This department welcomes short notes and problems believed to be new. Contributors should include solutions where known, or background to the problem in case the problem is unsolved. Send all communications concerning this department to Leo Moser, University of Alberta, Edmonton, Alberta. Alberta.

PROBLEMS FOR SOLUTION

P 7. vol. 1 (1958), 192. Correction. In (a) replace lim by lim sup .

P 16. (a) Prove that there is a polyhedron whose faces consist of 6 squares and (say) f hexagons where f may be greater than any given number.

(b) What is the sequence of possible values of f?

(c) What is the largest value of f for which the hexagons can all be regular?

(d) What is the largest value of f for which the hexagons can all be centrally symmetric?

H.S.M. Coxeter

P 17. If $\prod_{r=1}^{\infty} (1 + x^{r^3}) = \sum_{n=o}^{\infty} a_n x^n$, find the largest n such that $a_n = 0$.

Ron Graham

P 18. Prove that the number of integers $d < n$ which divide some $a! + 1$ is $< cn/\log n$.

P. Erdös

P 19. Let $a_1, \ldots, a_n$; $b_1, \ldots, b_n$ be real numbers and

$$\alpha = n^{-\frac{1}{2}} \sum a_i , \qquad \beta = n^{-\frac{1}{2}} \sum b_i .$$

Then

$$\sum a_i^2 \sum b_i^2 - (\sum a_i b_i)^2 \geqslant \alpha^2 \sum b_i^2 - 2\alpha\beta \sum a_i b_i + \beta^2 \sum a_i^2$$

with the sign of equality if and only if the three row vectors $a = (a_1, \ldots, a_n)$, $b = (b_1, \ldots, b_n)$, $e = n^{-\frac{1}{2}} (1, \ldots, 1)$ are linearly dependent.

H. Schwerdtfeger

P 20. Given a plane square lattice of side length one and a positive integer n. Form all the sets S of n lattice points. Let $L(S)$ denote the length of the boundary of the convex closure of S. Estimate $\min_S L(S)$ in terms of n. (cf. H.D. Block, Proc. Amer. Math. Soc. 8 (1957), 860-862.)

P. Scherk

P 21. It is possible to metrize an affine plane (with preservation of its natural topology) in such a way that on every affine line the metric is Euclidean, but that the whole plane does not become Euclidean, viz. by a Minkowski metric. Similarly, is it possible to metrize an affine space A^n $(n > 2)$ in such a way that in every affine plane the metric is Minkowskian, but that the whole space is not Minkowskian?

H. Helfenstein

SOLUTIONS

P 1. Let $f(x)$ be a Lebesgue integrable function on some interval $a - \varepsilon \leqslant x \leqslant b + \varepsilon$, $\varepsilon > 0$, and let $F_h(x) = h^{-1} \int_x^{x+h} f(t)dt$. An important theorem in the theory of Lebesgue integration states that $\lim_{h \to 0} F_h(x) = f(x)$ for almost all x. Show that we also have $\lim_{h \to 0} \int_a^b | F_h(x) - f(x) | dx = 0$.

W.A.J. Luxemburg