DO COMET GROUPS EXIST?

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ABSTRACT. The phenomena of comet groups, i.e. sets of comets that exhibit similarity in their orbital elements, is investigated. A computer program based on the D-criterion of orbital similarity is used to search for comet pairs and groups. The reality of the groups is tested by making computer searches in random samples of comet orbits.

The data base for the study is 599 long-period comet orbits. The degree of orbital similarity within a comet group was first assumed to be identical to that encountered in meteor streams. The computer search at this level produced five comet pairs plus two groups with four and seven members, respectively. The latter two represented the eleven known members of the Kreutz group of sun-grazing comets. A comparison with searches in random samples showed that the two Kreutz groups were significant. There is a probability of 0.2 that the five comet pairs found in the real sample could be accidental formations.

In a second study the orbital similarity parameter D_s was varied and the number of comet groups found in the real and synthetic comet populations was compared at each level of D_s. Apart from the Kreutz group of comets, the number of groups detected in the real comet sample was for all levels of orbital similarity only slightly higher than the average found in the random samples. At the 2σ confidence level we conclude that comet groups exhibit similarity in their orbital elements, that is no greater than might be expected by chance.

INTRODUCTION

Hock (1865, 1866, 1867) found that there exist different comets with nearly identical orbital elements (time of perihelion-passage excepted). Such comets form a comet group. A remarkable example of such a group is the sun-grazing group (Kreutz, 1888, 1891, 1901). The group consisted of comets 1843 I, 1880 I, 1882 II and 1887 I. All of these comets were bright with highly eccentric orbits and very small perihelion distances. It was obvious that no two of them could be appearances of the same comet. It was therefore assumed that they were individual parts of a primitive comet which has disrupted in the past. The existence of such

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comet groups is therefore a question of considerable interest to cometary physics.

Pickering (1911) in a study of long-period comet orbits listed 66 comet groups with from two to five members in each. Porter (1952) analysed about 500 long-period comet orbits and selected on the basis of similarity in the orbital elements and proximity of aphelia 19 "clearly-defined" comet pairs and groups with from two to six members in each. A revised list with 15 comet groups was presented in Porter (1963). Although Porter's groups showed similarity in the orientation of the orbital planes and major axes there was a considerable spread in the perihelion distances.

A fundamental but somewhat controversial question is the statistical significance of the pairs and groups. Öpik (1971) analysed 472 comets with aphelion distances beyond Saturn. He selected 97 groups that showed similarities in the angular orbital elements. Öpik calculated an overall probability of 10^{-39} that these similarities could have occurred by chance. Öpik's conclusions have been criticized by Whipple (1977). Whipple repeated the statistical analysis of Öpik and tested it on a random sample of cometary orbits. Approximately the same number of pairs and groups were found in the random sample as in the original sample. Whipple concluded that, except for a few pairs, the groups listed by Öpik exhibit similarity in their orbital elements that is no greater than might be expected by chance.

An obvious problem in the previous studies is the somewhat qualitative nature of the selection criteria for orbital similarity. The number of comet groups can be much enlarged, if the investigator imposes less severe restrictions as to the allowable spread in each orbital element. An objective measure of orbital similarity is needed. Such a criterion applied uniformly to a real and a random sample of comet orbits would provide a definitive test of the reality of the proposed comet groups.

The problem of classification based on orbital similarity is well known in meteor astronomy, where the study of meteor streams has necessitated the use of sophisticated computer techniques for the detection and classification of streams. A mathematical definition of orbital similarity, the so-called D-criterion, has been introduced by Southworth and Hawkins (1963). It has been used by Lindblad (1971a, 1971b, 1974), Sekanina (1973), Porubcan (1968, 1977) to search for meteor streams, and by Zausajev and Galimova (1982) to describe the evolution of a meteor stream. It has also been used by Lindblad and Southworth (1971) to determine the membership of the Hirayama asteroid families, by Kramer et al. (1979) to study clustering effects in Jupiter's family of short-period comets and by Kresák (1982a) to study the Tisserand invariant for various types of planetary orbits. In an investigation of comet groups Kresák (1982b) applied the D-criterion to all long period cometary orbits listed in Marsden's 1972 catalogue.

