

Space clocks to test relativity: ACES and SAGAS

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Abstract. Atomic clocks are an outstanding tool for the experimental verification of general relativity and more generally for fundamental astronomy (VLBI, pulsar timing, navigation, etc). Recent years have seen a rapid improvement in the performance of such clocks, promising new improved tests of relativity, in particular onboard terrestrial and interplanetary space missions. We present the scientific motivations of such tests taking the ACES Salomon *et al.* (2001) and SAGAS Wolf *et al.* (2009) projects as particular examples.

Keywords. tests of general relativity, atomic clocks, fundamental physics in space

1. Introduction

Gravitation is one of the fundamental interactions of physics. It is well described by General Relativity in the macroscopic world, where the gravitational interaction plays a dominant role. However General Relativity remains a classical theory incompatible with the standard techniques of quantization. Unification models lead to modifications of the theory that have observable consequences. They can become apparent, for example, in violations of Lorentz invariance, of the equivalence principle, of the distance dependence of gravitation, or in the variation of the fundamental constants. Additionally, general relativity is challenged by observations at larger galactic and cosmic scales which are presently taken care of through the introduction of “dark matter” or “dark energy”. As long as these components are neither detected through non gravitational means, nor explained as resulting from new physical phenomena, it remains of the uttermost importance to test general relativity at all experimentally accessible scales.

Experimental tests of fundamental physics, with continuously improving precision and in constantly enlarged distance domains, are one of the privileged methods we dispose of to try and uncover the first indications of the “new physics” expected beyond the standard model and general relativity. In this respect, experiments in the solar system provide information of utmost interest, not only because of their excellent precision, but also because of the a priori easier control of the real environment of the experiments. Thus space becomes an ideal laboratory for experiments in fundamental physics.

In this paper, we recall the basic elements of general relativity and briefly review the experimental evidences supporting it. We then discuss perspectives for new tests which are based on recent technological developments as well as new ideas. We focus our attention on tests performed with atomic clocks in particular the ACES Salomon *et al.* (2001) and SAGAS Wolf *et al.* (2009) projects.

2. Tests of general relativity

General relativity (GR) is built up on two basic ideas which have to be distinguished. The first one is the metrical (geometrical) interpretation of gravitation, which identifies the gravitational field with the space-time metric. This is the very core of GR, but it is not sufficient to fix the latter theory. In order to select GR out of the variety of metric theories of gravitation, it is necessary to fix the relation between the geometry of space-time and its matter content. In GR, this relation is given by the Einstein-Hilbert equation which was written only in 1915.

The metrical interpretation of gravity, often coined under the generic name of the “equivalence principle”, is one of the most accurately verified properties of nature Will (2001). Freely falling test masses follow the geodesics of the Riemannian space-time, that is also the curves which extremize the integral

$$\Delta s \equiv \int ds \quad , \quad ds^2 \equiv g_{\mu\nu} dx^\mu dx^\nu \quad , \quad (2.1)$$

$g_{\mu\nu}$ is the metric tensor characterizing the space-time and dx^μ the displacements in this space-time. As free motions obey a geometrical definition, they are independent of the compositions of the test masses. The potential violations of this “universality of free fall” property are parametrized by a relative difference in the accelerations undergone by two test bodies in free fall from the same location with the same velocity. Modern experiments constrain this parameter to stay below 10^{-12} , this accuracy being attained in laboratory experiments Adelberger *et al.* (2003), Schlamminger *et al.* (2008) as well as in space tests using lunar laser ranging Williams *et al.* (1996), Williams *et al.* (2004) or planetary probe tracking Anderson *et al.* (1996). These results do not preclude the possibility of small violations of the equivalence principle and such violations are indeed predicted by unification models Damour *et al.* (2002). Large improvements of this accuracy are expected in the future thanks to the existence of dedicated space projects MICROSCOPE Touboul *et al.* (2001) and, on a longer term, STEP Mester *et al.* (2001).

As another consequence of the geometrical interpretation of gravity, ideal atomic clocks operating on different quantum transitions measure the same time, because it is also a geometrical quantity, namely the proper time Δs integrated along the trajectory (see eq. 2.1). This “universality of clock rates” property has also been verified with an extreme accuracy. Its potential variations are measured as a constancy of relative frequency ratios between different clocks at a level of the order of 10^{-16} per year, recently a few 10^{-17} per year Bize *et al.* (2003), Peik *et al.* (2004), Fortier *et al.* (2007), Ashby *et al.* (2007), Rosenband *et al.* (2008). These results can also be interpreted in terms of a potential “variation of fundamental constants” Flambaum (2006), thus opening a window on the “new physics” expected to lie “beyond the standard model”. In this domain also, large improvements of the accuracy can be expected in the future with the space project ACES projects using optical clocks Wolf *et al.* (2009), Schiller *et al.* (2009).

