ONTARIO MATHEMATICAL MEETING

The twelfth Ontario Mathematical Meeting was held on Saturday, October 25, 1969, at McMaster University. Research papers were presented in the morning in three separate sessions: topology and analysis (Abstracts 69.16 to 69.21), algebra (Abstracts 69.22 to 69.27) and statistics, numerical mathematics and combinatorics (Abstracts 69.28 to 69.33). The invited address given by Professor D. Zelinsky was entitled: Galois theory of rings.

69.16 L.D. Nel (Carleton University)
Spaces Determined by their Lattices of Lower Semi-continuous Functions

Let $C_L(Y)$ denote the lattice of all lower semi-continuous functions on Y into $[0,1]$. Does the lattice $C_L(Y)$ determine the topological space Y? The problem can be reduced at once to T_o-spaces. THEOREM. *$C_L(Y)$ is isomorphic to $C_L(X)$ where X is the T_o-identification of Y.* LEMMA. *The topology of a T_o-space X is the smallest for which all $f \in C_L(X)$ are lower semi-continuous.* Call $p \in C_L(X)$ a prime if $p < 1$ and $p = u \wedge v$ only if $p = u$ or $p = v$. Define $e_{kA}(x) = k$ if $x \in A$ and $= 1$ if $x \in X - A$, for $0 \leq k < 1$ and irreducible closed sets $A \neq \phi$. LEMMA. *p is a prime if and only if p has the form e_{kA}.* This lemma establishes a link between the lattice and topological structures which can be exploited in T_o-spaces which have no irreducible closed sets other than point closures. Such spaces (call them pc-spaces) have been considered also by Blanksma [Doctoral thesis, Utrecht, 1968] in connection with the lattice $\mathcal{C}(X)$ of closed subsets of X. THEOREM. *If X is a pc-space, then $C_L(X)$ determines X.* THEOREM. *For any topological space Y the lattices $C_L(Y)$ and $\mathcal{C}(Y)$ determine each other.* It is known that $\mathcal{C}(X)$ determines X when X is a T_D-space. COROLLARY. *If X is a T_D-space, $C_L(X)$ determines X.*

69.17 L. Jonker (Queen's University)
A Theorem on Minimal Surfaces

Let M^2 be a connected two-dimensional manifold, R^n the n-dimensional Euclidean space, Δ the Laplacian on M^2, $< \ldots, \ldots >$ the inner product on R^n. Osserman [Remarks on minimal surfaces, Comm. Pure and Appl. Math., 12 (1959) 233-239] proved the following THEOREM. *If $x: M^2 \to R^3$ is an isometric immersion such that for some $a \in R^3$, we have $\Delta < a, x > = 0$, then M^2 is a minimal surface, or else a locally cylindrical surface with its generators parallel to a.* The present author generalized this to a higher codimension: THEOREM. *Suppose $x: M^2 \to R^n$ is an isometric immersion, and A an $n - 2$*

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dimensional linear subspace of R^n. Suppose further that $\Delta < a, x > = 0$ for all $a \in A$. Then there are only two possibilities: 1. M^2 is minimally immersed in R^n; or 2. There is a set $\{m_i\} \subset M^2$ of isolated points so that each $p \in M - \{m_i\}$ has a neighbourhood U so that the orthogonal projection of $x(U)$ on $A^{\perp}$ is a regular curve γ, and so that $x: U \to \gamma \times A$ is minimal.

69.18 R. J. Loy (Carleton University)
Uniqueness of the Complete Norm Topology and Continuity of Derivations on Banach Algebras

We are concerned with the following two propositions regarding a complex Banach algebra A:

$\mathfrak{U}$ A has unique complete norm topology.

$\mathfrak{D}$ Derivations on A are necessarily continuous.

