CORRESPONDENCE.

A FALLACY IN ELIMINATION.

To the Editor of the *Mathematical Gazette.*

SIR,—Can you spare me a little space in which to confess to a fallacy in the Introduction to my recent book? The following argument is used (*Jacobian Elliptic Functions*, p. 44) :

If $f(z)$ is any elliptic function, the functions $f(y+z)$, $f(z)$, regarded as functions of z, are on the same lattice, and are therefore connected by an algebraic equation, which can be written

$$\Phi\{f(y+z), f(z)\ ;\ y\}=0. \qquad \text{(1)}$$

The function Φ is polynomial in $f(y+z)$ and $f(z)$, and involves y parametrically. Interchanging y and z, we have also

$$\Phi\{f(y+z), f(y)\ ;\ z\}=0. \qquad \text{(2)}$$

I now proceed : " The two equations are identical, since otherwise we could eliminate $f(y+z)$ algebraically and obtain a relation satisfied identically by y and z, contradicting the assumption that y is independent of z."

The inference just is not valid, for the simple reason that in general (1) contains the two functions $f(y)$, $f'(y)$, and (2) contains the two functions $f(z), f'(z)$.

A relation like $\qquad \cos^2 x-\cos^2 y=\sin^2 y-\sin^2 x$

may easily be a condition of compatibility and not involve a threat to the independence of the variables. The point can be illustrated from the addition theorem for the sine. Write

$$X=\sin x,\ X'=\cos x,\ Y=\sin y,\ Y'=\cos y,\ Z=\sin(x+y).$$

The addition theorem in the ordinary form is

$$Z=XY'+YX'.$$

The simplest algebraic equation connecting Z and X is

$$Z^2-2Y'XZ+(1-Y^2)X^2=Y^2(1-X^2),$$

that is, $$Z^2-2XY'Z+(X^2-Y^2)=0. \qquad \text{(3)}$$

and the corresponding algebraic equation connecting Z and Y is

$$Z^2-2YX'Z-(X^2-Y^2)=0. \qquad \text{(4)}$$

These two equations are compatible, but not identical.

The two equations linear in Z derived from them in the ordinary way are

$$Z-(XY'+YX')=0, \quad (XY'-YX')Z-(X^2-Y^2)=0.$$

The first of these is the formula from which we began, and the condition of compatibility is the identity

$$X^2Y'^2-Y^2X'^2=X^2-Y^2.$$

If (1) happens to be of the form

$$\Phi\{f(y+z), f(z), f(y)\}=0,$$

instead of $$\Phi\{f(y+z), f(z)\ ;\ f(y), f'(y)\}=0,$$

the argument is perhaps—I am learning caution—sound. Two distinct equations $\Phi(Z, X, Y)=0$, $\Phi(Z, Y, X)=0$ may of course be compatible, as equations in Z, in virtue of an identity in X and Y—we have only to express

(3), (4) above in terms of $\tan \frac{1}{2}x$ and $\tan \frac{1}{2}y$ to have an example—but because the equations are distinct they can be combined to produce an equation of lower degree in Z which is still rational in X and Y, and the assumption is tacit that (1) is irreducible. It was unfortunate for me that the only application I attempted was to the function $\wp z$, for in this case $\wp' x$ and $\wp' y$ do disappear together and the result is accidentally true. The irreducible equation connecting ns u, ns v, ns w when $u+v+w \equiv 0$ is of the fourth degree in each function and of the eighth degree in all. E. H. NEVILLE.

Reading.

PS.—I take this opportunity to point out that on p. 38 of the book, g_2, g_3 should read $\frac{1}{4}g_2$, $\frac{1}{4}g_3$ in l. 3, and that on p. 205, three lines above 12·43, K_q should read K_p ; on p. 220, line below ·517, the reference should be to ·517, not to ·514 ; in the last line of p. 240, the relation between the variables should be $u=vv$. There are a number of typographical blemishes, but I have not yet found any other mistakes that could mislead or puzzle the reader.

QUEENSLAND BRANCH.

REPORT FOR THE YEAR 1943-1944.

THE Annual Report was presented to the Annual Meeting of the Branch held on 31st March, 1944. The statement of receipts and expenses for the year was presented and was adopted.

During the year three meetings have been held. The first was the Annual Meeting on 7th May, 1943, at which the President, Professor Simonds, gave an address on " Greek Mathematics ". At a General Meeting, held at the University on 6th August, Mr. J. P. McCarthy read a paper on " The signs and symbols of elementary mathematics " and at the second General Meeting, also held at the University on 5th November, Mr. E. W. Jones read a paper on " Harmonic analysis ".

The Branch has now a credit balance of £13 10s. 9d. The number of Branch members is 31, of whom 9 are members of the Mathematical Association. Four members of the Branch are on duty with the Forces.

The *Mathematical Gazette* comes to hand regularly and the copies are circulated as usual. The attendance at meetings throughout the year has been satisfactory. J. P. MCCARTHY, Hon. Sec.

BOOKS RECEIVED FOR REVIEW

E. H. Neville. *Jacobian elliptic functions.* Pp. xvi, 331. 25s. 1944. (Oxford University Press)

T. H. Turney. *Heaviside's operational calculus made easy.* Pp. vii, 96. 10s. 6d. 1944. (Chapman and Hall)

H. S. Uhler. *Exact values of the first 200 factorials.* Pp. 24. 80 cents. 1944. (Yale University)

H. S. Uhler. *Original tables to 137 decimal places of natural logarithms for factors of the form $1 \pm n \,.\, 10^{-p}$, enhanced by auxiliary tables of logarithms of small integers.* 1942. (Yale University)

Tables of Lagrangian interpolation coefficients. Prepared by the Mathematical Tables Project under the sponsorship of the National Bureau of Standards. Pp. xxxvi, 392. 5 dollars. 1944. (Columbia University Press)

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