CORRECTION TO "NULL TRIGONOMETRIC SERIES IN DIFFERENTIAL EQUATIONS"

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My paper, *Null trigonometric series in differential equations*, in this Journal, *5* (1953) 536-543, contains an error which I wish to correct.

The correction is to add to the statement of the theorem on page 541 the condition:

(iii) *the integrated series* $\Sigma_{n\neq 0}\, \gamma_n\, e^{inx}/n$ *be summable* (C, $k-1$) *or convergent, uniformly as in* (ii).

No change is needed in the remarks on page 542 as to the scope of the theorem. In case (a), assuming that the function $f(x)$ is represented by a (Lebesgue) Fourier series, a standard theorem (**4**, p. 30; **6**, p. 340) ensures that the integrated series is uniformly convergent. In case (b) the series arising from meromorphic functions necessarily have summable conjugates and correspondingly summable integrated series. In these cases condition (iii) is redundant.

The error arose in the discussion of §3 (p. 538) and in the proof of the theorem of §4 (p. 542). Using Σ' to denote summation from 1 to ∞, the known theorem on convergence factors (**2**, Theorem 76) states that if $\Sigma' u_n$ is summable (C, k) then $\Sigma' u_n/n^s$ is summable (C, $k-s$) if $0 < s < k+1$. This is applied with $s = 1, 2, \ldots, m$ to the trigonometric series which, expressed in conventional two-way form, is Σw_n, actually representing $\Sigma'(w_n + w_{-n})$ where

$$w_n = (in)^m\, c_n\, e^{inx}.$$

If s is even $\Sigma w_n/n^s$ represents $\Sigma'(w_n + w_{-n})/n^s$, but if s is odd it represents the conjugate series $\Sigma'(w_n - w_{-n})/n^s$. The error lies in ignoring this distinction between the odd and even cases.

The most direct way to validate the argument would be to assume conditions ensuring the summability (C, k) of the conjugate series $\Sigma w'_n$ where $w'_n = w_n \operatorname{sgn} n$. The theorem could then be applied to this conjugate series for odd values of s and to the original series for even values. But since the series used are Σw_n, $\Sigma w'_n/n$, etc., a better way is to assume conditions for the (C, $k-1$) summability of $\Sigma w'_n/n$, which is the integrated series of Σw_n.

In the theorem of §4 the coefficient c_n is given by 4.2, 4.3 and consists of two parts, arising respectively from the null series and from the series for $f(x)$. Write

$$w_n = (in)^m\, c_n\, e^{inx} = \lambda_n + \mu_n.$$

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The series $\Sigma\lambda_n$ arising from the null series, and its conjugate, are necessarily summable as desired. The desired summability of $\Sigma\mu_n$ and $\Sigma\mu_n \operatorname{sgn} n/n$ is ensured if we assume the (C, $k - 1$) summability of the integrated series in addition to condition (ii). The extra assumption is condition (iii) above.

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