(If A does not satisfy $\mathfrak{U}$ the validity of $\mathfrak{D}$ may depend on the choice of topology.) Although no formal relation is known between $\mathfrak{U}$ and $\mathfrak{D}$, a survey of the literature shows that positive results regarding $\mathfrak{U}$ for a class of Banach algebras have been paralleled by positive results for $\mathfrak{D}$ for the same class. The most general results are those of Johnson and Sinclair that semisimple Banach algebras satisfy both $\mathfrak{U}$ and $\mathfrak{D}$. For the non-semisimple case Lindberg has recently shown that certain algebraic extensions of commutative semisimple Banach algebras satisfy $\mathfrak{U}$, and it is not difficult to show they also satisfy $\mathfrak{D}$. We consider the case of a Banach algebra of power series, $\underline{\underline{A}}$, over a commutative Banach algebra A. Such algebras always satisfy $\mathfrak{D}$, and, provided A is semisimple, they also satisfy $\mathfrak{U}$. Whether or not $\mathfrak{U}$ is true in general is not known. The proofs make essential use of the coordinate projections which in a sense take the place of the multiplicative linear functionals in the work of Johnson. The results are only of interest if $\underline{\underline{A}}$ is not semisimple, and certain results concerning the semisimplicity of $\underline{\underline{A}}$ are given.

69.19 R.G. Lintz (McMaster University)
Homotopy for g-functions and Generalized Retraction

It can be shown, with simple examples, that the usual concepts of continuous deformations, i.e., homotopy, retraction, etc. are not adequate in many respects if we want to extend them to topological spaces whose points are not necessarily of countable type, i.e., with base of neighbourhoods which is not countable. To overcome those difficulties there was introduced by us, a few years ago, a generalization of the concept of continuous function, actually called g-functions [Annali di Mat. Pura ed Applicata (Italy) 67 (1965) 301-348; ib. 67 (1965) 215-234; ib. 72 (1966) 45-48; ib. 72 (1966) 97-104]. Finally, we succeeded in defining the concept of homotopy for g-functions and as a consequence a generalization of the concept of deformation retract was obtained. As a by-product we got a new proof of the classical result that homotopic maps induce the same homomorphism in the Cech homology groups. The definitions are too long to be given here.

69.20 I.M. Michael (McMaster University, University of Dundee)
The Spectrum of a Second Order Partial Differential Operator

The spectral properties of differential operators can be described either by Hilbert space theory or, to obtain detailed results, by analysis of the solutions of the corresponding differential equations. It is not always clear that the two sets of definitions of the various parts of the spectrum agree, but in a recent paper [Proc. Roy. Soc. of Edin. 68A (1969) 95-119] Chaudhuri and Everitt proved agreement in the case of certain ordinary differential expressions of the form $L[\cdot]$, where

$$L[f] = -(pf')' + qf .$$

We prove similar results for the corresponding partial differential operators.

Explicitly, we consider operators associated with the two-dimensional equation

$$(\alpha) \qquad \Delta \psi(\underset{\sim}{x}) + \{\lambda - q(\underset{\sim}{x})\} \psi(\underset{\sim}{x}) = 0 ,$$

where the equation holds over the whole plane and q is real-valued, has continuous first-order partial derivatives and is such as to ensure the symmetry of the associated maximal operator. (It is fairly straightforward to extend the theory to higher dimensions and to relax the continuity conditions on q.) This operator is essentially self-adjoint, and so has a unique associated spectral theory.

The constructive definition of the spectrum is in terms of the function H, where

$$H(\underset{\sim}{x}, \underset{\sim}{\xi}, \lambda) = \lim_{\nu \to 0} \int_0^{\lambda} \mathrm{im}\, G(\underset{\sim}{x}, \underset{\sim}{\xi}, \sigma + i\nu)\, d\sigma ,$$

and G is the Green's function, unique by our conditions on q, associated with (α). We prove that the standard definition of the spectrum agrees with the constructive one, while the eigenvalues of the self-adjoint operator associated with (α) are exactly the points of discontinuity of $H(\underset{\sim}{x}, \underset{\sim}{\xi}, \cdot)$.

69.21 D. Lovelock (University of Waterloo)
Degenerate Lagrange Densities involving Geometric Objects

Let g_{ij} and Γ^h_{ij} $(i, j, h, \ldots, = 1, \ldots, n)$ be respectively the components of a symmetric tensor and the components of a symmetric affine connection. With any given Lagrangian

(1) $$L = L\{g_{ij};\ \Gamma^h_{ij};\ \Gamma^h_{ij,k}\}$$

(where the comma denotes partial differentiation) we may associate the Euler-Lagrange equations

(2) $$\frac{\partial L}{\partial \Gamma^h_{ij}} - \frac{\partial}{\partial x^k}\left\{\frac{\partial \Gamma}{\partial \Gamma^h_{ij,k}}\right\} = 0 ,$$

(summation convention) in which the g_{ij} are regarded as auxiliary preassigned variables and are not to be varied. In general, these equations (2) represent <u>second</u> order partial differential equations in the Γ^h_{ij}.

