are financially weak and must further increase the job-cut rate to weather episodes of illiquidity in times of rapid technological progress, the adoption rate increases so that the firm-composition effect strengthens.

The relationship between a technology choice and creditor protection may also be important when incorporating financial imperfections into the analysis. Benmelech and Bergman (2011) use a panel of aircraft-level data around the world and find that airlines enjoying the benefits of higher creditor protection operate aircraft of a newer technology and younger vintage. They argue that improved investor protection and its associated reduction in financial frictions affect firms' tendency to invest in newer technologies as a key driver of productivity growth. This view seems to be helpful for uncovering a valuation in unemployment through a change in the adoption rate.

5. Conclusions

This study considered heterogeneous multiworker firms and examined the impact of technological progress on unemployment. Incorporating firm heterogeneity into the model uncovered endogenous variations in the exit cutoff and the technology cutoff with respect to firm-specific productivity. In other words, the surviving firms are divided into two broad types of technology choices: updating and obsolescing firms. Thus, the model incorporates the channel of a change in the aggregate composition of firms, referred to as the firm-composition effect, in addition to the canonical channels, the creative-destruction effect and the capitalization effect. The simulation results implied that the firm-composition effect is substantial and helps fill the gap between theory and data.

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Notes

1 In the Appendix, I review previous studies on the relationship between ICT and productivity growth.
2 Aghion and Howitt (1994) and Mortensen and Pissarides (1998) are foundational works in the literature.
3 This percent range is from using different targeted data moments in calibrating the model, with respect to the ratio of the unemployment flow value to the wage, based on Shimer (2005) or Hall and Milgrom (2008).
4 For example, see Elsby and Michaels (2013) and Acemoglu and Hawkins (2014) for the setting of multiworker firms.
5 Roughly speaking, the impact of growth on unemployment in a DMP model with disembodied technology is referred to as the capitalization effect, and that in a DMP model with embodied technology without updates is referred to as the creative-destruction effect.
6 This study avoids considering multiple update frequencies as in Mortensen and Pissarides (1998) because an aggregate adoption extent becomes intractable in that case. In other words, this study considers two broad types of consecutive-updating firms (updating firms) versus non-updating firms (obsolescing firms).
7 Specifically, the revenue function is derived under the following setup. Each consumer maximizes the utility function over the consumption of differentiated variety of goods:

$$\max_{q_{i,j}} \left[n^{-\frac{1}{\sigma}} \int q_{i,j}^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}$$

Here, $q_{i,j}$ denotes the consumption of good i by consumer j. Following the literature, one firm produces one differentiated good. The price index is defined as $P \equiv \left[(1/n) \int p_i^{1-\sigma} di \right]^{1/(1-\sigma)}$, where p_i denotes the price of good i. Without loss of generality, the price index is normalized to one. See Blanchard and Giavazzi (2003), Ebell and Haefke (2009), and Felbermayr and Prat (2011).
8 For example, when both sides of (5) are divided by $a(t)$, I show that $J^u(x, l, t)/a(t) = \ldots + [(1-\delta)/(1+r)] [a(t+1)/a(t)] J^u(x, l', t+1)/a(t+1) \Leftrightarrow J^u(x, l, t)/a(t) = \ldots + [(1-\delta)(1+g)/(1+r)] J^u(x, l', t+1)/a(t+1)$.
9 All proposition proofs are shown in the appendix.
10 I explicitly present the expression of $E(t)$ in Section 2.5.
11 This is a basic feature of models with multiworker firms. I avoid further discussion regarding this issue.
12 Technically, any downsizing through gradual obsolescence is described as a decrease in new vacancies for replenishing vacant jobs that occur with an exogenous separation shock.
13 Equation (25) exactly traces (22). See the appendix for a detailed derivation.
14 In essence, the definition of $\tilde{x}$ is unnecessary, but it is convenient to regard its term as a chunk.
15 Recall that revenue per worker in any firm equals the same value through vacancy optimization.
16 Because I define $\tilde{x}$ so that the average revenue equals the revenue evaluated with $x = \tilde{x}$ and $t - \tau = 0$, the weighted downsizing operator is conveniently arranged in its $\tilde{x}$.
17 For the canonical form of this effect, evaluate the model in Appendix I with ϕ^{α} set to 0.
18 Precisely speaking, there is one productivity normalization for $x_{\min}$.
19 Following the method in Nickell et al. (2003), I calculate the replacement rate under a 12-month unemployment duration averaged over family types in 2001, including social assistance benefits and housing benefits. Because there is no drastic change over time, I use the value as of 2001. It is similar to that used in Shimer (2005).
20 This empirical study considers not only the manufacturing and construction sectors but also the market services sector. The last sector tends to generate a higher markup relative to the first two.
21 Simulated changes in key endogenous variables are summarized in Appendix H.
22 Technically, I first compute the two steady-state outcomes under $g = 0.02$ and 0.0201. Then, by using these results, each value in Table 2 is calculated. For example, du/dg is obtained as $[u|_{g=0.0201} - u|_{g=0.02}]/[0.0201 - 0.02]$.
23 In fact, Miyamoto and Takahashi (2011) use this alternative data moment in their calibration. The reason for using $b/w = 0.37$ in this paper is that it is conservative.
24 I thank an anonymous referee for the information.
25 Because Hornstein et al. (2007) focus on the creative-destruction effect, their results are not directly comparable to the current results. However, the essence seems similar.
26 For example, see Hall (2017).
27 This is consistent with Pissarides (2009), who shows that such an independent element (regardless of its sources) magnifies business cycle fluctuations.
28 $\theta/m(\theta, 1)$ vacancies are posted on average to employ one worker.
29 Lower credit ratings are associated with higher interest rates (as a well-known fact) and lower average firm size (from their summary statistics).
30 Associated with this early productivity paradox, Robert Solow (1987) remarks that "You can see the computer age everywhere but in the productivity statistics." In addition, Greenwood and Yorukoglu (1997, Figure 2) show a productivity slowdown before 1990 and a simultaneous increase in information technology investment. One reason for the paradox is that ICT capital accumulation is fairly small relative to the total capital in the early period.