<u>D-criterion and rejection level D_S</u>. Southworth and Hawkin's criteria compares two sets of orbital elements. Let A and B represent two individual comets to be tested for orbital similarity. Let the orbital elements be represented by the five quantities q, e, i, Ω and π, where $\pi = \omega + \Omega$ is the longitude of perihelion. A quantitative measure of orbital simi-

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larity (or difference) is then given by the expression

$$D_{AB}^{2} = (e_A - e_B)^2 + (q_A - q_B)^2 + (2 \sin \tfrac{1}{2} I_{AB})^2 + + (e_A + e_B)^2 (\sin \tfrac{1}{2}\Pi_{AB})^2 \qquad (1)$$

where $(e_A + e_B)$ is a weight function, I_{AB} is the angle between the orbital planes and Π_{AB} is the difference between the longitudes of perihelion measured from the intersection of the orbital planes. The reason for using the perihelion distance q in the comparison instead of the semimajor axis a is that the perihelion distance q for meteor (and comet) orbits is better defined than a.

The D-criterion is an objective method of classification on the bases of the orbital elements, i.e. it can be used to search for concentrations in five-dimensional (q, e, i, ω, Ω) space. However, it is left to the investigator to choose the appropriate rejection level D_s. The rejection level D_s will vary with sample size and to some extent also with the accuracy of the orbits. For precise photographic meteor orbits Lindblad (1971a, 1971b) found

$$D_s = 0.80 \cdot N^{-1/4} \qquad (2)$$

where N is sample size. A preliminary study (Lindblad, 1970 unpublished) showed that it is possible to use eq. (2) for cometary orbits as well provided 1) these orbits have approximately the same distribution in five-dimensional (q, e, i, ω, Ω) space as meteors, 2) the errors in the cometary orbits are not larger than those encountered in meteor studies.

Since meteor streams originate from comets, the orbital distributions of the two populations should be rather similar. A comparison of the 1/a distribution of the meteor and comet populations, however, indicates a higher percentage of long-period orbits in the comet populations (Lindblad, 1974). A possible objection to the use of the D-criterion in a study of long-period comet orbits is thus the extremely high eccentricity of some of the comet orbits. This objection has been discussed in detail by Kresák (1982b), and is found to be irrelevant. However, it is not à priori known what value of the numerical constant in eq. (2) we should use if the sample is restricted only to long-period orbits.

PART I. SEARCH FOR COMET GROUPS AT FIXED D_s-VALUE

Data base. The data base for the present study was an updated tape version of the comet orbits listed in Marsden (1982). The tape included 1139 cometary apparitions of 724 individual comets, the remainder representing earlier appearances of periodic comets. Of the individual comets 125 were of short period (P < 200 years) and 599 were parabolic or of long period (P > 200 years).

Search at D_s = 0.155 in real comet sample. We now assume that equation (2) defines the appropriate rejection level to use in a computer search

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for comet groups. In order to retain a strict similarity with previous meteor stream searches, we included in the search both short-period and long-period orbits. Inserting N = 724 in equation (2) we obtain $D_S = 0.155$. The computer search at $D_S = 0.155$ produced a total of twelve comet groups. We found ten groups with 2 members each, one group with 4 members and one group with 7 members. Five of the ten comet groups with only 2 members consisted of short-period comets. For the purpose of the present study these groups were rejected. The remaining five two--member groups consisted of long-period comets as did also the two groups with 4 and 7 members, respectively. The detected groups are listed in Tables I and II. In passing we note that the comet pairs listed in Table I are the same as the first five pairs listed in Table IV of Kresák (1982b).

All eleven members of the two largest groups (Table II) were identified as belonging to the sun-grazing group of comets. It is interesting to note that the computer search at $D_s = 0.155$ clearly separates the Kreutz group into two subgroups. This division has been discussed in some detail by Marsden (1967).