In one approach to the field equations of general relativity [1, page 106 et seq.] a specific Lagrangian (which is also a scaler density) of the type (1) is used. However the resulting Euler-Lagrange equations (2) are not of second order in Γ^h_{ij}, but are indeed independent of both $\Gamma^h_{ij,rs}$ and $\Gamma^h_{ij,r}$ i.e. they are of <u>zero</u> order in Γ^h_{ij}. In this case the Euler-Lagrange equations are precisely those equations which ensure that the Γ^h_{ij} are the Christoffel symbols of the second kind (for $n > 2$).

In this report we assume that L given by (1) is a scalar density which therefore ensures that (2) are tensorial in character [2]. Necessary and sufficient conditions for the second order equations (2) to degenerate into first and zero order equations are obtained. Furthermore we show that (for $n > 2$) if the g_{ij} may be regarded as independent variables then the only Euler-Lagrange equations which arise from (1) and which are of zero order in Γ^h_{ij} are those which ensure that the Γ^h_{ij} are the Christoffel symbols of the second kind.

Certain other degenerate Lagrange densities of importance in general relativity have also been treated [3; 4; 5].

REFERENCES

1. E. Schrödinger, Space-time structure. (Cambridge University Press, 1963).

2. H. Rund, Invariant theory of variational problems for geometric objects. Tensor (N.S.) 18 (1967) 239-258.

3. D. Lovelock, The uniqueness of the Einstein field equations in a four-dimensional space. Arch. Rational Mech. Anal. 33 (1969) 54-70.

4. D. Lovelock, Divergence-free tensorial concomitants. Aequationes Mathematicae (to appear, 1970).

5. D. Lovelock, Degenerate Lagrange densities for vector and tensor fields. Joint Math. Coll., Universities of South Africa and Witwatersrand. (1967) 237-269.

69.22 Ahmad Shafaat (Carleton University)
Quasivarieties and Structure of Algebras

For a definition of quasivarieties of Ω-algebras see, for example, P.M. Cohn: Universal Algebra, Harper and Row, 1965. Many varieties and quasivarieties $\underline{K}$ of algebras have the following property:

($\cdot$) There exist in $\underline{K}$ finitely many finite algebras $A_1, \ldots, A_n$ such that every algebra in $\underline{K}$ is isomorphic to a subcartesian product of some of the algebras $A_1, \ldots, A_n$.

Semilattices, distributive lattices, Stone algebras and normal idempotent semigroups are only few of the examples of varieties satisfying ($\cdot$). In this paper we prove that if $\underline{K}$ is a locally finite quasivariety with finitely many subquasivarieties, then $\underline{K}$ satisfies ($\cdot$). This result is an example of how a study of the lattice of subquasivarieties of a quasivariety can give useful information about the structure of its algebras.

69.23 K.B. Lee (McMaster University)
Equational Classes of Distributive Pseudo-complemented Lattices

A pseudo-complemented lattice is a lattice L with zero such that for each $a \in L$, there exists $a^* \in L$ such that for all $x \in L$, $a \wedge x = 0$ if and only if $x \leq a^*$. One can construct a distributive pseudo-complemented lattice $\overline{B}$ from a Boolean algebra B by adjoining a new unit 1. Let $\mathcal{B}_n$ be the equational classes of distributive pseudo-complemented lattices generated by $\overline{B}_n$, where B_n are 2^n-element Boolean algebras ($n \geq 0$); $\mathcal{B}_{-1}$ be the class of all one-element pseudo-complemented lattices; $\mathcal{B}_F$ the class of all finite distributive pseudo-complemented lattices; $\mathcal{B}_\infty$ the class of all distributive pseudo-complemented lattices.