As already stated, GR is selected out of the large family of metric theories of gravity by the Einstein-Hilbert equation which fixes the coupling between curvature on one hand, matter on the other one, by setting the Einstein curvature tensor $G_{\mu\nu}$ to be proportional to the stress tensor $T_{\mu\nu}$

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu} \quad . \quad (2.2)$$

The coefficient is fixed by the Newtonian limit and determined by the Newton constant G and the velocity of light c . At this point, it is worth emphasizing that it is not possible to deduce the relation (2.2) only from the geometrical interpretation of gravity. In other

words, GR is one member of the family of metric theories of gravity which has to be selected out of this family by comparing its predictions to the results of observations or experiments. The tests performed in the solar system effectively show a good agreement with GR, as shown in particular by the so-called “parametrized post-Newtonian” (PPN) approach Will (2001).

The main idea of the PPN approach can be described by writing down the solution of GR with a simple model of the solar system where the gravity sources are reduced to the Sun treated as a point-like motion-less mass M (but the formalism can be generalized to the realistic N-body case). Using a specific gauge convention where spatial coordinates are isotropic, the metric element thus takes the following form (coordinates are $x^0 \equiv ct$, the radius r , the colatitude and azimuth angles θ and φ)

$$ds^2 = g_{00}c^2dt^2 + g_{rr}(dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)). \quad (2.3)$$

With this simple model, an exact solution can be found for the metric. It is convenient to write it as a power series expansion in the reduced Newton potential ϕ (the length scale GM/c^2 has a value of the order of 1.5km, with M the mass of the Sun, so that ϕ is much smaller than unity everywhere in the solar system)

$$g_{00} = 1 + 2\phi + 2\phi^2 + \dots, \quad g_{rr} = -1 + 2\phi + \dots, \quad \phi \equiv -\frac{GM}{rc^2}. \quad (2.4)$$

The family of PPN metrics can then be introduced by inserting a constant β in front of ϕ^2 in g_{00} and a constant γ in front of ϕ in g_{rr} (with $\beta = \gamma = 1$ in GR). The values of these PPN parameters affect the predicted motions, and can therefore be confronted to the observations. Experiments on the propagation of light have led to more and more stringent bounds on $|\gamma - 1|$, with the best current results corresponding to deflection measurements using VLBI astrometry Shapiro *et al.* (2004) and Doppler tracking of the Cassini probe during its 2002 solar occultation Bertotti *et al.* (2003). Meanwhile, analysis of lunar laser ranging data Williams *et al.* (2004) have led to bounds on linear superpositions of β and γ , and then in constraints on $|\beta - 1|$. The current status of these tests clearly favors GR as the best description of gravity in the solar system.

Let us emphasize that the common presentation of this status under the form “general relativity is confirmed by the tests” is a bit too loose. The tests discussed so far do not answer by a final “yes” answer to a mere “yes/no” question. They rather select a vicinity of GR as the best current description of gravity within the family of PPN metrics. This warning is not a mere precaution, it is rather a pointer to possible future progress in the domain. There indeed exist theoretical models (see for example Damour *et al.* (2002)) which deviate from GR while staying within the current observational bounds. Furthermore, as discussed below, extensions of GR do not necessarily belong to the PPN family.

This last point is in particular emphasized by the so-called “fifth force” tests which are focused on a possible scale-dependent deviation from the gravity force law Fischbach (1998). Their main idea is to check the r -dependence of the gravity potential, that is also of the component g_{00} . Hypothetical modifications of its standard expression, predicted by unification models, are often parametrized in terms of a Yukawa potential added to the standard g_{00} . This potential depends on two parameters, an amplitude α measured with respect to Newton potential and a range λ related through a Yukawa-like relation to the mass scale of the hypothetical new particle which would mediate the “fifth force”. A recent update of the status of such a fifth force is shown on Fig. 1 in Jaekel (2005). It shows that the Yukawa term is excluded with a high accuracy at ranges tested in lunar laser ranging Williams *et al.* (1996) and tracking of martian probes Anderson *et al.*

(1996). At the same time, it also makes it clear that windows remain open for large corrections ($\alpha > 1$) at short ranges as well as long ranges.

The short range window is being actively explored, with laboratory experiments reaching an impressive accuracy at smaller and smaller distances. In the long range window, a test of the gravity force was initiated by NASA as the extension of Pioneer 10/11 missions after their primary planetary objectives had been met. This led to the largest scaled test of gravity ever carried out, with the striking output of a signal that disagreed with the expectations from the physical models (thermal radiation, solar pressure, signal propagation, etc...) and the known laws of gravity Anderson *et al.* (1998), Anderson *et al.* (2002).