31 In addition, the authors compare their results with those reported in the other seminal studies (Jorgenson and Stiroh (2000), and Whelan(2000)) and suggest that, as a robust result, computer hardware makes a substantially larger contribution to output growth during the second half of the 1990s than during the first half.
32 See also Brynjolfsson and Hitt (2000) for a summary of microlevel studies, Gordon and Sayed (2020) for longer period analyses, and Timmer and van Ark (2005) and Ark et al. (2008) for the productivity growth difference between the USA and Europe.
33 The computation method for the values in Table 7 is the same as in the body text so that du/dg is evaluated near the point $g=0.02$. However, a previous draft computes du/dg as $[u|_{g=0.03} - u|_{g=0.02}]/[0.03 - 0.02]$ by computing the steady-state unemployment rates at $g=0.02$ and $g=0.03$. In this case, the positive magnitudes of du/dg for low ϕ^{α} become much larger.

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A. Empirical Implications for the Relationship between ICT and Productivity

In this section, I review some results to see that ICT has contributed to labor productivity growth in the USA, at least until approximately 2000.

Oliner and Sichel (2000) consider the three periods, 1974–1990, 1991–1995, and 1996–1999, and find that the contribution of ICT capital deepening (as an increase in the total amount of hardware, software, and communication equipment per hour worked) to labor productivity growth is modest from the first to the second period and is strong from the second to the third period. The former result is consistent with studies conducted before the early 1990s failing to confirm the relationship between ICT and aggregate productivity growth.[30] The latter result is specified so that 68% of the increase in labor productivity growth from the second to the third period occurs via the sum of both ICT capital deepening, 43%, and computer and computer-related semiconductor productions, 25%.[31] Interestingly, by investigating the residual productivity in their growth accounting, the authors show that approximately half of the residual productivity growth is achieved via the computer and semiconductor sectors in all three periods.

Greenwood et al. (1997) build a model in which neutral technological progress and investment-specific technological progress are considered separately. The former technological progress is measured as the growth rate of the total factor productivity directly incorporated into the production function. The latter technological progress is measured as the growth rate of the extent of how efficiently the amount of equipment investment is converted into its capital accumulation. The authors use equipment price data for a measure of investment-specific technological progress, and neutral technological progress is measured as the remaining component of labor productivity growth. The authors show that 58% of labor productivity growth is via investment-specific technological progress between 1954 and 1990.

