LETTERS TO THE EDITOR

Dear Editor,

In his paper 'Asymptotic behaviour of a stopping time related to cumulative sum procedures and single-server queues' (*J. Appl. Prob.* **24** (1987), 200–214), Wolfgang Stadje proved the following theorem:

For the simple symmetric random walk $\{S_n\}$ with $p = 1/2$

$$\lim_{n\to\infty} P\left\{N_n < xn^2 \mid S_0 = \frac{\alpha}{n}\right\} = \int_0^x f_\alpha(t)dt, \qquad \alpha\in[0, 1), \tag{1}$$

where

$$f_\alpha(t) = (2\pi t^3)^{-1/2} \sum_{-\infty}^{\infty} (-1)^{n+1}(2n+\alpha-1)\exp\{-(2n+\alpha-1)^2/2t\}, \quad t > 0,$$

and

$$N_x = \inf\left\{k \geqq 0 : S_k - \min_{0\leqq i\leqq k} S_i > x\right\}, \qquad x > 0.$$

The author also conjectures that the theorem remains true for arbitrary random walk with zero expectation and finite variance. Indeed this is correct and actually it is a simple and well-known consequence of the central limit theorem.

In the context of queueing, $S_k - \min_{0\leqq i\leqq k} S_i = W_k$ is the waiting time of the kth customer, and hence

$$(2)\qquad \begin{aligned} P\left\{N_n > xn^2 \mid S_0 = \frac{\alpha}{n}\right\} &= P\left\{\max_{1\leqq k\leqq n^2x} W_k < n \mid W_0 \frac{\alpha}{n}\right\} \\ &\xrightarrow{\text{d}} P_{\alpha/\sigma}\left\{\max_{0\leqq t\leqq x} |B|(t) < \sigma^{-1}\right\}, \end{aligned}$$

where σ is the deviation of the random walk and $|B|$ is reflected Brownian motion.

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Starting from the reflection principle for Brownian motion given by

$$P_0\left\{b \leqq \inf_{0 \leqq t \leqq x} B(t) \leqq \sup_{0 \leqq t \leqq x} B(t) < a\right\} \tag{3}$$

$$= \int_b^a (2\pi x)^{-1/2} \sum_{-\infty}^{\infty} \{\exp[-(y+2nc)^2/2x]$$

$$- \exp[-(y-2b+2nc)^2/2x]\}dy, \quad c = a-b,$$

it is not difficult to obtain (1) in full generality.

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Yours sincerely,
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Dear Editor,

In my paper in Volume 24, No. 1 of the *Journal of Applied Probability* I studied the asymptotic behaviour of the stopping time N_x for the case of negative drift. The case of a non-negative drift is only considered in the binomial example of Section 3, since in this example the Laplace transform of N_x can be given in the same closed form for arbitrary drift. From the explicit formula (3.10)–(3.12) the asymptotics of all three cases (negative, zero and positive drift) are simultaneously derived. Surely this is the reason for having overlooked the general argument given by Dr Hooghiemstra which is applicable only to the case of zero drift. I should like to thank Dr Hooghiemstra for calling my attention to this point.

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Yours sincerely,
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