THEOREM 1. Let L be a distributive pseudo-complemented lattice. Then the following conditions are equivalent ($n \geq 1$):

(1) L satisfies the equation

$$(E_n) \quad \left(\bigwedge_{i=1}^{n} x_i\right)^* \vee \bigvee_{i=1}^{n} (x_1 \wedge \ldots \wedge x_i^* \wedge \ldots \wedge x_n)^* = 1;$$

(2) $L \in \mathcal{B}_n$;

(3) <u>every prime filter in</u> L <u>is contained in at most</u> n <u>distinct maximal filters</u>;

(4) <u>every (proper) prime ideal in</u> L <u>contains at most</u> n <u>distinct minimal prime ideals</u>;

(5) $L = \bigvee_{i=1}^{n+1} Q_i$ <u>for any</u> $n+1$ <u>distinct minimal prime ideals in</u> L.

THEOREM 2. <u>Let</u> $\mathfrak{G}$ <u>be an equational class of distributive pseudo-complemented lattices</u>. <u>Then</u> $\mathfrak{G} \not\subseteq \mathfrak{B}_n \rightarrow \mathfrak{B}_{n+1} \subseteq \mathfrak{G}$ $(n \geq -1)$.

THEOREM 3. $\mathfrak{B}_\infty = HSP(\mathfrak{B}_F)$.

THEOREM 4. $\mathfrak{B}_{-1} \subset \mathfrak{B}_0 \subset \mathfrak{B}_1 \subset \ldots \subset \mathfrak{B}_n \subset \ldots \subset \mathfrak{B}_\infty$ <u>is the whole lattice of equational classes of distributive pseudo-complemented lattices</u>.

69.24 V.D. Belousov, (Institute of Math. of the Academy of Sciences of Moldavian S.S.R., Kishinev, U.S.S.R. and University of Waterloo)
<u>A Functional Equation of Generalized Associativity on Quasigroups</u>

Following G. Čupona we denote by x_i^j the sequence $x_i, x_{i+1}, \ldots, x_{j-1}, x_j$, $(i \leq j)$. If $i > j$, then x_i^j is an empty set. A n-ary quasigroup operation (shortly quasigroup) A defined on the set Q is the mapping $A : Q^n \rightarrow Q$ such that $A(a_1^{i-1}, x, a_{i+1}^n) = b$ has a unique solution for all $a_1^n \in Q^n$, $b \in Q$. The integer n is called the arity of A and is denoted by $|A|$. The functional equation

$$(1) \qquad A\left(x_1^{i-1}, B\left(x_i^{i+h-1}\right), x_{i+n}^r\right) = C\left(x_1^{j-1}, D\right), \left(x_j^{j+q-1}\right), x_{j+q}^r\right)$$

where $|B|, |D| > 1$ is called generalized associativity. If A and B are given quasigroups then the left side of (1) defines a binary operation $(\overset{i}{+})$ on the set of all quasigroups: $(A \overset{i}{+} B)(x_1^2) = A(x_1^{i-1}, B(x^{i+n-1}), x_{i+n}^r)$. The operation $(\overset{i}{+})$ is partial, $A \overset{i}{+} B$ exists only if $|A| \geq i$. The set of all quasigroups including nullary operations (i.e. the operations $(\overset{i}{+})$, $i = 1, 2, \ldots$ becomes a partial algebra with some properties. These properties taken as axioms define an abstract partial algebra in which the equality

$$(2) \qquad A \overset{i}{+} B = C \overset{j}{+} D$$

corresponds to the functional equation (1).

In order to solve the equation (2) five possible cases ($I_{1,2}$, $II_{1,2,3}$) are considered. The most important is the case $II_3: i < j, j < i + n < j + q$, where $n = |B|$, $q = |D|$. The solution in this case is

$$A = K \overset{i}{+} \gamma \circ F, \; B = \gamma^{-1}(E \circ S), \; C = K \overset{i}{+} E \circ \theta, \; D = \theta^{-1}(S \circ F)$$

where $\gamma, \theta, E, F, S, K$ are arbitrary operations of arity $1, 1$, $k = j - i$, $q - n + k$, $n - k$, $p - k$ correspondingly. The (o) is a group operation, i.e. $(o) \overset{1}{+} (o) = (o) \overset{2}{+} (o)$ and $A \circ B$ is defined by $A \circ B = ((o) \overset{2}{+} B) \overset{1}{+} A$. If $i < j$, $i + n \leq j$ (Case II_1) the solution is given by $A = K \overset{j-n+1}{+} D$, $C = K \overset{i}{+} B$, where B, D, K are arbitrary operations of arity $n, q, p - n + 1$. The solutions for other cases are similar to II_1. The case $i = 1$, $j = q$, $j \leq n$, considered by M. Hosszu is a particular case of II_3.