TABLE I. Comet pairs at D = 0.155

1532	1911 VI	1973 VII	1881 IV	1979 X
1661	1790 III	1846 V	1898 X	1770 II

Search at $D_S = 0.155$ in random samples. To test the reality of the proposed comet groups the D-criterion was applied to searches in a random sample of cometary orbits. The main constraints we impose on a random sample are: 1) the sample should have the same size as the real sample, 2) the sample should have the same overall distribution of the orbital elements as the real sample, i.e. the frequency functions f(a), f(e), f(i), f(ω) and f(Ω) should be very nearly the same in the synthetic and in the real comet population, and 3) existing correlations between the orbital elements a, e and i should not be destroyed by the randomization procedure. The latter constraint is, of course, mainly of importance when both short-period and long-period orbits are included in a search. A fourth requirement is that the randomization process should be carried out in such a way that it is feasible to develop a large number of synthetic samples.

To develop a random sample we first excluded from the data base the eleven members of the Kreutz group. The remaining 713 orbits were then randomized by jumbling the nodes, i.e. the longitudes of nodes Ω were randomly distributed amongst the orbits. In order to preserve the frequency functions f($\Omega + \omega$) and f($\Omega - \omega$) the longitude of perihelion ω was simultaneously distributed. The randomization procedure was repeated on a number of data decks, which were shuffled and cut in various ways. In all twenty different random samples were produced.

The twenty random samples were searched for comet groups at the same rejection level D_S as in the real comet population. No comet groups were found which included both short-period and long-period orbits. The detected short-period comet groups were removed from our study. Table III lists the number of long-period comet groups found in the random comet

TABLE II. Kreutz comet group

Comet	Y M D of perihelion passage	q	e	i	ω	Ω	Ω−ω	Sub-group	L	B	D
1843 I	43.02.28	0.0055	0.999914	144°.35	82°.64	2°.83	280°.19	1	281°.86	35°.31	0.025
1880 I	80.01.28	0.0055	1.0	144.66	86.25	7.08	280.83	1	281.67	35.25	0.019
1887 I	87.01.12	0.0048	1.0	144.38	83.51	3.89	280.38	1	281.86	35.36	0.014
1963 V	63.08.24	0.0051	0.999946	144.58	86.16	7.24	281.08	1	281.95	35.36	0.020
1882 II	82.09.18	0.0078	0.999907	142.00	69.59	346.96	277.37	2	282.24	35.24	0.006
1945 VII	45.12.28	0.0075	1.0	141.87	72.06	350.50	278.44	2	282.87	35.97	0.033
1965 VIII	65.10.21	0.0078	0.999915	141.86	69.05	346.30	277.25	2	282.26	35.22	0.013
1970 VI	70.05.14	0.0089	1.0	139.07	61.29	336.32	275.03	2	282.26	35.07	0.133
1979 XI	79.08.31	0.0016	1.0	142.68	72.07	350.10	278.03	2	282.24	35.23	0.031
1981 I	81.01.27	0.0049	1.0	142.74	71.98	349.97	277.99	2	282.20	35.15	0.030
1981 XIII	81.07.20	0.0043	1.0	143.18	73.73	352.23	278.50	2	282.26	35.12	0.055

Note

Comets 1979 XI, 1981 I and 1881 XIII have been observed by the SOLWIND coronagraph onboard an Earth-orbiting satellite. Parabolic orbits have been computed on the assumption that the comets perihelion direction agrees with that of the Kreutz group. (IAU Circ. no:s 3647, 3716 and 3719). The orbit of comet 1887 I has been computed under a similar assumption.

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populations. There is a surprisingly large dispersion in the number of cometary groups detected in the various random samples. The number of comet pairs found varied from 0 to 7, with a mean number of 3.4. It follows that a study of comet groups which is based on only one or two random samples may produce very misleading results.

TABLE III. Number of comet groups at D_s = 0.155 in real and random samples.

	No. of pairs	No. of triplets	Total groups
	2		2
	5		5
	4		4
	3	1	4
	2		2
	4	1	5
	3		3
	5		5
	3		3
	4		4
	4		4
	2		2
	6		6
	2		2
	4		4
	3		3
	2		2
	0		0
	4		4
	7		7
Mean random sample	3.4	0.1	3.5
Real comet sample	5	0	7*

* Including two groups with 4 and 7 members

Comparison. It is interesting to note that none of the twenty random searches produced a group with four or more long-period comets. We therefore conclude that the two Kreutz groups listed in Table II are significant, i.e. they cannot be accidental formations. In view of the extremely small perihelion distances of these comets, this result is not very surprising.