Similarly, Cummins and Violante (2002) use a model consistent with Greenwood et al. (1997) but additionally consider a change in the labor quality associated with rising educational attainment over time. They show that 60% of labor productivity growth is via investment-specific technological progress between 1948 and 1999. Moreover, from their estimates by asset, the information processing equipment and software category, including computers and peripheral equipment, is the driving force. Implied productivity improvements in computers and peripheral equipment increase at an annual average of 23.5% between 1948 and 2000.[32]

B. Proof of Proposition 1 (Wage)

First, I obtain the first-order and envelope conditions based on (4) and (5) as follows:

$$-a(t)c+\frac{m(\theta,1)}{\theta}\frac{1-\delta}{1+r}\frac{\partial J^{o}(x,l',\tau,t+1)}{\partial l}=0, \tag{B1}$$

$$-a(t)c+\frac{m(\theta,1)}{\theta}\frac{1-\delta}{1+r}\frac{\partial J^{u}(x,l',t+1)}{\partial l}=0, \tag{B2}$$

$$\frac{\partial J^{o}(x,l,\tau,t)}{\partial l}=\frac{\partial R(x,l,\tau,t)}{\partial l}-\frac{\partial w(x,l,\tau,t)}{\partial l}l-w(x,l,\tau,t) \tag{B3}$$

$$+(1-\lambda)\frac{1-\delta}{1+r}\frac{\partial J^{o}(x,l',\tau,t+1)}{\partial l},\ \text{and} \tag{B4}$$

$$\begin{aligned}\frac{\partial J^{u}(x,l,t)}{\partial l}&=\frac{\partial R(x,l,t,t)}{\partial l}-\frac{\partial w(x,l,t,t)}{\partial l}l-w(x,l,t,t)\\&\quad+(1-\lambda)\frac{1-\delta}{1+r}\frac{\partial J^{o}(x,l',t+1)}{\partial l}.\end{aligned} \tag{B5}$$

These equations are very similar between firm types. Using (6), (8), (B1), and (B3), equation (9) is written as follows:

$$\begin{aligned}&\beta\left[\begin{array}{c}\frac{\partial R(x,l,\tau,t)}{\partial l}-\frac{\partial w(x,l,\tau,t)}{\partial l}l-w(x,l,\tau,t)\\ +\frac{1-s}{1+r}\frac{\partial J^{o}(x,l',\tau,t+1)}{\partial l}\end{array}\right]\\&=(1-\beta)\left[\begin{array}{c}w(x,l,\tau,t)-a(t)b-\frac{m(\theta,1)}{1+r}\left[\tilde{W}(t+1)-U(t+1)\right]\\ +\frac{1-s}{1+r}\left[W^{o}(x,l',\tau,t+1)-U(t+1)\right]\end{array}\right].\end{aligned} \tag{B6}$$

In addition, equation (B6) takes the following form when using the equality $m(\theta,1)[\tilde{W}(t+1)-U(t+1)]/(1+r)=a(t)\beta c\theta/[(1-\beta)(1-\delta)]$, which follows from the first-order conditions and (9).

$$\frac{\partial w(x,l,\tau,t)}{\partial l}+\frac{w(x,l,\tau,t)}{\beta l}-\frac{1}{l}\left[\frac{\partial R(x,l,\tau,t)}{\partial l}+\frac{1-\beta}{\beta}a(t)\omega(\theta)\right]=0, \tag{B7}$$

where $\omega(\theta)\equiv b+\beta c\theta/[(1-\beta)(1-\delta)]$ is defined as the reservation wage. It increases with θ.

Finally, by using a general solution shown by Bertola and Garibaldi (2001, p. 343), the ordinary differential equation (B7) is solved as follows:

$$\begin{aligned}w(x,l,\tau,t)&=l^{-\frac{1}{\beta}}\int_{0}^{l}k^{\frac{1}{\beta}-1}\frac{\partial R(x,k,\tau,t)}{\partial l}dk+(1-\beta)a(t)\omega(\theta)\\&=\frac{\beta(\sigma-1)}{\sigma-\beta}\frac{R(x,l,\tau,t)}{l}+(1-\beta)a(t)\omega(\theta),\end{aligned} \tag{B8}$$

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where $\partial R(x, l, \tau, t)/\partial l = [(\sigma-1)/\sigma]\,(E(t)/n)^{\frac{1}{\sigma}}\,(a(\tau)x)^{\frac{\sigma-1}{\sigma}} l^{-\frac{1}{\sigma}}$.

C. Proof of Proposition 2 (Optimal Labor Demand and Revenue)

I have the following Euler equation for employment by using (B1) and (B3):

$$\frac{1+r}{1-\delta}\frac{a(t-1)c\theta}{m(\theta,1)} = \frac{\partial R(x,l,\tau,t)}{\partial l} - \frac{\partial[w(x,l,\tau,t)l]}{\partial l} + (1-\lambda)\frac{a(t)c\theta}{m(\theta,1)}. \tag{C1}$$

Because $\partial[w(x,l,\tau,t)l]/\partial l = \beta[(\sigma-1)/(\sigma-\beta)]\partial R(x,l,\tau,t)/\partial l + (1-\beta)a(t)\omega(\theta)$ holds, the above equation leads to the following:

$$\frac{R(x,l,\tau,t)/a(t)}{l} = \frac{\sigma-\beta}{\sigma-1}\kappa(\theta), \tag{C2}$$