This so-called “Pioneer anomaly” was recorded on Doppler tracking data of the Pioneer 10 & 11 probes by the NASA deep space network. It is thus a result of radionavigation techniques, which are based on the performances of the accurate reference clocks located at reception and emission stations Asmar *et al.* (2005). The Doppler observable can equivalently be interpreted as a relative velocity of the probe with respect to the station, which contains not only the effect of motion but also relativistic and gravitational effects. The anomaly has been registered on the two deep space probes showing the best navigation accuracy. This is not an impressive statistics when compared to the large number of tests confirming GR. In particular, when the possibility of an artefact onboard the probe is considered, this artefact could be the same on the two probes. A number of mechanisms have been considered as attempts of explanations of the anomaly as a systematic effect generated by the spacecraft itself or its environment (see the references in Nieto *et al.* (2005)) but they have not led to a satisfactory understanding to date.

The Pioneer anomaly constitutes an intriguing piece of information in a context where the status of gravity theory is challenged by the puzzles of dark matter and dark energy. If confirmed, this signal might reveal an anomalous behavior of gravity at scales of the order of the size of the solar system and thus have a strong impact on fundamental physics, solar system physics, astrophysics and cosmology. It is therefore important to use as many investigation techniques as might be available for gaining new information.

The Pioneer data which have shown an anomaly have been re-analyzed by several independent groups which have confirmed the presence of the anomaly Markwardt (2002), Olsen (2007), Levy *et al.* (2009). As data covering the whole period of Pioneer 10 & 11 missions from launch to the last data point have been saved, it is worth investigating not only the Doppler tracking data, but also the telemetry data Turyshev *et al.* (2006). These efforts should lead to an improved control of systematics and produce new information of importance on several properties of the effect, for example its direction, long-term variation as well as annual or diurnal modulations, spin dependence, a question of particular interest being that of the onset of the anomaly.

In the meantime, new missions have been designed to further test GR eg. Touboul *et al.* (2001), Salomon *et al.* (2001), Christophe *et al.* (2009), Wolf *et al.* (2009). In the following, we will focus our attention on atomic clocks, as used in the projects ACES Salomon *et al.* (2001) and SAGAS Wolf *et al.* (2009). We will also discuss their applications, not only for tests in fundamental physics, but also for other important purposes such as earth sciences, solar system physics or navigation.

3. Atomic Clock Ensemble in Space (ACES)

The measurement of time intervals has experienced spectacular progress over the last centuries. In the middle of the 20th century, the invention of the quartz oscillator and the first atomic clocks opened a new era for time keeping, with inaccuracies improving from 10 microseconds per day in 1967 to 100 picoseconds per day for the primary cesium

atomic clocks using laser cooled atomic fountains. The best atomic fountains approach 10 picosecond error per day, i.e a frequency stability of 1 part in 10^{16} Bize *et al.* (2005) while the most recent atomic clocks, operating in the optical domain, reach 2 picoseconds per day Rosenband *et al.* (2008), Ludlow *et al.* (2008) and improve at a fast pace.

Because time intervals and frequencies can be measured so precisely, applications of atomic clocks are numerous and diverse. Most precision measurements and units of the SI system can be traced back to frequencies. With the redefinition of the meter 25 years ago and the choice of a conventional value for the speed of light in vacuum, distance measurements have been simply translated into time interval measurements. In other words, the development of quantum technologies has led to large progress in the investigation of our spatio-temporal environment. This is illustrated by the impressive improvement of the measurement of the Einstein redshift effect Vessot *et al.* (1980), Pound (1999), which will be pushed further by the ACES project (see the discussion below). This is also made clear by the measurements of distances in the solar system, with the astronomical unit now connected to atomic units and consequently known with a much better accuracy than previously Shapiro (1999).

The most visible consequence of these ideas is given by the Global Navigation Satellite Systems (GNSS), which are a spectacular and initially unexpected application of the resources of quantum physics (the precision of the system comes from the atomic clocks in the satellites) and general relativity. The US Global Positioning System (GPS) enables today any user with a small receiver or modern cell phone to locate its position on the globe with meter accuracy. In a few years from now, the GALILEO system will be the european contribution to GNSS and its vast uses for various applications, in particular in the scientific domain in geodesy, Earth monitoring, and time metrology.

In the following, we review some of the properties of space clocks in the context of the ACES mission (Atomic Clock Ensemble in Space). ACES is an ESA mission in fundamental physics Salomon *et al.* (2001), Cacciapuoti *et al.* (2007), Salomon *et al.* 2007. ACES aims at flying a new generation of atomic clocks onboard the International Space Station (ISS) and comparing them to a network of ultra-stable clocks on the ground. The ISS is orbiting at a mean elevation of 400 km with 90 min. of rotation period and an inclination angle of 51.6° . ACES will be transported on orbit by the Japanese transfer vehicle HTV, and installed at the external payload facility of the Columbus module using the ISS robotic arm.