The significance of the five long-period comet pairs detected in the real comet population was investigated as follows. From Table III we derived a histogram which depicts the number of cases vs the number of

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comet pairs. Cumulative relative numbers as derived from the random searches are plotted in Fig. 1 vs the number of comet pairs detected. From Fig. 1 we conclude that there is a twenty per cent probability that a collection of five or more comet pairs could be accidental formations. It is thus very doubtful if any of the pairs listed in Table I should be considered as real.

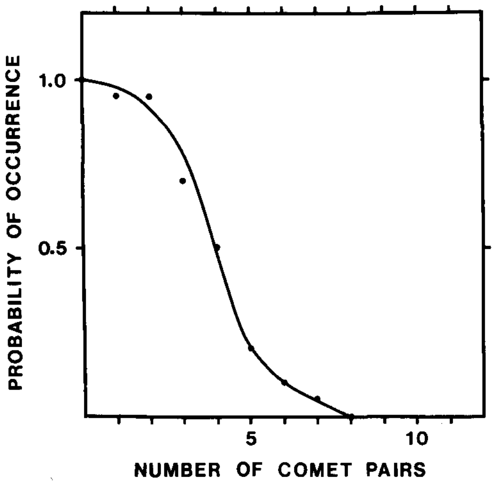

Figure 1. Probability of occurrence of N or more comet pairs as determined from searches in 20 random samples of long-period comet orbits.

PART II. SEARCH FOR COMET GROUPS AT VARIOUS D_S-VALUES

A possible objection to the previous analysis is that equation (2) is not appropriate for cometary studies, and that one should accept less severe restrictions as to the allowable spread in the orbital elements within a comet group. It is obvious that most of the previous investigators have been very tolerant in this respect. In view of these considerations the reality of comet groups was next investigated without making any à priori assumptions as to the appropriate rejection level D_S. In this study we only considered long-period comets, i.e. comets with periods > 200 years. For simplicity in the data handling the eleven members of the Kreutz group were removed. The data base for part II thus consisted of 588 parabolic or long-period orbits. This sample was searched for groups using D_S-values in the interval 0.12 -0.30. The value 0.12 corresponds approximately to the lowest D_S-value used in meteor stream searches - a larger value of D_S indicating a more liberal definition of orbital similarity. For comparison studies twenty random samples were developed and searched at the same sequence of D_S-values.

Fig. 2 compares the number of comet groups found in the real and synthetic samples. Searches were made at nine different values of the rejection level D_s as indicated in the diagram. The solid curve depicts the number of comet groups detected in the real comet population. At the lowest D_s-value (0.12) only three comet pairs were detected. Thus only 1 per cent of the comets were in groups at the lowest rejection level. At the highest D_s-value (0.30) 93 comets formed 41 groups of various sizes. Thus at this level 16 per cent of the comets were in groups.