The more general equation

$$(A \overset{i}{+} B)(x_1^r) = (C \overset{j}{+} D)(y_1^r),$$

where y_1^r is a permutation of x_1^r is also solved. The obtained results are used to give a description of (i, j)-associative quasigroups A, i.e. A satisfies (2) for $A = B = C = D$.

69.25 John D. Dixon (Carleton University)
The Number of Steps required in Applying the Euclidean Algorithm

The object of this paper is to give a sketch of the proof of the following THEOREM. *For all positive integers* u, v *with* $u \leq v$ *we define* $L(u, v)$ *to be the number of steps required in applying the Euclidean algorithm to find the greatest common divisor of* u *and* v. *Then for each* $\varepsilon > 0$

$$(*) \qquad |L(u, v) - (12\pi^{-2} \log 2) \log v| < (\log v)^{\frac{1}{2} + \varepsilon}$$

for almost all pairs u, v. *Indeed the proportion of pairs* $u \leq v \leq x$ *which fail to satisfy this condition is certainly less than any power* $(\log x)^{-C}$ *as* $x \to \infty$.

The only result of this kind that had been known earlier is a theorem of H. Heilbronn (1968) which showed that for each v the average of $L(u, v)$ over the u relatively prime to v is asymptotic to $(12\pi^{-2} \log 2) \log v$ with error term at worst $O((\log \log v)^4)$; his result is proved by purely arithmetic methods. In contrast our theorem is based on results in the metric theory of continued fractions (initiated by Gauss), and makes use of a paper by W. Philipp. [Das Gesetz vom iterierten Logarithmus mit Anwendungen auf die Zahlentheorie. Math. Ann. 180 (1969) 75-94].

69.26 V. Dlab (Carleton University)
Structure of Perfect Rings

The concept of a perfect ring was introduced by S. Eilenberg in [3]. Later, H. Bass [1] gave several characterizations of perfect rings. To his list, we may add the following one:

A ring R is (right) perfect if and only if R possesses a (left transfinite) socle sequence and $R = \bigotimes_{i=i}^{r} L_i$ with indecomposable (left) ideals L_i. As a consequence, each L_i contains a unique (left) ideal K_i of R which is maximal in L_i.

By means of the standard matrix representation of R, one can show that there is a one-to-one correspondence between non-isomorphic perfect rings and non-isomorphic finite additive categories $\underline{\underline{A}}$ such that [A, A] are local perfect rings for all $A \in \underline{\underline{A}}$. Consequently, particular types of perfect rings can be characterized either intrinsically or in terms of matrices or additive categories (cf. [2]).

REFERENCES

1. H. Bass, Finistic dimension and a homological generalization of semi-primary rings. Trans. Amer. Math. Soc. 95 (1960) 466-488.

2. V. Dlab, Matrix representation of perfect rings.

3. S. Eilenberg, Homological dimension and syzygies. Ann. of Math. 64 (1956) 328-336.

69.27 Tae Ho Choe (McMaster University)
Locally Compact Lattices with Small Lattices

In the Symposia Pure Math. (Lattice theory) Vol. II, A.M.S., L. Anderson conjectured that any locally compact connected topological lattice has a base consisting of open sublattices (i.e. has small lattices). We first discuss that this conjecture is not true in general. Recently, J.D. Lawson has given an example of a compact connected metrizable distributive lattice which admits no nontrivial lattice homomorphisms into the unit interval. We shall see that this lattice does not have any small lattices, and it is infinite dimensional.

However, if a lattice is finite dimensional, we are able to show that any locally compact connected lattice of finite codimension has small lattices.