The ACES payload accommodates two atomic clocks: PHARAO, a primary frequency standard based on samples of laser cooled cesium atoms, and the active hydrogen maser SHM. The performances of the two clocks are combined to generate an on-board timescale with the short-term stability of SHM and the long-term stability and accuracy of the cesium clock PHARAO. A GNSS receiver installed on the ACES payload and connected to the on-board time scale will provide precise orbit determination of the ACES clocks. The ACES clock signal will be transferred on ground by a time and frequency transfer link in the microwave domain (MWL). MWL compares the ACES frequency reference to a set of ground clocks, enabling fundamental physics tests and applications in different areas of research.

The planned mission duration is 18 months with a possible extension to 3 years. The launch is planned for 2013. In microgravity, the linewidth of the atomic resonance of the PHARAO clock will be reduced by two orders of magnitude, down to sub-Hertz values (from 11 Hz to 110 mHz), 5 times narrower than in Earth based atomic fountains. After clock optimization, performances in the 10^{-16} range are expected both for frequency instability and inaccuracy. Then the on-board clocks will be compared to a network of ground atomic clocks operating both in the microwave and optical domain.

The scientific objectives of the ACES mission cover a wide spectrum. The frequency comparisons between the space clocks and ground clocks will be used to test general relativity to high accuracy. As recalled above identical clocks located at different positions in gravitational fields experience a frequency shift that depends directly on the component g_{00} of the metric, i.e. to the Newtonian potential at the clock position. The comparison between the ACES onboard clocks and ground-based atomic clocks will measure the frequency variation due to the gravitational red-shift with a 70-fold improvement on the best previous experiment Vessot *et al.* (1980), testing the Einstein prediction at the 2 ppm uncertainty level. Time variations of fundamental constants will be measured by comparing clocks based on different transitions or different atomic species Fortier *et al.* (2007). Any transition energy in an atom or a molecule can be expressed in terms of the fine structure constant α and the two dimensionless constants m_q/Λ_{QCD} and m_e/Λ_{QCD} , involving the quark mass m_q , the electron mass m_e and the QCD mass scale Λ_{QCD} Flambaum (2006). ACES will perform crossed comparisons of ground clocks both in the microwave and in the optical domain with a resolution of 10^{-17} after a few days of integration time. These comparisons will impose strong and unambiguous constraints on time variations of fundamental constants reaching an uncertainty of 10^{-17} /year in case of a 1-year mission duration and $3 \cdot 10^{-18}$ /year after three years. ACES will also perform tests of the Local Lorentz Invariance (LLI), according to which the outcome of any local test experiment is independent of the velocity of the freely falling apparatus. In 1997, LLI tests based on comparing clocks on-board GPS satellites to ground hydrogen masers Wolf and Petit (2007) have been performed. ACES will perform a similar experiment with 10-fold improved uncertainty.

Developed by CNES, the cold atom clock PHARAO will combine laser cooling techniques and micro-gravity conditions to significantly increase the interaction time and consequently reduce the linewidth of the clock transition. Improved stability and better control of systematic effects will be demonstrated in the space environment. PHARAO will reach a fractional frequency instability of $1 \cdot 10^{-13} \cdot \tau^{-1/2}$, where τ is the integration time expressed in seconds, and an inaccuracy of a few parts in 10^{16} . The engineering model of the PHARAO clock has been completed and is presently under test at CNES premises in Toulouse. Design and first results are presented in Laurent *et al.* 2006.

Developed by SPECTRATIME under ESA coordination, SHM provides ACES with a stable fly-wheel oscillator. The main challenge of SHM is represented by the low mass and volume figures required by the space clock with respect to ground H-masers. SHM will provide a clock signal with fractional frequency instability down to $1.5 \cdot 10^{-15}$ after only 10^4 s of integration time. Two servo-loops will lock together the clock signals of PHARAO and SHM generating an on-board time scale combining the short-term stability of the H-maser with the long-term stability and accuracy of the cesium clock. The Frequency Comparison and Distribution Package (FCDP) is the central node of the ACES payload. Developed by ASTRIMUM and TIMETECH under ESA coordination, FCDP is the on-board hardware which compares the signals delivered by the two space clocks, measures and optimizes the performances of the ACES frequency reference, and finally distributes it to the ACES microwave link MWL.

With the microwave link MWL, frequency transfer with time deviation better than 0.3 ps at 300 s, 7 ps at 1 day, and 23 ps at 10 days of integration time will be demonstrated. The gravitational shift measurement requires fully relativistic modeling and precise orbit determination and knowledge of the Earth gravitational potential as described in Duchayne *et al.* 2009. MWL is developed by ASTRIMUM, KAYSER-THREDE and TIMETECH under ESA coordination. The proposed MWL concept is an upgraded

version of the Vessot two-way technique used for the GP-A experiment Vessot *et al.* (1980) and the PRARE geodesy instrument.

