CORRESPONDENCE

To the Editor of the *Transactions of the Faculty of Actuaries.*

SIR,

"The Investment Value of a Varying Life Interest."

"Interesting," "stimulating," and "practical" are the terms which, I think, best describe Mr. Lidstone's recent paper (*T.F.A.*, xvii. p. 106). Interesting, because it is a model of that perfect clarity and that admirable ingenuity which stamp all his contributions to our literature; stimulating, on account of the "unexpected and rather startling" conclusion which he reaches; and practical, because one could readily conceive of an important estate having to be dealt with for, say, estate duty purposes on the lines of the paper.

After a careful study has been made of the paper, the question which naturally occurs to one is: "What is the relation between the old (as represented by the classic Jellicoe formulae) and the new (as represented by the formulae of the paper)?" Mr. Lidstone himself obviously visualises the relationship clearly when he states (p. 117): ". . . and the same indeterminateness which we found in our general problem really also arises in the simpler classic case, the ordinary solution of which is tacitly based on a postulated relation exactly parallel with that which we have adopted." As, however, I think that an explicit analysis of the relationship between the old and the new might be of interest, I set out below a brief demonstration. The analysis is simple but, in justification of its publication, I would quote the remarks of an eminent mathematician made some years ago at one of our sessional meetings in connection with the question of fitting the old within the compass of the new: "That is an important thing to do, and not until it is done can we really understand how the old method is related to the new, or whether the latter has any advantages over the old."

The fundamental general formula is (adopting the notation of the paper):

$$V = \alpha - \beta P.$$

Now in the classic case of a uniform life interest we have (for convenience writing d for $\underset{j}{d}$):

$$\alpha = \frac{K}{A} = \frac{\underset{i}{\Sigma} v^n(1+j)^n Q_n a_{\overline{n-1}|}}{\underset{i}{\Sigma} v^n(1+j)^n Q_n} = \frac{\frac{1}{j}\{A'_x - (1+j)A_x\}}{A'_x} = \frac{1}{d}\left(1 - \frac{A_x}{A'_x}\right) - 1$$

$$\text{and}\quad \beta = \frac{B - k\underset{i}{a_x}}{A} = \frac{\frac{1}{d}(A'_x - A_x) - k\underset{i}{a_x}}{A'_x} = \frac{1+j}{j}\left(1 - \frac{A_x}{A'_x}\right) - \frac{k\underset{i}{a_x}}{A'_x}.$$

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Also in this case G_n is constant $(=1)$ and therefore the minimum value of P is obtained from the equation

$$ja + (1+j-j\beta)P = 1\,;$$

i.e.

$$P = \frac{1-ja}{1+j-j\beta}$$

$$= \frac{(1+j)\dfrac{A_x}{A'_x}}{(1+j)\dfrac{A_x}{A'_x} + \dfrac{\dfrac{kja_x}{i}}{A'_x}}$$

$$= \frac{1}{1+\dfrac{kd}{\pi_x}}$$

$$= \frac{\pi_x}{\pi_x + kd}$$

$$= \frac{P_x}{P_x + d} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Hence we have

$$V = a - \beta P$$

$$= \frac{1}{d}\left(1 - \frac{A_x}{A'_x}\right) - 1 - \left\{\frac{1}{d}\left(1 - \frac{A_x}{A'_x}\right) - \frac{\dfrac{ka_x}{i}}{A'_x}\right\}\frac{P_x}{P_x + d}$$

$$= \frac{1}{d}\left(1 - \frac{A_x}{A'_x}\right)\frac{d}{P_x + d} + \frac{\dfrac{ka_x}{i}}{A'_x} \cdot \frac{P_x}{P_x + d} - 1$$

$$= \frac{1}{A'_x(P_x + d)}\left(A'_x - kP_x\frac{a_x}{i} + kP_x\frac{a_x}{i}\right) - 1$$

$$= \frac{1}{P_x + d} - 1 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

also S_n

$$= (1+j)^n[V + a_{\overline{n}|}P - U_{n-1}]$$

$$= \frac{(1+j)^n}{P_x + d}(1 - P_x - d + P_x a_{\overline{n}|} - a_{\overline{n-1}|}P_x - a_{\overline{n-1}|}d)$$

$$= \frac{(1+j)^n}{P_x + d}\{1 - d(1 + a_{\overline{n-1}|})\}$$

$$= \frac{1}{P_x + d} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

From (1), (2), and (3) above it is seen that if the general formulae of Mr. Lidstone's paper be applied to the particular case of a uniform life interest the results obtained are identical with the usual Jellicoe solutions, so that the Jellicoe set of formulae is a particular case of the general formulae of the paper.

It may be suggested that in the first line of para. (v) of the paper, the words "more than," though correct in themselves, go a little beyond what is proved in para. (iii), and that the omission of these words might, without weakening the argument, make it easier for readers to link up the two paragraphs.

Your obedient servant,

A. R. Reid.

3 George Street,
7th May 1942.

CORRIGENDA

This volume, p. 116. The last line should read:

$$S_n = (1+j)a_x - (s_{\overline{n}|} - 1)$$

p. 217. Line 3 should read:

$$V_x = \sum a_n (1+i-x)^{-n}$$

p. 217. Middle of page. Expression for ξ should read:

$$\xi \doteqdot [n + \tfrac{1}{2}t(t+1)]/(t+2).$$

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