BOOK REVIEWS

BERGE, C., *Topological Spaces*, translated by E. M. Patterson (Oliver and Boyd, 1963), xiii+270 pp., 50s.

It is difficult to detect a consistent purpose behind the writing of this book, or a substantial class of readers for whom it is intended. The first half of the book is in some respects an excellent introduction to general topology, and I particularly like its thoroughness over elementary matters and its unusually explicit use of quantifiers. On the other hand, its utility for the beginner is surely greatly reduced by the author's insistence on allowing functions to be many-valued. The main feature of the second half of the book is a long chapter on convex sets and convex functions in Euclidean space. This is an excellent account of many of the fundamental properties of convex sets, and is virtually independent of the rest of the book. The final chapter on topological vector spaces is so short that I doubt its utility. I think that this book should have been written as three books instead of one: an elementary text book on general topology, a research monograph on multi-valued functions, and a short text book on convex sets and convex functions.

Apart from this possible lack of single-minded purpose, the book is a model of clear mathematical writing, and it will be highly valued for the elegant treatment of many points of detail. The translation and printing are of the highest possible standard.

F. F. BONSALL

GRAY, J. F., *Sets, Relations and Functions* (Holt, Rinehart and Winston, London, 1962), 143 pp., 20s.

This elementary account of the subjects mentioned in the title, and of related topics, was written primarily for secondary school teachers and students, but also for a wider group of readers such as " liberal adults who are tired of merely reading *about* modern mathematics and seek some readable and illustrative contact with modern mathematics ". It fulfils these functions excellently. The pace is leisurely and the number of examples is so extensive that no student should find any difficulties. This has its own dangers, however, and it is to be hoped that students will not attempt to solve *all* the exercises included. The multiplication of artificial examples is, if anything, even less illuminating and valuable in set theory than in old-fashioned subjects such as trigonometric identities.

In addition to the usual standard terminology, the author introduces the useful word " co-domain ", which is the set *into* (not *onto*) which a function maps its domain. As a minor criticism one may doubt whether the basic reason why human beings have the same number of fingers on each hand is that they can be put into one-to-one correspondence with each other. The printing is excellent (except for the " contained in " sign for which an unsuitable fount of type is used) and there are numerous diagrams.

R. A. RANKIN

KUBILYUS, Ĭ. P. AND KUBILIUS, J., *Veroyatnostnye metody v teorii čisel* (2-oe Izd.) (Gos. Izd. politiceskoĭ i naučnoĭ literatury Litovskoĭ SSR, Vil'nyus (Vilna), 1962), 220 pp., 82 Kopeks.

At first no two subjects could seem more essentially remote than numbertheory and statistics. The number-theoretician tries in vain to prove results for which there

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is overwhelming evidence (e.g. Goldbach's conjecture); it is the business of the statistician to facilitate decisions in the light of inadequate evidence. In fact, however, there are grave philosophical difficulties in applying probability theory to the " real world " even when to the non-philosophical eye the evidence points unmistakably in one direction (as the tobacco manufacturers have recently pointed out!); but there are none in applying it to numbertheory. Both numbertheory and probability theory have an axiomatic basis, and so if the hypotheses of a theorem in probability theory apply to a number-theoretical situation then so do the consequences.

(It should perhaps be mentioned that the literature contains also " applications " of probability theory to numbertheory of a quite different (and as usually presented fairly meaningless) nature, often to the theory of primes. One considers the set S of all sequences s: $a_1 < a_2 < \ldots < a_n <$ of positive integers which have certain properties P (usually not explicitly stated) enjoyed by the sequence of prime numbers. In the set S a probability measure m is (usually tactitly) introduced and it is then shown that m-almost all sequences s in S have a certain property Q (e.g. that every sufficiently large integer is the sum of two distinct members of s). It is then concluded that it is " very probable " that the prime numbers have Q. It is held to be a deep mystery when two practitioners of this art come to contradictory conclusions (on the basis of different-unstated-P's and m's). Needless to say, the remarks above do not apply to these arguments!)

The author, who has made important contributions of his own, treats exhaustively one only of several types of application of probability theory to numbertheory, namely to the asymptotic distribution of multiplicative and additive number-theoretical functions. Fairly detailed reviews of the first edition (1959) are available in, e.g. *Mathematical Reviews*; the second edition is similar in scope but contains considerable new material. The following may be taken as a typical special case of what is proved: Let $\omega(n)$ denote the number of prime factors of the integer n. For any real number ξ and positive integer N denote by $F_N(\xi)$ the number of $n < N$ for which

$$\omega(n) < \log\log n + \sqrt{(\log\log n)}.$$

Then

$$\lim_{N\to\infty} F_N(\xi) = \frac{1}{\sqrt{(2\pi)}} \int_{-\infty}^{\xi} \exp\left(-\tfrac{1}{2}u^2\right) du,$$

where the right hand side is the " error-function ". Estimates are given of the rapidity of the approach to the limit. Here $\omega(n)$ is an additive function in the number-theoretical sense. Most of the important number-theoretical functions or their logarithms are additive, and the result quoted above is typical for a class of additive functions. The case of $\omega(n)$ is, however, in some ways particularly simple and more precise results are given for it than can be obtained generally.

The reviewer understands that a translation into English is under preparation.

J. W. S. CASSELS

BOURBAKI, N., *Éléments de mathématique*, *Fascicule XX* (Theorie des ensembles, Chapitre 3—Ensembles ordonnés, cardinaux, nombres entiers) (Hermann, 1963), 152 pp., 36 F.

This is the second edition of Fascicule XX of the well known Bourbaki series. Compared with the first edition, the layout of this book has been altered: the exercises for each section are collected together at the end. Also, the indices of notation and terminology are more convenient. The changes in the text are mainly minor ones which remove obscurities, but part of section 1 from the first edition has been rewritten and expanded and has become section 7 of the second edition.