The frequency resolution that the ACES microwave link should reach in operational conditions is surpassing by one to two orders of magnitude the existing satellite time transfer comparison methods based on the GPS system and TWSTFT (Two Way Satellite Time and Frequency Transfer) Bauch *et al.* (2006). It may be compared with T2L2 (Time Transfer by Laser Link) Exertier *et al.* (2008) which is now flying onboard the JASON-2 satellite and taking science data since June 2008. The assessment of its time transfer capability is presently ongoing with the interesting prospect to reach a time resolution of $\simeq 10$ ps after one day of averaging for clock comparisons in common view. For the ACES mission, due to the low orbit of the ISS and the limited duration of each pass (300–400 s) over a given ground station, the common view technique will be suitable for comparing ground clocks over continental distances, for instance within Europe or USA or Japan. In a common view comparison, the frequency noise of the onboard clock cancels out to a large degree so that only the link instability remains. ACES mission will also demonstrate the capability to compare ground clocks in non common view with a resolution better than $10^{-13} \Delta t^{1/2}$ for $\Delta t > 1000$ s, that is 3 ps and 10 ps for space-ground comparisons separated by 1000 s and 10000 s respectively. This science objective takes full benefit of the excellent onboard time scale realized by the combination of SHM and PHARAO.

These performances will enable common view and non-common view comparisons of ground clocks with 10^{-17} frequency resolution after few days of integration time. ACES will take full advantage of the recent progress of optical clocks Rosenband *et al.* (2008), Ludlow *et al.* (2008), reaching today instability and inaccuracy levels of 2 parts in 10^{17} . Multiple frequency comparisons between a variety of advanced ground clocks will be possible among the 35 institutes which have manifested their interest to participate to the ACES mission. This is important for the tests of the variability of fundamental physical constants, and international comparisons of time scales.

Other applications of the ACES clock signal are currently being developed. ACES will demonstrate a new “relativistic geodesy” based on a differential measurement of the Einstein’s gravitational red-shift between distant ground clocks. It will take advantage of the accuracy of ground-based optical clocks to resolve differences in the Earth gravitational potential at the $\simeq 10$ cm level. A 10 cm change in elevation on the ground amounts to a frequency shift of 1 part in 10^{17} , comparable to today’s optical clock accuracy of 2 parts in 10^{17} . The GNSS receiver on-board the ACES payload will allow to monitor the GNSS networks and develop interesting applications in Earth remote sensing. This includes studies of the oceans surface via GNSS reflectometry measurements and the analysis of the Earth atmosphere with GNSS radio-occultation experiments.

In addition, ACES will deliver a global atomic time scale with 10^{-16} accuracy, it will allow clocks synchronization at an uncertainty level of 100 ps, and contribute to international atomic time scales. At the 10^{-18} level, ground clocks will be limited by the fluctuations of the Earth potential suggesting that future high precision time references will have to be placed in the space environment where these fluctuations are significantly reduced. From this point of view, ACES is the pioneer of new concepts for global time keeping and positioning based on a reduced set of ultra-stable space clocks in orbit.

4. SAGAS

The SAGAS mission will study all aspects of large scale gravitational phenomena in the Solar System using quantum technology, with science objectives in fundamental

physics and Solar System exploration. The large spectrum of science objectives makes SAGAS a unique combination of exploration and science, with a strong basis in both programs. The involved large distances (up to 53 AU) and corresponding large variations of gravitational potential combined with the high sensitivity of SAGAS instruments serve both purposes equally well. For this reason, SAGAS brings together traditionally distant scientific communities ranging from atomic physics through experimental gravitation to planetology and Solar System science.

The payload will include an optical atomic clock optimized for long term performance, an absolute accelerometer based on atom interferometry and a laser link for ranging, frequency comparison and communication. The complementary instruments will allow highly sensitive measurements of all aspects of gravitation via the different effects of gravity on clocks, light, and the free fall of test bodies, thus effectively providing a detailed gravitational map of the outer Solar System whilst testing all aspects of gravitation theory to unprecedented levels.

The SAGAS accelerometer is based on cold Cs atom technology derived to a large extent from the PHARAO space clock built for the ACES mission discussed in the preceding section. The PHARAO engineering model has recently been tested with success, demonstrating the expected performance and robustness of the technology. The accelerometer will only require parts of PHARAO (cooling and trapping region) thereby significantly reducing mass and power requirements. The expected sensitivity of the accelerometer is $1.3 \cdot 10^{-9} \text{m/s}^2 / \sqrt{\text{Hz}}$ with an absolute accuracy (bias determination) of $5 \cdot 10^{-12} \text{m/s}^2$.