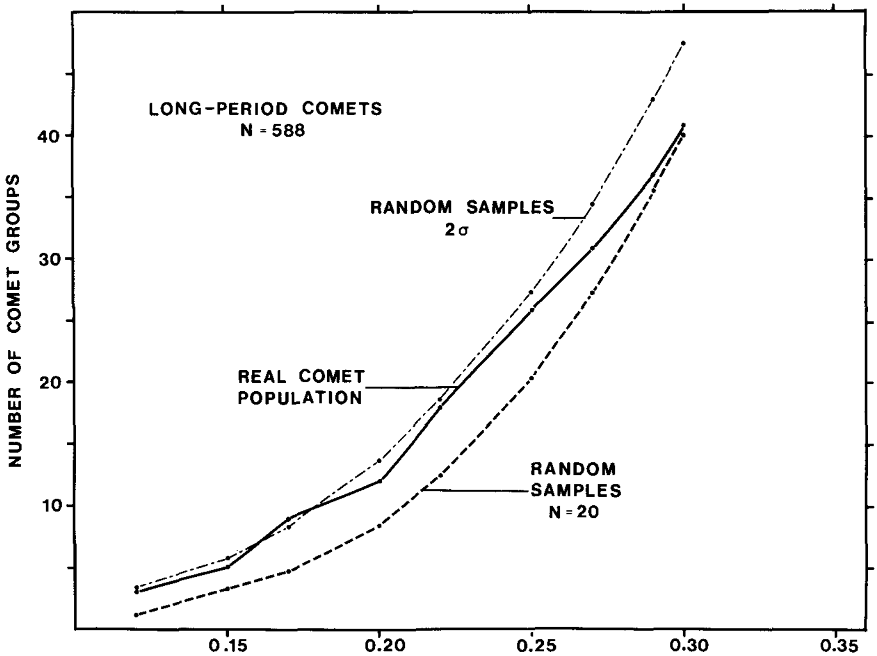

Figure 2. Solid curve: Number of comet groups found in real comet population (Kreutz group removed). Dashed curve: Mean number of comet groups found in 20 random samples of comet orbits. Dot-dashed curve: Upper 2σ confidence limit of number of comet groups as determined from searches in 20 random samples.

The lower, dashed curve in Fig. 2 depicts the mean number of comet groups detected in searches in the twenty synthetic samples. We note that the number of groups is slightly higher (2-5 groups) in the real comet population than in the average of the random samples. We further note that this difference does not vary appreciably with the degree of orbital similarity that is imposed. Although the number of comet groups increases drastically when less severe restrictions are imposed as to the spread in the orbital elements (increased value of D_s), it is evident that the number of possibly significant groups does not increase.

The upper, dot-dashed curve in Fig. 2 shows the 2σ variation in the random samples. We note that the number of comet groups in the real sample generally does not reach the upper 2σ confidence level. Hence, at any level of orbital similarity there are no significant comet groups at the 0.05 probability level. By ordinary statistical procedures we must therefore reject the existence of comet groups. In agreement with previous work of Whipple and Kresák we conclude that all comet groups (except the Kreutz groups) are accidental.

Discussion. Our search at $D_S = 0.30$ was compared with Kresák (1982b). In Kresák's investigation the discriminant D was calculated for all possible pairs of orbital elements by computer techniques, while the various possible combinations of orbits with $D_S < 0.30$ were selected manually. This was very time consuming, and there was an obvious risk that some pairs and groups could be missed. Kresák's Table IV lists 25 pairs, three groups with 3 members and two groups with 4 members. Our search confirmed 24 of the 25 pairs and also the five larger groups. In addition our search identified 11 groups that were not included in Kresák's table, owing either to new or improved orbits in our data base or to omissions in Kresák's table.

Kresák tested his analysis on three independent synthetic samples, which were based on a random distribution of the angular elements and the observed distribution of perihelion distances. Statistics of D_S--values less than 0.30 was computed for each random sample and the results compared with those obtained in the real comet population. After excluding four pairs in the real population Kresák found that the number of cases with $D_S < 0.30$ was exactly the same in the real population as in the mean of the random samples. Kresák hence concluded that the proposed comet groups were accidental formations. Although Kresák's results are based on a precariously small number of random samples his conclusions are entirely confirmed by our study.

KREUTZ GROUP OF COMETS

The two largest groups detected by the search at $D_S = 0.155$ consisted of the 11 known members of the Kreutz family. Table II compiles values of the orbital elements (1950.0 equinox), perihelion latitudes and longitudes and the D-values (computed from the mean orbit of each group). With one exception the two sun-grazing groups persisted unchanged in all searches at all rejection levels, i.e. no members were added or subtracted to the two Kreutz groups, when the orbital similarity parameter D_S was varied. Comet 1668, which sometimes has been suggested as a member of the Kreutz group was not classified as a member in our searches.

The D-values listed in Table II represent the individual deviations from the mean orbits of the two subgroups. The mean D-values for the subgroups are 0.020 and 0.043, respectively. If one outlying member (1970 VI) is excluded, the mean D-value of the second Kreutz group is lowered to 0.028. These mean D-values are markedly lower than those of other comet groups, i.e. the degree of orbital similarity in the Kreutz group is much higher than in any of the other comet groups detected in our searches. The positions of the perihelion points of the Kreutz comet orbits are located in a limited sky area of about $1^{\circ} \times 1^{\circ}$. However, it is important to note that four of the orbit computations have been based

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on the à priori assumption that the perihelion direction agrees with that of the mean of the Kreutz group.

