multifractal decomposition of a measure, which involves looking at the dimensions of sets of the form

$$\{x : \underline{\alpha}(\mu, x) = \alpha\} \quad \text{or} \quad \{x : \overline{\alpha}(\mu, x) = \alpha\}.$$

The final chapter of the book gives a brief but lively introduction to the rôle of fractals in the study of differential equations, including discussions of how to estimate dimensions of attractors and how to solve the heat equation on domains with a nice self-similar boundary. This involves a nice application of the Renewal theorem discussed earlier in the book.

To summarise: this is a lovely book and a worthy successor to *Fractal Geometry*. It presents concisely many of the techniques currently used in the study of fractals and is a good survey of results which have appeared since the publication of the last book.

The book, unfortunately, does possess many minor errors which may be occasionally distracting (this is especially embarrassing to this reviewer since he was involved with proof-reading . . .) but even the most serious error I found (in the chapter on tangent measures, where the set of tangent measures of log 2/log 3-dimensional Hausdorff measure restricted to the usual $\frac{1}{3}$-Cantor set is misdescribed) is not particularly bad.

The author's clear, lucid style of writing is a pleasure to read and his decision to concentrate on the essential ideas rather than to obtain the greatest generality pays off handsomely. Anyone who reads this book will, by the end of it, be able to pick up a current research paper on fractals and have a good chance of understanding it.

T. C. O'NEIL

REFERENCES

1. K. J. Falconer, *Fractal geometry* (Wiley, 1989).

2. P. Mattila, *Geometry of sets and measures in euclidean spaces* (Cambridge, 1995).

Aschbacher, M. *3-Transposition Groups* (Cambridge Tracts in Mathematics Vol. 124, Cambridge, 1997), vii + 260pp., 0 521 57196 0 (hardback), £35.00 (US$49.95).

A 3-transposition group is a finite group G which is generated by a normal set D of elements of order 2 such that any two elements of D either commute or have a product of order 3. The symmetric groups S_n ($n > 2$) are 3-transposition groups. Less obviously, several classical groups in characteristics 2 and 3 are 3-transposition groups. Part I of the book under review is devoted to a proof of B. Fischer's beautiful theorem, proved around 1970, which characterized finite 3-transposition groups G in which $Z(G) = 1$, G' is simple and D is a single conjugacy class. This characterization led directly to the discovery of three new "sporadic" simple groups, and Fischer groups also turn out to be closely related to several other sporadic simple groups. Parts of the proof of Fischer's theorem had remained unpublished prior to the appearance of this book.

Fischer's work was one of the most imaginative chapters in the classification of the finite simple groups, and the author of this book was and is the foremost exponent of the use of geometries and graphs derived from internal group-theoretic structure in characterization theorems which may be viewed (at least in part) as a natural development of that work. The author gives the reader the benefit of his own deep insight as he develops the material in an assured manner.

In the second and third parts of the book the author considers the questions of existence and uniqueness of the Fischer groups and the local structure of the Fischer groups. Both of these are viewed in the wider context of developing a reasonably uniform theory of sporadic simple groups.

https://doi.org/10.1017/S0013091500020149 Published online by Cambridge University Press

In principle, this book should be accessible to a reader with a fairly rudimentary knowledge of group theory, especially part I . The development is fairly rapid and the later parts certainly require the careful attention of the reader. I recommend the book strongly to two types of potential reader: the reader who wishes to see a proof of a beautiful and key theorem in the classification theorem explained by a master and the reader who is already expert in finite group theory and who wishes to gain detailed insight into the current programme of placing the theory of sporadic simple groups in a conceptual framework.

G.R. ROBINSON

HUGHES, B. and RANICKI, A. *Ends of complexes* (Cambridge Tracts in Mathematics Vol. 123, Cambridge, 1996), xxv+353 pp., 0 521 57625 3 (hardback), £45.00 (US$64.95).

The subject of ends is the study of topological properties of spaces determined by the complements of compact subsets. Approaches to the subject of ends include the study of ends as a choice of components in the complements of compact subsets, the study of the number of ends, and the applications of the homotopy theory of proper maps (for which inverse images of compact sets are compact). The book under review concentrates on the topology of the end space of a non-compact manifold, where the *end space* $e(W)$ of a space W is the space of all paths $\omega : ([0, \infty], \infty) \to (W^{\infty}, \infty)$ such that $\omega^{-1}(\infty) = \{\infty\}$ with the compact-open topology. Since, if W is compact, its one-point compactification W^{∞} is disconnected, the end space of a compact space is empty.

In 1965 Siebenmann introduced the notion of a tame end of a manifold satisfying a geometric condition that ensures that the topology of the end space is reasonably well-behaved. The main result obtained in the volume under review describes tame manifold ends in dimensions ≥ 6 and does so in very precise terms, which we may summarise by saying that such ends look like infinite cyclic covers of compact manifold bands. A *band* (W, c) consists of a compact space W with a map $c : W \to S^1$ such that the pullback infinite cyclic cover $\overline{W}$ is finitely dominated (i.e., it is a homotopy retract of a finite CW complex).

The introduction gives fair warning:

"The proof . . . occupies most of Parts One and Two (Chapters 1–20)."

The chapters are short, but we are still offered a trip of 254 pages. This prospect struck fear into the heart of the timorous reviewer, but there was nothing for it but to begin.

In the early chapters the scene is set with some care. The invariants of the end space are introduced along with a swift but thorough account of homotopy limits and colimits – material for which a good introduction is hard to find elsewhere. The authors expect a certain bravado from the reader, who is definitely assumed to be a working rather than a lazy mathematician, but the ingredients are there to be worked upon.

The flavour and delicacy of the subject are best sampled in an example. A singular r-chain in a space W is (as usual!) a formal $\mathbb{Z}$-linear sum $\sum_{\alpha\in\mathbb{Z}} n_\alpha\sigma_\alpha$ of singular r-simplices. A *locally finite* singular r-chain is a formal $\mathbb{Z}$-linear product $\prod_{\alpha\in\mathbb{Z}} n_\alpha\sigma_\alpha$ such that each point of W has a neighbourhood meeting only finitely many of the σ_α. The *locally finite homology* of W is then defined in the obvious way, and the homology at ∞ is defined, via a mapping cone, to measure the difference between the locally finite and ordinary singular homology and to link them in a long exact sequence. If we regard the natural numbers $\mathbf{N}$ as a discrete space, then $H_0(\mathbf{N})$ is of course a free abelian group of countably infinite rank, which we identify with the polynomial ring $\mathbb{Z}[z]$. But the locally finite version $H_0^{lf}(\mathbf{N})$ is the product of countably many copies of $\mathbb{Z}$, identified with the power series ring $\mathbb{Z}[[z]]$. The non-zero quotient $\mathbb{Z}[[z]]/\mathbb{Z}[z]$ is the homology $H_{-1}^{\infty}(\mathbf{N})$ at ∞. But $e(\mathbf{N}) = \emptyset$, and so the homology at ∞ is not the homology of the end space.

This sort of bad behaviour is eliminated by the imposition of tameness restrictions, and Part One of the book proceeds with a careful development of tameness and its desirable consequences.

https://doi.org/10.1017/S0013091500020149 Published online by Cambridge University Press