$\kappa(\theta)$ is defined as the employment cost, namely,

$$\kappa(\theta) = \omega(\theta) + \frac{c\theta}{(1-\beta)m(\theta,1)}\left[\frac{1+r}{(1-\delta)(1+g)} - 1 + \lambda\right]. \tag{C3}$$

Finally, the optimal labor demand and revenue are solved by using revenue's definition $R(x,l,\tau,t) = (E(t)/n)^{\frac{1}{\sigma}}(a(\tau)xl)^{\frac{\sigma-1}{\sigma}}$ and (C2).

D. Proof of Proposition 3 (Entry Values)

First, I transform the wage function as follows:

$$\begin{aligned}
w(x,l,\tau,t) &= \frac{\beta(\sigma-1)}{\sigma-\beta}\frac{a(t)R(x,t-\tau)}{l(x,t-\tau)} + (1-\beta)a(t)\omega(\theta)\\
&= \frac{\beta(\sigma-1)}{\sigma-\beta}\frac{a(t)R(x,t-\tau)}{l(x,t-\tau)}\\
&\quad + (1-\beta)a(t)\left[\frac{\sigma-1}{\sigma-\beta}\frac{R(x,t-\tau)}{l(x,t-\tau)} - \frac{c\theta}{(1-\beta)m(\theta,1)}\left(G_1^{-1} - 1 + \lambda\right)\right]\\
&= a(t)\left[\frac{\sigma-1}{\sigma-\beta}\frac{R(x,t-\tau)}{l(x,t-\tau)} - \frac{c\theta}{m(\theta,1)}\left(G_1^{-1} - 1 + \lambda\right)\right],
\end{aligned} \tag{D1}$$

where the second equality follows from (C2) and (C3). By using (D1), the operational value of an obsolescing firm is obtained as follows:

$$\begin{aligned}
J^o(x, l(x,t-\tau), \tau, t) &= \max_T \sum_{i=t}^{T+\tau}\left(\frac{1-\delta}{1+r}\right)^{i-t}\left[\begin{array}{c} a(i)R(x,i-\tau) - w(x, l(x,i-\tau), \tau, i)l(x,i-\tau) \\ -a(i)cv^o(x,i-\tau)\end{array}\right]\\
&= \max_T a(t)\sum_{i=t}^{T+\tau} G_1^{i-t}G_2^{i-\tau}\left[R(x,0)\frac{1-\beta}{\sigma-\beta} + l(x,0)\frac{c\theta}{m(\theta,1)}\left(G_1^{-1} - G_2\right)\right]\\
&= a(t)G_2^{t-\tau}\left[\frac{1}{1-G_1G_2}R(x,0)\frac{1-\beta}{\sigma-\beta} + \frac{1}{G_1}l(x,0)\frac{c\theta}{m(\theta,1)}\right],
\end{aligned} \tag{D2}$$

where $v^o(x,t-\tau) \equiv \left[l(x,t+1-\tau) - (1-\lambda)l(x,t-\tau)\right]\theta/m(\theta,1)$. The optimal firm-shutdown period T becomes ∞ because $G_1^{-1} - G_2 = G_1^{-1} - 1 + 1 - G_2 > 0$. Similarly, the operational value of an updating firm is as follows:

$$\begin{aligned} J^u(x, l(x,0), t) &= a(t)R(x,0) - w(x, l(x,0), t, t)l(x,0) \\ &\quad - a(t)cv^u(x) - a(t)I + \frac{1-\delta}{1+r}J^u(x, l(x,0), t+1) \\ &= a(t)\left[\frac{1}{1-G_1}\left[R(x,0)\frac{1-\beta}{\sigma-\beta} - I\right] + \frac{1}{G_1}l(x,0)\frac{c\theta}{m(\theta,1)}\right], \end{aligned} \tag{D3}$$

where $v^u(x) \equiv \left[l(x,0) - (1-\lambda)l(x,0)\right]\theta/m(\theta,1)$.

Finally, the entry values are given as follows:

$$J^u(x,0,t) = a(t)\left[-cv_0(x) - I + G_1 J^u(x, l(x,0), t)\right] \text{ and} \tag{D4}$$

$$J^o(x,0,t,t) - a(t)I = a(t)\left[-cv_0(x) - I + G_1 J^o(x, l(x,0), t, t)\right], \tag{D5}$$

where the initial number of vacancies corresponds to $v_0(x) \equiv l(x,0)\theta/m(\theta,1)$. From (D2)-(D5), the entry values are calculated as in the proposition.