The SAGAS clock will be an optical clock based on trapped and laser cooled single ion technology as pioneered in numerous laboratories around the world. In the present proposal it will be based on a Sr^+ ion with a clock wavelength of 674 nm. The expected stability of the SAGAS clock is $1 \cdot 10^{-14} / \sqrt{\tau}$ (with τ the integration time), with an accuracy in realising the unperturbed ion frequency of $1 \cdot 10^{-17}$. The best optical single ion ground clocks presently show stabilities below $4 \cdot 10^{-15} / \sqrt{\tau}$, slightly better than the one assumed for the SAGAS clock, and only slightly worse accuracies $2 \cdot 10^{-17}$. So the technology challenges facing SAGAS are not so much the required performance, but the development of reliable and space qualified systems, with reduced mass and power consumption.

The optical link is using a high power (1 W) laser locked to the narrow and stable frequency provided by the optical clock, with coherent heterodyne detection on the ground and on board the spacecraft. It serves the multiple purposes of comparing the SAGAS clock to ground clocks, providing highly sensitive Doppler measurements for navigation and science, and allowing data transmission together with timing and coarse ranging. It is based on a 40 cm space telescope and 1.5 m ground telescopes (similar to lunar laser ranging stations). The short term performance of the link in terms of frequency comparison of distant clocks will be limited by atmospheric turbulence at a few $10^{-13} / \tau$, thus reaching the clock performance after about 1000 s integration time, which is amply sufficient for the SAGAS science objectives. The main challenges of the link will be the required pointing accuracy ($0.3''$) and the availability of space qualified, robust 1 W laser sources at 674 nm. Quite generally, laser availability and reliability will be the key to achieving the required technological performances, for the clock as well as the optical link.

For this reason a number of different options have been considered for the clock/link laser wavelength, with several other ions that could be equally good candidates (e.g. Yb^+ at 435 nm and Ca^+ at 729 nm). Given present laser technology, Sr^+ was preferred, but this choice could be revised depending on laser developments over the next years. We also acknowledge the possibility that femtosecond laser combs might be developed for

space applications in the near future, which would open up the option of using either ion with existing space qualified 1064 nm Nd:YAG lasers for the link.

More generally, SAGAS technology takes advantage of the important heritage from cold atom technology used in PHARAO and laser link technology designed for LISA (Laser Interferometric Space Antenna). It will provide an excellent opportunity to develop those technologies for general use, including development of the ground segment (Deep Space Network telescopes and optical clocks), that will allow such technologies to be used in many other mission configurations for precise timing, navigation and broadband data transfer throughout the Solar System.

SAGAS will carry out a large number of tests of fundamental physics, and gravitation in particular, at scales only attainable in a deep space experiment. The unique combination of onboard instruments will allow 2 to 5 orders of magnitude improvement on many tests of special and general relativity, as well as a detailed exploration of a possible anomalous scale dependence of gravitation. It will also provide detailed information on the Kuiper belt mass distribution and determine the mass of Kuiper belt objects and possibly discover new ones. During the transits, the mass and mass distribution of the Jupiter system will be measured with unprecedented accuracy. The science objectives are discussed in the following, based on estimated measurement uncertainties of the different observables.

SAGAS will provide three fundamental measurements: the accelerometer readout and the two frequency differences (measured on ground and on board the satellite) between the incoming laser signal and the local optical clock. Auxiliary measurements are the timing of emitted/received signals on board and on the ground, which are used for ranging and time tagging of data. The high precision science observables will be deduced from the fundamental measurements by combining the measurements to obtain information on either the frequency difference between the clocks or the Doppler shift of the transmitted signals. The latter gives access to the relative satellite-ground velocity, from which the gravitational trajectory of the satellite can be deduced by correcting non-gravitational accelerations using the accelerometer readings.

In the following, we assume that Earth station motion and its local gravitational potential can be known and corrected to uncertainty levels below 10^{-17} in relative frequency (10 cm on geocentric distance), which, although challenging, are within present capabilities. For the Solar System parameters this requires 10^{-9} relative uncertainty for the ground clock parameters (GM and r of Earth), also achieved at present, and less stringent requirements for the satellite.

For long term integration and the determination of an acceleration bias, the limiting factor will then be the accelerometer noise and absolute uncertainty (bias determination). More generally, modelling of non-gravitational accelerations will certainly allow some improvement on the long term limits imposed by the accelerometer noise and absolute uncertainty, but is not taken into account in our preliminary evaluation Wolf *et al.* (2009). Also, depending on the science goal and corresponding signal, the ground and on-board data can be combined in a way to optimise the S/N ratio (see Reynaud *et al.* (2008) for details). For example, in the Doppler observable, the contribution of one of the clocks (ground or space) can be made negligible by combining signals such that one has coincidence of the “up” and “down” signals on board or on the ground.

We will use a mission profile with a nominal mission lifetime of 15 years and the possibility of an extended mission to 20 years if instrument performance and operation allow this. In that time frame, the trajectory allows the satellite to reach a heliocentric distance of 39 AU in nominal mission and 53 AU with extended duration.