Detailed studies of the Kreutz group have been made by Marsden (1967) and Sekanina (1967). The discovery in a period of less than two years of three new members which apparently have collided with the sun (Michels et al., 1982, Sheeley et al., 1982 and IAU Circ. no:s 3647, 3716 and 3719), is an unusual event and suggests that the group may consist of far more comets than was previously assumed. A difficult problem in celestial mechanics is to explain how the perihelion distances could be perturbed to values less than the solar radius. Possible mechanisms have been discussed by Weissman (1983).

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REFERENCES

Hock, M.: 1865, Mon. Not. R. Astr. Soc. 24, 243.
Hock, M.: 1866, Mon. Not. R. Astr. Soc. 26, 204.
Hock, M.: 1867, Astr. Nachr., 70, 203.
Kramer, E.N., Musij, V.L. and Shestaka, I.S.: 1979, Astron. Vestn., 13, 42.
Kresák, L.: 1982a, Bull. Astron. Inst. Czechosl., 33, 104.
Kresák, L.: 1982b, Bull. Astron. Inst. Czechosl., 33, 150.
Kreutz, H.: 1888, Publ. Sternwarte Kiel, no. 3.
Kreutz, H.: 1891, Publ. Sternwarte Kiel, no. 6.
Kreutz, H.: 1901, Astron. Abhandl., 1, 1.
Lindblad, B.A.: 1971a, Smithson. Contr. Astrophys. 12, 1.
Lindblad, B.A.: 1971b, 'Meteor Streams', in *Space Res. XI*, Akad. Verlag, Berlin.
Lindblad, B.A.: 1974, in *Asteroids, Comets, Meteoric Matter*, IAU Coll. 22 (Nice), Cristescu and Klepczynski (Eds.), Bukarest, pp. 269-281.
Lindblad, B.A. and Southworth, R.B.: 1971, in Gehrels (Ed.), *Physical Studies of Minor Planets*, Washington, pp. 337-352.
Marsden, B.G.: 1967, Astron. J., 72, 1170.
Marsden, B.G.: 1982, Catalogue of Cometary Orbits, Fourth Ed., Cambridge.
Michels, D.J., Sheeley, N.R., Howard, R.A. and Koomen, M.J.: 1982, Science, 215, 1097.
Öpik, E.S.: 1971, Irish Astron. J., 10, 35.
Pickering, W.H.: 1911, Harvard Ann., 61, 163.
Porter, J.G.: 1952, 'Comets and Meteor Streams', Chapman and Hall, London.
Porter, J.G.: 1961, Mem. British. Astron. Assoc., 39, No. 3.
Porter, J.G.: 1963, in Middlehurst and Kuiper (Eds.), *The Solar System IV*, Chicago Univ. Press. pp. 550-572.

Porter, J.G.: 1966, Mem. British. Astron. Assoc., 40, No. 2.
Porubcan, V.: 1968, Bull. Astron. Inst. Czechosl., 19, 327.
Porubcan, V.: 1977, Bull. Astron. Inst. Czechosl., 28, 257.
Sekanina, Z.: 1967, Acta Universitatis Carolinae - Mathematica et Physica, No. 2, 33-84, (Publ. Astron. Inst. Charles Univ., no. 51).
Sekanina, Z.: 1973, Icarus 18, 253.
Sheeley, N.R., Howard, R.A., Koomen, M.J. and Michels, D.J.: 1982, Nature 300, 239.
Southworth, R.B. and Hawkins, G.S.: 1963, Smithson. Contr. to Astrophys. 7, 261.
Weissman, P.R.: 1983, Icarus 55, 448.
Whipple, F.L.: 1977, Icarus 30, 736.
Zausajev, A.F. and Galimova, A.G.: 1982, Bull. Astrophys. Inst. Acad. Sci., Tadjik SSR, 72, 20.