Since finite dimensionality of a lattice is clearly not a necessary condition for the lattice to have small lattice, (for example, I^{ω}) it may not be easy to obtain a condition in terms of dimension of the lattice. We shall next show that if L is a locally compact connected lattice, then L has small lattices if and only if for each pair of elements x and y with $x \not\geq y$, every neighbourhood U of x with $U \not\geq y$ ($u \not\geq y$ for all $u \in U$) has an element z such that $x \in (z \wedge L)^{o}$, and dually.

For a complemented lattice we have the following: any locally compact relatively complemented lattice which has small lattices is totally disconnected. This yields that any locally compact orthomodular lattice which has small lattices is totally disconnected.

For the dimension and the center of a lattice, we have the following: if L is a locally compact connected lattice with 0 and 1 and if the codimension of L is n, then $\mathrm{Card.}(\mathrm{Cen}(L)) \leq 2^{n}$. Moreover, if the lattice L is not compact, then $\mathrm{Card.}(\mathrm{Cen}(L)) \leq 2^{n-1}$.

69.28 D.R. Beuerman (Queen's University)
On the Limit Distribution of the Time of First Passage Over a Curvilinear Boundary

Let $X_1, X_2, X_3, \ldots$ be a sequence of independent and identically distributed random variables which belong to the domain of attraction of a stable law of index α and distribution function G. Let B_n be the norming constants and $S_n = \sum_{i=1}^{n} X_i$.

Take the constant $\beta \in [0, 1)$. Our primary interest is in the random variable, $T_\beta(X) = \min[k : S_k > xk^{\beta}]$, which represents the first passage time for the random walk S_n over the curvilinear boundary xn^{β}. Several related random variables may be considered in terms of $T_\beta(x)$. For example, if we define

$$M_{n,\beta} = \max \{0, S_1, 2^{-\beta} S_2, \ldots, n^{-\beta} S_n\}$$

we note that, for $x \geq 0$, $M_{n,\beta} > x$ if and only if $T_\beta(x) \leq n$.

The main results are the following. Take $1 \leq \alpha \leq 2$, $\mu = \mathbb{E}[X_i] > 0$, this being one of the cases of "drift" for the random walk S_n. Then we have these limit results.

$$(1) \qquad \lim_{n\to\infty} P\left(\frac{M_{n,\beta} - n^{1-\beta}\mu}{n^{-\beta} B_n} \le a\right) = G(a)$$

$$(2) \qquad \lim_{x\to\infty} P\left(\frac{T_\beta(x) - \lambda}{\mu^{-1} B(x)} \ge \frac{-a}{1-\beta}\right) = G(a)$$

where $\lambda = (\mu^{-1} x)^{1/(1-\beta)}$, $B(x) \sim B_n$.

Siegmund [Ann. Math. Stat., 39, pages 1493-1497] has obtained similar results for the normal ($\alpha = 2$) case, but with the condition that the X_i be identically distributed relaxed to common mean and variance.

For $\beta = 0$, (1) reduces to a result of Heyde [J. Appl. Prob., Vol. 4, pages 144-150].

Dr. C. C. Heyde, now at the Australian National University, guided me in this work while we were at the Manchester-Sheffield School of Probability and Statistics.

69.29 I. S. Chorneyko and S. G. Mohanty (McMaster University)
On the Enumeration of Pseudo-search Codes

A codeword c is a finite sequence of non-negative integers. Any finite set of codewords is called a code C. A codeword b is a prefix of the codeword c if there exists a codeword a such that $c = ba$, where ba is the concatenation of b and a. The empty set and the code consisting of the empty codeword are called the empty and trivial codes respectively. Denote by Z the set of all codewords. Furthermore, for any code C and $a \varepsilon Z$, C_a is the set of all codewords $b \varepsilon Z$ such that $ab \varepsilon C$.

We make the following definitions:

(1) A code C is branched if and only if one of the following occurs: (i) C is the empty code; (ii) C is the trivial code; (iii) C does not contain the empty codeword and there exists an integer $b(C) \ge 1$ such that for k, the codeword consisting of the single letter k, $k = 0, 1, 2, \ldots,$ the code C_k is empty or not according as $k \ge b(C)$ or $< b(C)$.

(2) $b(C)$ is the branching number of the code C.

(3) A code C is a pseudo-search code if C_a is branched for every $a \varepsilon Z$.