In General Relativity (GR), the frequency difference of two ideal clocks is given by (see eq.(2.1) to first order in the weak field approximation, with $g_{00} \simeq 1 - 2w/c^2$ and $g_{rr} \simeq -1$; w is the Newtonian gravitational potential and v the coordinate velocity)

$$\frac{ds_g}{cdt} - \frac{ds_s}{cdt} \simeq \frac{w_g - w_s}{c^2} + \frac{v_g^2 - v_s^2}{2c^2}. \quad (4.1)$$

In theories different from GR this relation is modified, leading to different time and space dependence of the frequency difference. This can be tested by comparing two clocks at distant locations (different values of w and v) via exchange of an electromagnetic signal. The SAGAS trajectory (large potential difference) and low uncertainty on the observable (4.1) allows a relative uncertainty on the redshift determination given by the 10^{-17} clock bias divided by the maximum value of $\Delta w/c^2$. For a distance of 50 AU this corresponds to a test with a relative uncertainty of $1 \cdot 10^{-9}$, an improvement by almost 5 orders of magnitude on the uncertainty obtained by the most sensitive experiment at present Vessot *et al.* (1980).

Additionally, the mission also provides the possibility of testing the velocity term in (4.1), which amounts to a test of Special Relativity (Ives-Stilwell test), and thus of Lorentz invariance. Towards the end of the nominal mission, this term is about $4 \cdot 10^{-9}$, and can therefore be measured by SAGAS with $3 \cdot 10^{-9}$ relative uncertainty. The best present limit on this type of test is $2 \cdot 10^{-7}$ Saathoff *et al.* (2003), so SAGAS will allow an improvement by a factor ~ 70 . Considering a particular preferred frame, usually taken as the frame in which the 3K cosmic background radiation is isotropic, one can set an even more stringent limit. In that case a putative effect will be proportional to $(\mathbf{v}_s - \mathbf{v}_g) \cdot \mathbf{v}_{\text{Sun}}/c^2$ (see Saathoff *et al.* (2003)), where \mathbf{v}_s and \mathbf{v}_g are the velocity vectors of the satellite and ground while \mathbf{v}_{Sun} is the velocity of the Sun through the CMB frame (~ 350 km/s). Then SAGAS will allow a measurement with about $5 \cdot 10^{-11}$ relative uncertainty, which corresponds to more than 3 orders of magnitude improvement on the present limit.

Spatial and/or temporal variations of fundamental constants constitute another violation of Local Position Invariance and thus of GR. Over the past few years, there has been great interest in that possibility (see e.g. Uzan (2003) for a review), spurred on the one hand by models for unification theories of the fundamental interactions where such variations appear quite naturally, and on the other hand by recent observational claims of a variation of different constants over cosmological timescales Murphy *et al.* (2003), Reinhold *et al.* (2006). Such variations can be searched for with atomic clocks, as the involved transition frequencies depend on combinations of fundamental constants and in particular, for the optical transition of the SAGAS clock, on the fine structure constant α . Such tests take two forms: searches for a drift in time of fundamental constants, or for a variation of fundamental constants with ambient gravitational field. The latter tests for a non-universal coupling between ambient gravity and non-gravitational interactions (clearly excluded by GR) and is well measured by SAGAS, because of the large change in gravitational potential during the mission.

SAGAS also offers a possibility to improve the test of the PPN family of metric theories of gravitation Will (2001). The two most common parameters of the PPN framework are the Eddington parameters β and γ . Present limits on γ are obtained from measurements on light propagation such as light deflection, Shapiro delay and Doppler velocimetry of the Cassini probe during the 2002 solar occultation Bertotti *et al.* (2003). SAGAS will carry out similar measurements during solar conjunctions, with improved sensitivity and at optical rather than radio frequencies, which significantly minimizes errors due to the solar corona and the Earth's ionosphere. When combining the on board and ground

measurements such that the “up” and “down” signals coincide at the satellite (classical Doppler type measurement), the noise from the on-board clock cancels to a large extent and one is left with noise from the accelerometer, the ground clock, and the atmosphere. Details on these noise sources, and on the effects of atmospheric turbulence, variations in temperature, pressure and humidity are presented in Wolf *et al.* (2009). The resulting improvement on the estimation of γ is of the order of 100, with some potential for further improvement if several occultations may be analysed.

As already discussed, experimental tests of gravity have shown a good overall agreement with GR, but most theoretical models aimed at inserting GR within the quantum framework predict observable modifications at large scales. The anomalies observed on the Pioneer probes, as well as the phenomena commonly ascribed to “dark matter” and “dark energy”, suggest that it is extremely important to test the laws of gravity at interplanetary distances. This situation has motivated an interest in flying new probes to the distances where the anomaly was first discovered, that is beyond the Saturn orbit, and studying gravity with the largely improved spatial techniques which are available today.