E. Proof of Proposition 4 (Technology Adoption Rate and Exit Cutoff)

The reservation rules (20) and (21) characterize the revenue cutoffs $R(x_0,0)$ and $R(x_1,0)$ as follows:

$$\frac{1-\beta}{\sigma-\beta}R(x_0,0) = \frac{1-G_1G_2}{G_1}I \text{ and} \tag{E1}$$

$$\frac{1-\beta}{\sigma-\beta}R(x_1,0) = \frac{1-G_1G_2}{G_1(1-G_2)}I. \tag{E2}$$

Moreover, the following relationship can be obtained by calculating the ratio of the revenue cutoffs as follows:

$$\phi^{\sigma-1} = \frac{R(x_0,0)}{R(x_1,0)} = 1 - G_2. \tag{E3}$$

On the other hand, the free entry condition (22) is calculated as follows:

$$\begin{aligned} f_e &= \int_{x_0}^{x_1}\left[\frac{G_1}{1-G_1G_2}\frac{1-\beta}{\sigma-\beta}R(x_0,0)\frac{R(x,0)}{R(x_0,0)} - I\right]dF(x) \\ &\quad + \int_{x_1}^{\infty}\left[\frac{G_1}{1-G_1}\frac{1-\beta}{\sigma-\beta}R(x_0,0)\frac{R(x,0)}{R(x_0,0)} - \frac{I}{1-G_1}\right]dF(x) \\ &= I\left[\begin{array}{c} \frac{\alpha x_{\min}^{\alpha}}{\alpha-\sigma+1}\left(x_0^{-\alpha} - x_1^{-\alpha}\phi^{1-\sigma}\right) - [F(x_1) - F(x_0)] \\ +\frac{1-G_1G_2}{1-G_1}\frac{\alpha x_{\min}^{\alpha}}{\alpha-\sigma+1}x_1^{-\alpha}\phi^{1-\sigma} - \frac{1-F(x_1)}{1-G_1} \end{array}\right], \end{aligned} \tag{E4}$$

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where the second equality uses (E1). After dividing both sides by $I[1-F(x_0)] = I\,(x_{\min}/x_0)^{\alpha}$, expression (E4) is further transformed as follows:

$$
\begin{aligned}
\frac{f_e}{I}\left(\frac{x_{\min}}{x_0}\right)^{-\alpha} &= \frac{\alpha}{\alpha-\sigma+1}\phi^{\alpha}\phi^{1-\sigma}\frac{(1-G_2)G_1}{1-G_1} \\
&\quad + \frac{\alpha}{\alpha-\sigma+1} - 1 - \frac{G_1}{1-G_1}\phi^{\alpha} \\
&= \left(\frac{\alpha}{\alpha-\sigma+1} - 1\right)\left(\frac{G_1}{1-G_1}\phi^{\alpha} + 1\right), \qquad \text{(E5)}
\end{aligned}
$$

where the second equality uses the relationship $\phi^{1-\sigma}(1-G_2) = 1$ from (E3). Equation (E5) is equivalent to (25).

F. Demand Shifter and Average Productivity

Equation (26) can be arranged as follows:

$$
\begin{aligned}
\frac{E(t)/a(t)}{n} &= \int_{x_0}^{x_1}\sum_{t-\tau=0}^{\infty} R(x',0)\frac{R(x,t-\tau)}{R(x',0)}\left(1-\phi^{\alpha}\right)(1-\delta)^{t-\tau}\frac{\delta dF(x)}{F(x_1)-F(x_0)} \\
&\quad + \int_{x_1}^{\infty} R(x',0)\frac{R(x,0)}{R(x',0)}\phi^{\alpha}\frac{dF(x)}{1-F(x_1)} \\
&= R(x',0)\left(\frac{1}{x'}\right)^{\sigma-1}\left[\begin{array}{c}(1-\phi^{\alpha})\frac{\delta}{1-G_2(1-\delta)}\int_{x_0}^{x_1} x^{\sigma-1}\frac{dF(x)}{F(x_1)-F(x_0)} \\ +\phi^{\alpha}\int_{x_1}^{\infty} x^{\sigma-1}\frac{dF(x)}{1-F(x_1)}\end{array}\right], \qquad \text{(F1)}
\end{aligned}
$$

where I use $R(x,t-\tau)/R(x',0) = \left(x/x'\right)^{\sigma-1} G_2^{t-\tau}$ from (12). Notably, this expression holds for any x'.