(4) If C is a pseudo-search code, any $a \in Z$ for which $b(C_a) \geq 1$, is called the branching point of C and $b(C_a)$ is the branching number of a.

These definitions and the following results are motivated by an unpublished paper of A. Renyi, presented at McMaster University in June, 1969.

Let $S(q_1, q_2, \ldots, q_k)$ be the set of pseudo-search codes such that the ith branching point (in lexicographic order) has q_i as its branching number. A one-to-one correspondence is established between $S(q_1, \ldots, q_k)$ and the set of lattice paths in the plane from $(0, 0)$ to (a_k, k), not crossing the lattice path determined by the vector $(0, a_1, \ldots, a_{k-1})$, where $a_j = \sum_{i=1}^{j} q_i - j$, $j = 1, 2, \ldots, k-1$ is the horizontal distance from $(a_k, k-j-1)$ to the path and $a_k = \sum_{i=1}^{k} q_i - k + 1$. Using this correspondence, the number in $S(q_1, \ldots, q_k)$ is obtained as $\det(d_{ij})_{(k-1)\times(k-1)}$ where

$$d_{ij} = \begin{cases} 0 & \text{if} \quad i > j + 1, \\ \dbinom{a_{k-j} + 1}{j - i + 1} & \text{otherwise}. \end{cases}$$

Other similar results follow from this correspondence.

69.30 A.K. Basu (Queen's University)

<u>On Distinguishability of Sets of Distribution Functions</u>

According to Hoeffding and Wolfowitz [Ann. Math. Stat. 29 (1958) 700-718] two subsets $\mathcal{E}$ and $\mathcal{H}$ of $\mathfrak{F}$ of distribution functions are distinguishable in the given class $\mathfrak{L}$ of tests if there exists a test with maximum error probability in $\mathcal{E} \cup \mathcal{H}$ less than any preassigned positive number. We have extended these concepts to the k-decision problems when the chance variables are independently and identically distributed. The subsets $\mathfrak{F}_1, \mathfrak{F}_2, \ldots, \mathfrak{F}_k$ of $\mathfrak{F}$ will be called "k-distinguishable" if there exists a test $\phi \equiv (\phi_1, \ldots, \phi_k)$ such that the sum of error probabilities in $\mathfrak{F}_1 \cup \mathfrak{F}_2 \ldots \cup \mathfrak{F}_k$ is less than one. If we restrict ourselves to the class of fixed sample size tests then the above distinguishability is called "finite distinguishability"; otherwise it is called "sequential distinguishability".

Like Hoeffding and Wolfowitz we have tried to find necessary and/or sufficient conditions for existence of such tests. We have shown that "k-distinguishability" is stronger than pairwise distinguishability.

Then we have extended "distinguishability of sets of distributions" to Two-Sample Problems. We have shown that two-sample problem about equality of two medians is not "finitely distinguishable" but "sequentially distinguishable". Then we have constructed a practical non-parametric sequential test for equality of two medians which arises naturally in the course of proving sequential distinguishability of two-sample median problem. This sequential procedure converges with probability one. Upper bounds of two types of error probabilities are also calculated here. This test is similar to Mathiesen's two-sample test.

69.31 D.N. Behara

A Computational Matrix Algorithm for the Shortest Route Problem

This paper proposes a computational matrix algorithm for the shortest route problem. The problem deals with determination of the shortest route from one point to another in a network, $G = [N, A]$ subject to non-negative distances, d_{ij} associated with each arc $(i, j) \in A$ where

$$d_{ij} = \begin{cases} k_{ij} & i \neq j,\ k_{ij} > 0 \\ 0 & i = j \\ \infty & \text{if no single arc connects node } i \text{ to node } j. \end{cases}$$

The algorithm considers $n \times n$ distance matrix $[d_{ij}]$ for the network where columns and rows represent the nodes of the network. Origin and destination are labelled. Minimum d_{ij}s are selected beginning with row 1, and the remaining entries appearing in the selected d_{ij} column are cancelled. Then a minimum d_{ij} is selected from row $i = k$ if the d_{ij} selected above lies in the column, $j = k$ and the respective chain lengths for each pair of consecutive d_{ij}s are written down beside their respective rows. Among the subsequent minimum d_{ij}s the one which connects the minimum chain length noted above is selected. The procedure is repeated until the destination is reached. The algorithm is illustrated by a numerical example.