SAGAS has the capability to improve our knowledge of the law of gravity at the scale of the solar system, and to confirm or infirm the presence of a Pioneer anomaly (PA). With one year of integration, all SAGAS observables allow a measurement of any effect of the size of the PA with a relative uncertainty of better than 1%. This will allow a “mapping” of any anomalous scale dependence over the mission duration and corresponding distances. Furthermore, the complementary observables available on SAGAS allow a good discrimination between different hypotheses thereby not only measuring a putative effect but also allowing an identification of its origin. SAGAS thus offers the possibility to constrain a significant number of theoretical approaches to scale dependent modifications of GR. Given the complementary observables available on SAGAS the obtained measurements will provide a rich testing ground for such theories with the potential for major discoveries.

Let us emphasize at this point that this gravity law test is important not only for fundamental physics but also for solar system science. The Newton law of gravity is indeed a key ingredient of the models devoted to a better understanding of the origin of the solar system. The Kuiper belt (KB) is a collection of masses, remnant of the circumsolar disk where giant planets of the solar system formed 4.6 billion years ago. Precise measurements of its mass distribution would significantly improve our understanding of planet formation not only in the solar system but also in recently discovered planetary systems. The exceptional sensitivity and versatility of SAGAS for measuring gravity can be used to study the sources of gravitational fields in the outer solar system, and in particular the class of Trans Neptunian Objects (TNOs), of which those situated in the KB have been the subject of intense interest and study over the last years Morbidelli (2007). Observation of KB objects (KBOs) from the Earth is difficult due to their relatively small size and large distance, and estimates of their masses and distribution are accordingly inaccurate. Estimates of the total KB mass from the discovered objects (~ 1000 KBOs) range from 0.01 to 0.1 Earth masses, whereas in-situ formation of the observed KBOs would require three orders of magnitude more solid material in a dynamically cold disk.

A dedicated probe like SAGAS will help discriminating different models of the spatial distribution of the KB and for determining its total mass. The relative frequency shift between the ground and space clock due to the KB gravitational potential is indeed a sensitive probe of the spatial distribution of this mass (see Wolf *et al.* (2009) for details). The SAGAS frequency observable is well suited to study the large, diffuse, statistical mass distribution of KBOs essentially due to its sensitivity directly to the gravitational potential ($1/r$ dependence), rather than the acceleration ($1/r^2$ dependence). The large

diffuse signal masks any signal from individual KBOs. When closely approaching one of the objects, the crossover between acceleration sensitivity (given by the $5 \cdot 10^{-12} \text{ m/s}^2$ uncertainty on non gravitational acceleration) and the frequency sensitivity (10^{-17} uncertainty on w/c^2) for an individual object is situated at about 1.2 AU. Below that distance the acceleration measurement is more sensitive than the frequency measurement. This suggests a procedure to study individual objects using the SAGAS observables: use the satellite trajectory (corrected for the non gravitational acceleration) to study the gravity from a close object and subtract the diffuse background from all other KBOs using the frequency measurement. Investigating several known KBOs within the reach of SAGAS Bernstein *et al.* (2004) shows that their masses can be determined at the % level when approaching any of them to 0.2 AU or less. Of course, this also opens the way towards the discovery of new such objects, too small to be visible from the Earth. Similarly, during a planetary flyby the trajectory determination (corrected for the non gravitational acceleration) will allow the determination of the gravitational potential of the planetary system. The planned Jupiter flyby with a closest approach of $\sim 600000 \text{ km}$ will improve present knowledge of Jovian gravity by more than two orders of magnitude.

Doppler ranging to deep space missions provides the best upper limits available at present on gravitational waves (GW) with frequencies of order c/L where L is the spacecraft to ground distance i.e. in the 0.01 to 1 mHz range Armstrong *et al.* (1987), Armstrong *et al.* (2003), and even down to 1 μHz , albeit with lower sensitivity Anderson & Mashoon (1985), Armstrong *et al.* (2003). The corresponding limits on GW are determined by the noise PSD of the Doppler ranging to the spacecraft for stochastic GW backgrounds, filtered by the bandwidth of the observations when looking for GW with known signatures. In the case of SAGAS data, this yields a strain sensitivity of $10^{-14}/\sqrt{\text{Hz}}$ for stochastic sources in the frequency range of 0.06 to 1 mHz with a f^{-1} increase at low frequency due to the accelerometer noise. When searching for GW with particular signatures in this frequency region, optimal filtering using a corresponding GW template will allow reaching strain sensitivities as low as 10^{-18} with one year of data. This will improve on best present upper limits on GW in this frequency range by about four orders of magnitude. Even if GW with sufficiently large amplitudes are not found, still the results might serve as upper bounds for astrophysical models of known GW sources Reynaud *et al.* (2008).

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