The terms in brackets in (F1) are associated with the average firm-specific productivity $\tilde{x}$, which is defined as follows:

$$
\tilde{x}^{\sigma-1} \equiv (1-\phi^{\alpha})\frac{\delta}{1-G_2(1-\delta)}\int_{x_0}^{x_1} x^{\sigma-1}\frac{dF(x)}{F(x_1)-F(x_0)} + \phi^{\alpha}\int_{x_1}^{\infty} x^{\sigma-1}\frac{dF(x)}{1-F(x_1)}.
$$

This definition is equivalent to the following relationship:

$$
\begin{aligned}
\frac{E(t)/a(t)}{n} &= R(x',0)\left(\frac{\tilde{x}}{x'}\right)^{\sigma-1} \\
&= R(\tilde{x},0).
\end{aligned}
$$

G. Firm Size Density

Let $S(l)$ be the firm size density, L be the random variable of l, and X be the random variable of x. Let o_b be an obsolescence count, which equals $t-\tau$, and O_b be the random variable of o_b.

First, consider a range of l more than $l(x_1, 0)$. In this range, there is no effect of obsolescence, and the following equality holds:

$$
P(l \le L \le l + dl) = P(x(l) \le X \le x(l+dl)), \qquad \text{(G1)}
$$

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where $x(l) \equiv \left(l(1,0)^{-1}l\right)^{1/(\sigma-1)}$ from (13). Equation (G1) is equivalent to the following:

$$S(l)dl = \frac{f(x(l))}{1-F(x_0)}[x(l+dl)-x(l)].$$

Thus, I obtain the following:

$$S(l)|_{l>l(x_1,0)} = \frac{f(x(l))}{1-F(x_0)}\frac{\partial x(l)}{\partial l} \propto l^{-\frac{\alpha}{\sigma-1}-1}. \tag{G2}$$

Second, focus on a range of l less than $l(x_0,0)$. For this range, any level of l less than $l(x_0,0)$ stems from obsolescence. Thus, the following equality holds in this range:

$$P(l \le L \le l+dl) = \int_{x_0}^{x_1} P(o_b(l+dl,x) \le O_b \le o_b(l,x))\frac{dF(x)}{1-F(x_0)}, \tag{G3}$$

where $o_b(l,x) \equiv \ln\left[l(1,0)^{-1}x^{1-\sigma}l\right]/\ln G_2$ from (13). Based on (G3), I obtain the following:

$$S(l)|_{l<l(x_0,0)} = \int_{x_0}^{x_1} \delta(1-\delta)^{o_b(l,x)}\left|\frac{\partial o_b(l,x)}{\partial l}\right|\frac{dF(x)}{1-F(x_0)} \propto l^{\frac{\delta}{(\sigma-1)g}-1}, \tag{G4}$$

where the approximation values $1-\delta \approx \exp(-\delta)$ and $\ln G_2 \approx (1-\sigma)g$ are used.
Finally, consider the range $l(x_0,0) \le l \le l(x_1,0)$. In this range, the following equality holds:

$$P(L \le l) = P(L \le l(x_1,0)) - P(l \le L \le l(x_1,0)),$$

where

$$P(L \le l(x_1,0)) = 1-\phi^{\alpha}$$

and

$$P(l \le L \le l(x_1,0)) = \int_{x(l)}^{x_1} P(0 \le O_b \le o_b(l,x))\frac{dF(x)}{1-F(x_0)}.$$

I obtain

$$\begin{aligned} S(l)|_{l(x_0,0)\le l\le l(x_1,0)} &= \frac{\partial P(L\le l)}{\partial l} \\ &= -\frac{\partial}{\partial l}\left[\int_{x(l)}^{x_1} P(0 \le O_b \le o_b(l,x))\frac{dF(x)}{1-F(x_0)}\right] \\ &= -\frac{\partial}{\partial l}\left[\int_{x(l)}^{x_1}\sum_{o_b=0}^{o_b(l,x)}\delta(1-\delta)^{o_b}\frac{dF(x)}{1-F(x_0)}\right] \\ &\propto l^{-\frac{\alpha}{\sigma-1}-1} - l^{\frac{\delta}{(\sigma-1)g}-1}l(1,0)^{-\frac{\alpha}{\sigma-1}-\frac{\delta}{(\sigma-1)g}}x_1^{-\frac{\delta}{g}-\alpha}, \end{aligned} \tag{G5}$$

where the discrete time interval is assumed to be infinitesimal to obtain the fourth line from the third equality.

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The firm size density in the current economy is, therefore, summarized as in (G2), (G4), and (G5) for each range. Notably, the growth rate g does not affect the density within the sufficiently large firm size range.