69.32 J. Csima and B.A. Datta (McMaster University)

The DAD Theorem for Symmetric Nonnegative Matrices

A matrix is nonnegative if its entries are nonnegative real numbers. A nonnegative matrix is doubly stochastic if its row and column sums are all equal to 1. If $A = (a_{ij})$ is a nonnegative square matrix of order n we say that the entry a_{ij} belongs to a positive diagonal if there exists a permutation σ on the first n natural numbers such that $\prod_{i=1}^{n} a_{i\sigma(i)} > 0$.

Let A be a symmetric nonnegative matrix. We prove that a sufficient and necessary condition for the existence of a diagonal matrix D such that DAD is doubly stochastic is the following: every nonzero entry of A belongs to a positive diagonal. Equivalently, in view of the Perfect-Mirsky characterization of doubly stochastic patterns, such D exists if and only if there exists a doubly stochastic matrix whose nonzero places coincide with the nonzero places of A.

Previously known sufficient conditions for the existence of a D (D and A as above) are the following: (i) A is strictly positive or positive semidefinite without a zero row (Marcus and Newman); (ii) A has a strictly positive main diagonal (Brualdi, Parter and Schneider). Neither of these conditions is necessary as illustrated by $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

REFERENCES

1. R.A. Brualdi, S.V. Parter and H. Schneider, The diagonal equivalence of a nonnegative matrix to a stochastic matrix. J. Math. Anal. and Appl. 16 (1966) 31-50.

2. M. Marcus and H. Minc, A survey of matrix theory and matrix inequalities. (Allyn and Bacon, Boston, 1964).

3. Hazel Perfect and L. Mirsky, The distribution of positive elements in doubly stochastic matrices. J. Lon. Math. Soc. 40 (1965) 689-698.

4. R. Sinkhorn and P.V. Nopp, Concerning nonnegative matrices and doubly stochastic matrices. Pacific J. Math. 21 (1967) 343-348.

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On an Analytic Method in the Combinatorial Study of Finite Linear Spaces

By a non-trivial finite linear space, we mean a set of $p(\geq 2)$ points and $q(\geq 2)$ lines with an incidence relation between them such that

(1) There is just one line passing through any two distinct points.

(2) Every line passes through at least two points.

It may be noted that in the following the condition (2) can be weakened to:

(2') Every line passes through at least one point.

If any two lines meet, we call the space quasi-projective (and the property QP for short). It is well known that in this case $p = q$. Let now a_σ denote the number of points on the σ-th line, and b_α the number of lines through the α-th point. The following results formed the background for the present paper:

(a) Trivially: $\Sigma b_\alpha = \Sigma a_\sigma$.

(b) By a theorem due to de Bruijn and Erdős [Indag. Math. 10 (1948) 421-423]:

(b.1) $p \leq q$.

(b.2) $p = q$ implies QP.

(c) By a theorem of mine [Bull. Soc. Math. de Belgique 18 (1966) 430-438]:

(c.1) $\Sigma b_\alpha^2 \geq \Sigma a_\sigma^2$.

(c.2) $\Sigma b_\alpha^2 = \Sigma a_\sigma^2$ implies QP.

Let us now introduce the function F defined by

$$F(x) = \Sigma b_\alpha^{x+1} - \Sigma a_\sigma^{x+1} .$$

It is obvious that the above results can be neatly expressed by means of $F(0)$, $F(-1)$ and $F(+1)$. This suggested the following analytic method: derive combinatorial results as those above from the behaviour of F. More precisely, prove that F has only the trivial root nought unless it is identically zero, in which case QP must hold. This conjecture had been corroborated by numerical evidence gathered by Mr. Nico Benschop on the Waterloo computer in 1967-1968 (work supported by NRC-Grant A-4748).

This and more has now been established by proving the THEOREM. Both F and all its derivatives satisfy the property in f : $f(x) \geq 0$ and $y \geq x$ together imply $f(y) \geq f(x)$. And, if QP does not hold, one even has the stronger property in f : $f(x) \geq 0$ and $y > x$ together imply $f(y) > f(x)$.