H. Simulated Impacts of Growth on Key Variables

Figure H1 demonstrates how the growth rate g affects key variables, while using the benchmark parameter values in table 1.

The total impact of growth on unemployment is monotonically negative. Along with the discussion in the body text, the creative-destruction effect, associated with an increase in the job-cut rate, is overwhelmed by the firm-composition effect, mainly through an increase in the adoption rate.

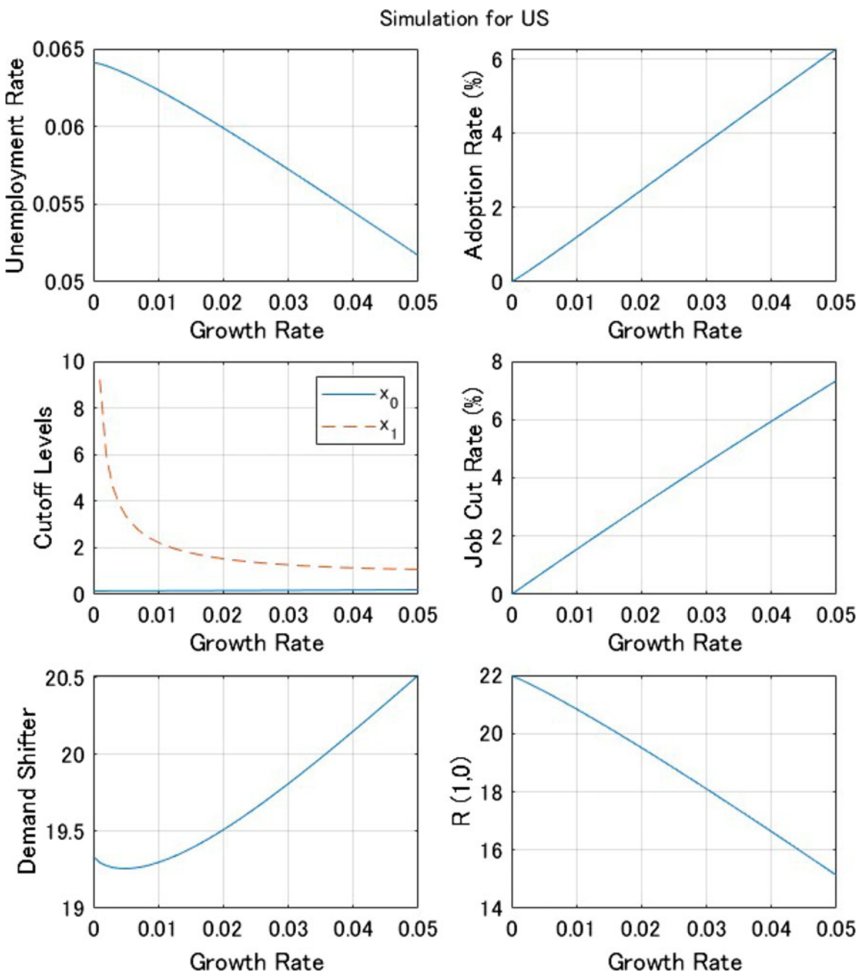

Figure H1. Demonstrates how the growth rate g affects key variables, while using the benchmark parameter values in Table 1.

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Behind these results, the demand shifter increases as being almost monotonic. In addition, Fig. H1 also depicts $R(1,0)$, as the common revenue component for each firm, which positively depends on the demand shifter and negatively depends on the employment cost $\kappa(\theta)$. Because an increase in the growth rate decreases $R(1,0)$, the employment cost increases dominantly in comparison to the increase in the demand shifter.

I. DMP Model with the Creative-Destruction Effect and Capitalization Effect

In this section, I describe a simple DMP model with both canonical effects. The notation here is the same as in the body text, but the adoption rate ϕ^α is set to be exogenous as the ratio of disembodied technology.

Following Pissarides and Vallanti (2007) and Hornstein et al. (2007), the output per worker in a job is defined as $a(t)^{\phi^\alpha} a(\tau)^{1-\phi^\alpha}$, where t is the current period and τ is the initial production date of the job (as a measure of the job's vintage). There are two components for this expression. The first component $a(t)^{\phi^\alpha}$, associated with disembodied technology, grows at rate $g\phi^\alpha$ because $a(t+1)^{\phi^\alpha}/a(t)^{\phi^\alpha} = (1+g)^{\phi^\alpha} \approx \exp(g\phi^\alpha)$. The second component $a(\tau)^{1-\phi^\alpha}$, associated with embodied technology, is constant within the job. However, the economy-wide output per worker grows at rate g because the second component is not fixed in the total economy.

The value functions are specified as follows:

$$V(t) = -a(t)c + \frac{1}{1+r}\left[\frac{m(\theta,1)}{\theta}J(t+1,t+1) + \left(1 - \frac{m(\theta,1)}{\theta}\right)V(t+1)\right], \tag{I1}$$

$$J(\tau,t) = \max\left[a(t)^{\phi^\alpha} a(\tau)^{1-\phi^\alpha} - w(\tau,t) + \frac{1-s}{1+r}J(\tau,t+1), 0\right], \tag{I2}$$

$$W(\tau,t) = \max\left[w(\tau,t) + \frac{1}{1+r}\left[(1-s)W(\tau,t+1) + sU(t+1)\right], U(t)\right], \text{and} \tag{I3}$$

$$U(t) = a(t)b + \frac{1}{1+r}\left[m(\theta,1)W(t+1,t+1) + (1-m(\theta,1))U(t+1)\right], \tag{I4}$$

where $V(t)$ denotes the value of a vacancy at time t. The model assumes the free entry of vacancies, that is, $V(t) = 0$. The wage function is obtained under the following surplus-sharing rule:

$$\beta J(\tau,t) = (1-\beta)[W(\tau,t) - U(t)]. \tag{I5}$$

The equality condition between job creation and destruction for pinning down equilibrium unemployment is:

$$JC \equiv m(\theta,1)u = (1-u)s + JC(1-s)^{o_b^*+1} \equiv JD, \tag{I6}$$

where JC equals the number of aggregate jobs created in each period and JD equals that of jobs destroyed. o_b^* denotes the maximum distance between t and τ such that each employer–employee match dissolves when $o_b = t - \tau$ exceeds o_b^*. The first term of JD represents the exogenous separations that occur with probability s. The second term of JD expresses separations via complete obsolescence with which a job surplus becomes smaller than the outside option value of each worker.

Under the setup of the model environment above, the *key endogenous variables*, each wage $w(\tau,t)$, maximum obsolescence in each job o_b^*, labor market tightness θ, and unemployment u, are solved by the following equations:

$$\frac{w(\tau,t)}{a(t)} = \beta(1+g)^{-(t-\tau)(1-\phi^\alpha)} + (1-\beta)\left[b + \frac{\beta c\theta}{1-\beta}\right], \tag{I7}$$

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Table 7. du/dg in the DMP model

Exogenous ϕ^{α}	du/dg
0.99	-0.0072
0.9	-0.0018
0.8	$+0.0041$
0.5	$+0.0210$
0.1	$+0.0388$

$$(1+g)^{-o_b^*(1-\phi^{\alpha})} = b + \frac{\beta c\theta}{1-\beta}, \tag{I8}$$

$$\frac{1+r}{1+g}\frac{c}{m(\theta,1)/\theta} = \sum_{o_b=0}^{o_b^*}\left[\frac{(1+g)(1-s)}{1+r}\right]^{o_b}(1-\beta)\left[(1+g)^{-o_b(1-\phi^{\alpha})} - b - \frac{\beta c\theta}{1-\beta}\right], \text{and} \tag{I9}$$

$$u = \frac{s}{m(\theta,1)\left[1-(1-s)^{o_b^*}\right]+s}. \tag{I10}$$

The first equation is derived from the surplus-sharing rule. For convenience, I define the normalized value $J(t-\tau)$ so that $J(t-\tau) \equiv J(\tau,t)/a(t)$. The second equation follows from $J(o_b^*) = 0$. The third equation equalizes the cost and benefit of each new job, through the free entry of vacancies; the right-hand side in the third equation equals $J(0)$. The fourth equation holds from the equality between job creation and destruction. The average wage is given as $[\sum_{o_b=0}^{o_b^*} w(o_b)(1-s)^{o_b}]/[\sum_{o_b=0}^{o_b^*}(1-s)^{o_b}]$, where $w(t-\tau) \equiv w(\tau,t)/a(t)$.

The model calibration method is the same as in the body text. Because the DMP model has the nine exogenous parameters, β, g, b, c, r, s, m_0, η, and ϕ^{α}, the targeted data moments are reduced to the three: job-finding probability, labor market tightness, and the replacement rate for determining b, c, and m_0. The parameter values except for b, c, m_0, and ϕ^{α} are the same as those in Table 1.

Table 7 shows the impact of growth on unemployment du/dg. The first column is the set of different exogenous adoption rates ϕ^{α}, and the second column reports the implied du/dg.[33]

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